PROCESS OF CONVERGENCE IN EU

AUGMENTED SOLOW MODEL

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AUGMENTED SOLOW MODEL

Basic neoclassical Solow model can be **augmented** by the incorporation of **other factors** that can possibly influence growth process and convergence

- ❑ Technology
- ❑ Human capital

SOLOW MODEL WITH TECHNOLOGY

- **Basic version** of Solow models describes how output per worker depends on capital per worker:

$$Y = f(K, L)$$

$$Y = K^{\alpha} L^{\beta}, \alpha + \beta = 1; 0 < \alpha, \beta < 1$$

$$y = K / L --> y = k^{\alpha}$$

- If **technology** is taken into account then:

$$Y = f(K, L, A)$$

$$Y = AK^{\alpha} L^{\beta}, \alpha + \beta = 1; \quad 0 < \alpha, \beta < 1$$

$$y = K / L --> y = Ak^{\alpha}$$

The Solow model does not explain technological progress but, instead, takes it as given and shows how it interacts with other variables in the process of economic growth.

Solow model with technology

$$Y(T) = F\big(K(T), L(T), A(T)\big)$$

$A(t)$ is technology at time *t*.

$$\frac{\dot{A}}{A} = g$$

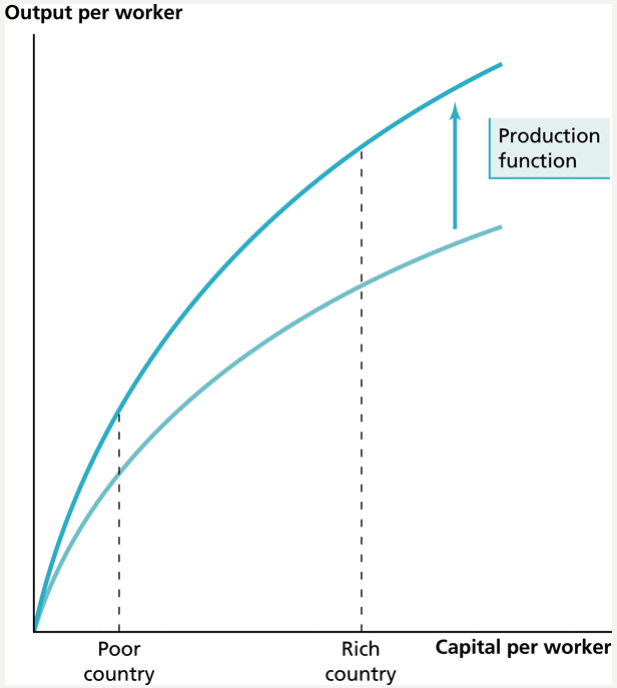
Constant technological progress **(Is it realistic?)**

$$Y = AF(K, L) = AK^{\alpha} L^{1-\alpha}$$
$$Y = F(AK, L) = (AK)^{\alpha} L^{1-\alpha}$$
$$Y = F(K, AL) = K^{\alpha} (AL)^{1-\alpha}$$

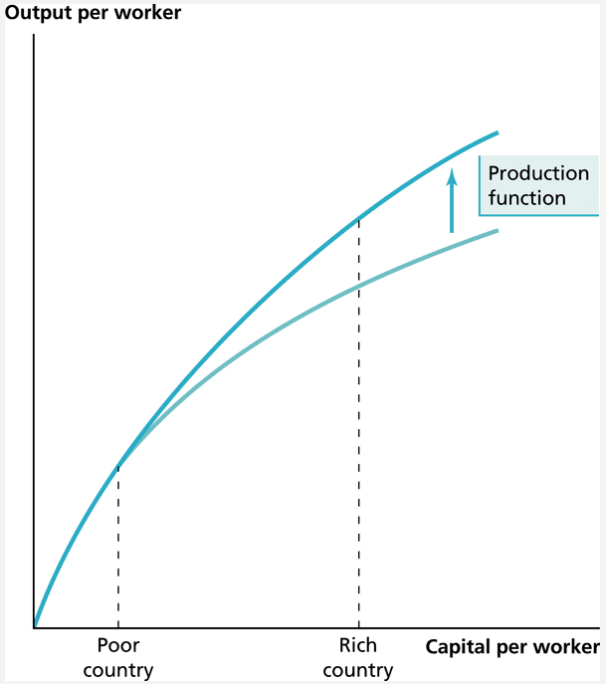
Where A should be put in the production function ?

TECHNOLOGICAL CHANGE

Neutral tech.change



Capital biased tech.change



Weil D., Economic Growth, (2009) Pearson International Edition, Figure 8.6&8.7

PRODUCTION FUNCTION PER EFFECTIVE WORKER:

$$k(t) = \frac{K(t)}{A(t)L(t)}$$ Capital per effective worker (1)

$$y = \frac{Y(t)}{A(t)L(t)}$$ Product per effective worker (2)

$$y = \frac{Y(t)}{A(t)L(t)} = \frac{AF(K,L)}{AL} = AF\left(\frac{K}{AL}, \frac{L}{AL}\right) = A\left(\frac{K}{AL}\right)^{\alpha}\left(\frac{L}{AL}\right)^{1-\alpha} = Ak^{\alpha}$$

Find capital and output per effective worker at the steady state

HINT: REMEMBER ABOUT THE TRICK:

$$\ln k(t) = \ln\left[\frac{K(t)}{A(t)L(t)}\right] = \ln K(t) - \ln L(t) - \ln A(t)$$

$$\frac{d(\ln k(t))}{dt} = \frac{d\left[\ln\left(\frac{K(t)}{A(t)L(t)}\right)\right]}{dt} = \frac{d\left(\ln(K(t)) - \ln(L(t)) - \ln(A(t))\right)}{dt} \Rightarrow$$

$$\frac{\dot{k}(t)}{k(t)} = \frac{\dot{K}(t)}{K(t)} - n - g$$

SUBSTITUTING $\dot{K}(T)$ FOR $K(T)$ from equation (1) we derive:

$$\frac{\dot{k}(t)}{k(t)} = \frac{sY(t) - \delta K(t)}{K(t)} - n - g = \frac{sY(t)}{K(t)} - \delta - n - g =$$

$$= \frac{syAL}{kAL} - (\delta + n + g) = \frac{sy}{k} - (\delta + n + g)$$

$$\boxed{\dot{k} = sy - (\delta + n + g)k}$$

,

We define steady state by the condition that $\dot{k}(t) = 0$ then setting equation (5) to zero

$$\boxed{sy = (\delta + n + g)k}$$

$$y = \frac{Y(t)}{A(t)L(t)} = \frac{AF(K,L)}{AL} = AF\left(\frac{K}{AL}, \frac{L}{AL}\right) = A\left(\frac{K}{AL}\right)^{\alpha}\left(\frac{L}{AL}\right)^{1-\alpha} = Ak^{\alpha}$$

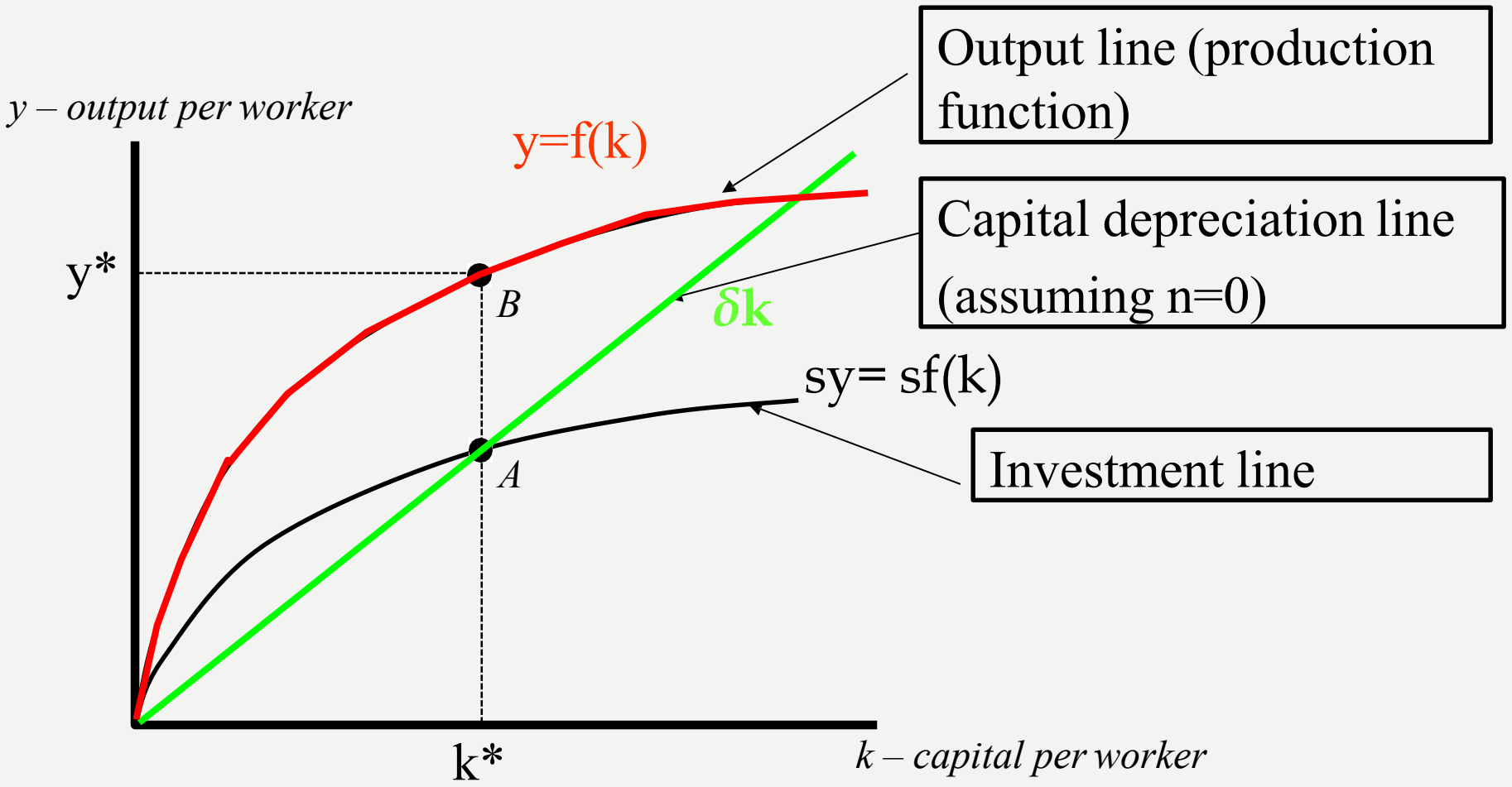
$$k^{ss} = \left(\frac{As}{\delta + n + g}\right)^{1/1-\alpha}$$

Capital per effective worker at steady state (3)

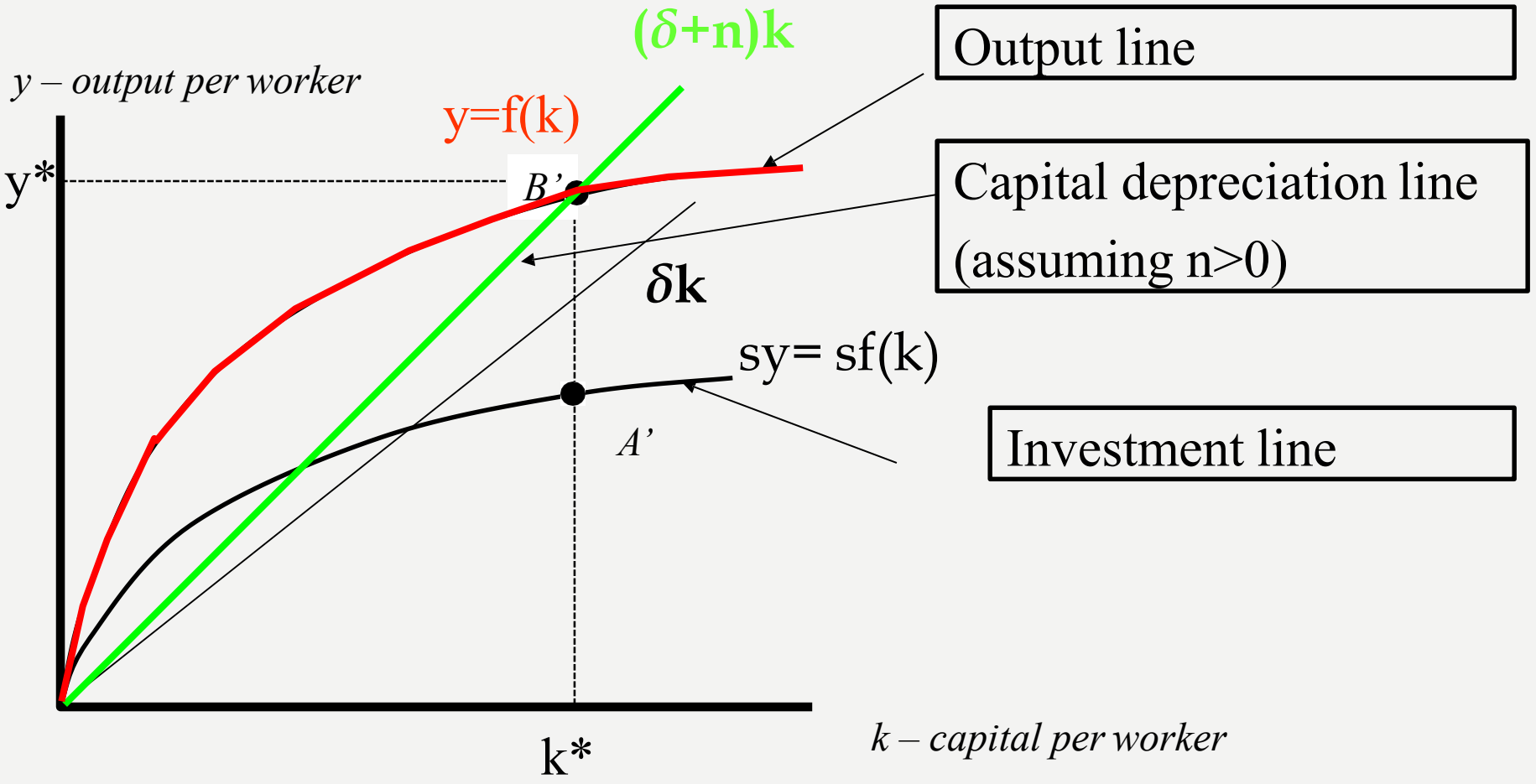
$$y^{ss} = Ak^{\alpha} = A^{1/1-\alpha}\left(\frac{s}{\delta + n + g}\right)^{\alpha/1-\alpha}$$

Output per effective worker at steady state (4)

Solow Diagram(when n=0, g=0)



Solow Diagram(when n>0, g=0)

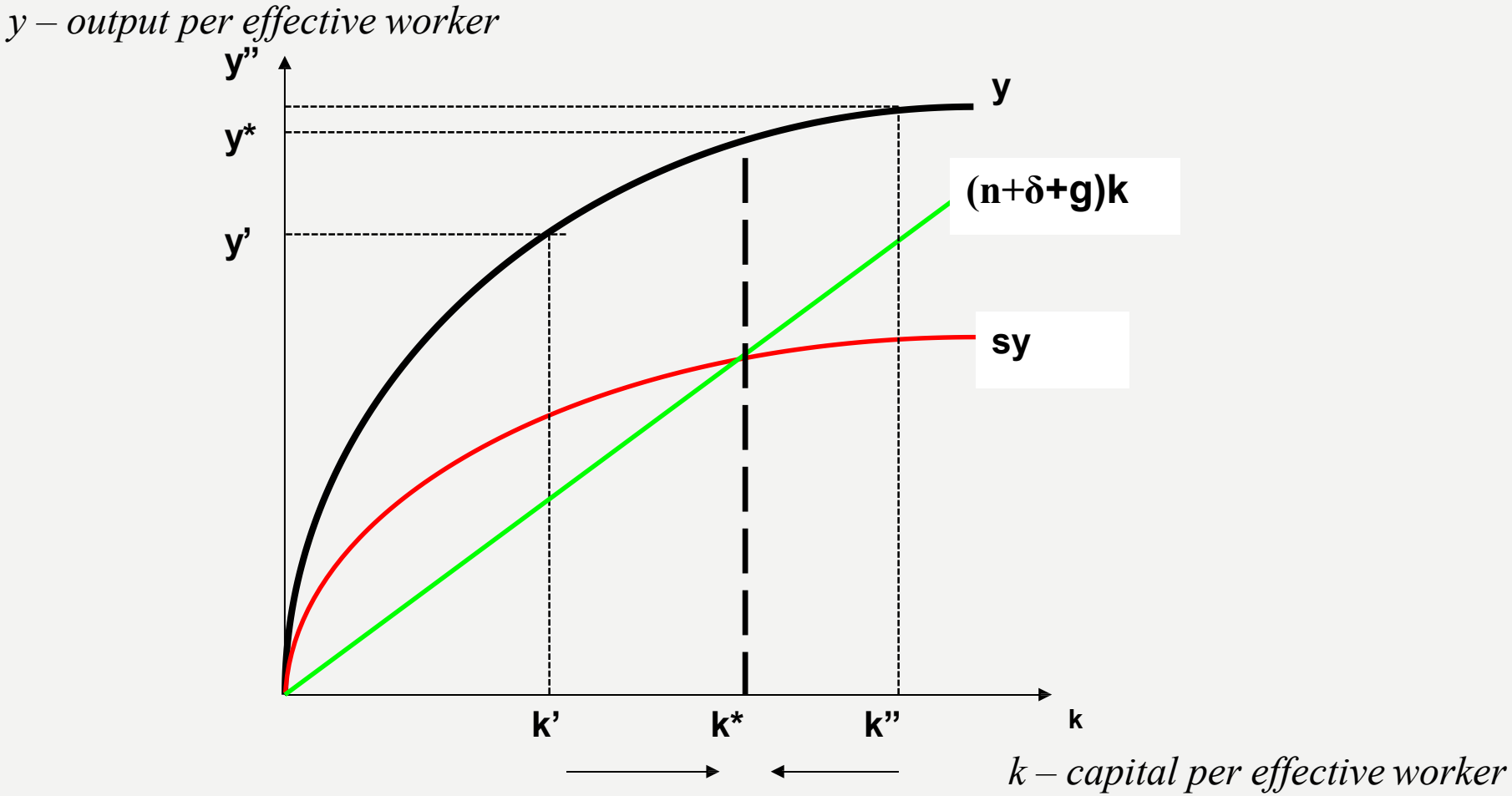


Output line

Capital depreciation line (assuming n>0)

Investment line

SOLOW DIAGRAM (WHEN N>0, G>0)



When the g term is added, gk is needed to provided capital to new "effective workers" created by technological progress

THE EFFECT OF CHANGE IN TECHNOLOGY GROWTH

- From eq. (4) raising the growth rate of technology will lower the steady state level of output per effective worker

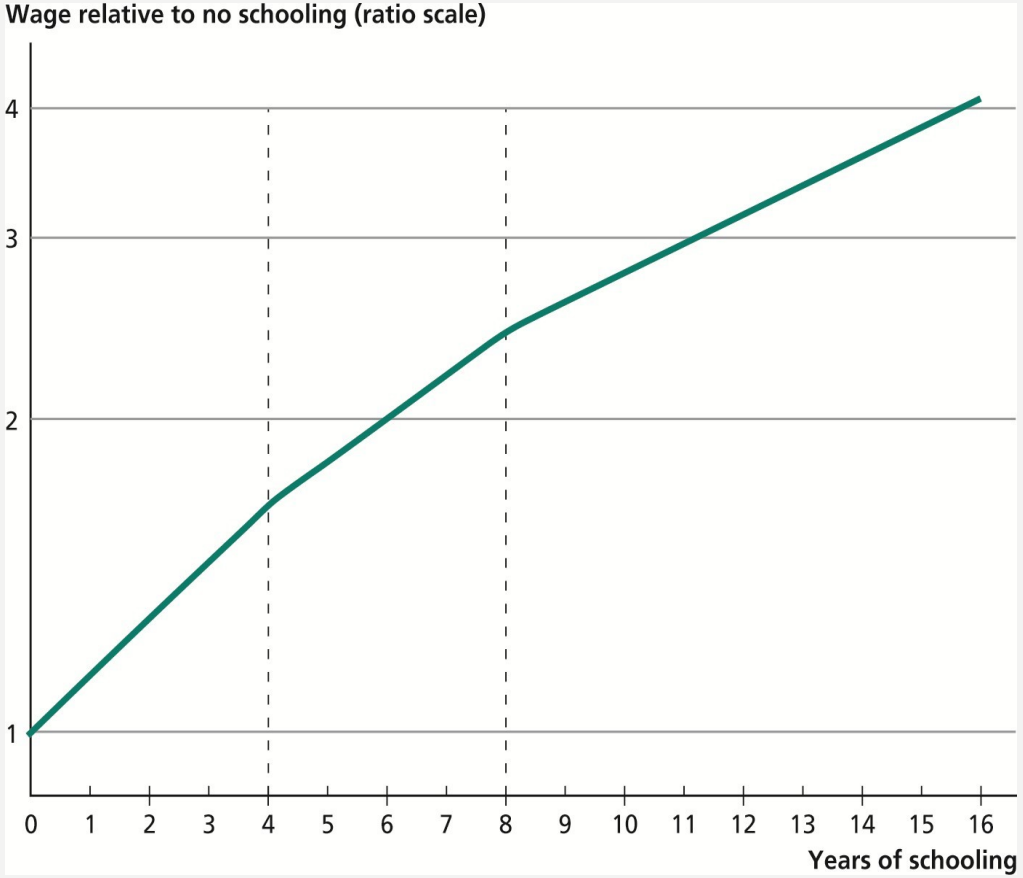
- Technological progress raises the output per worker even as it lowers output per effective worker and raises total output

SOLOW MODEL EXTENSION – THE ROLE OF HUMAN CAPITAL

Human capital is the stock of knowledge, habits, social and personality attributes, including creativity, embodied in the ability to perform labor so as to produce economic value

- Human capital earns a return (private and public)
- Investment in human capital increases its stock (education, training and health)
- Human capital depreciates

EFFECT OF EDUCATION ON WAGES



Weil (2013), p. 183

TABLE 1. BREAKDOWN OF THE POPULATION BY SCHOOLING AND WAGES

Highest Level of Education	Years of schooling	Wage Relative to No Schooling	Percentage of the Population	
			Developing Countries	Advanced Countries
No Schooling	0	1.00	20.8	2.5
Incomplete Primary	4	1.65	10.4	3.4
Complete Primary	8	2.43	18.0	12.3
Incomplete Secondary	10	2.77	19.3	17.8
Complete Secondary	12	3.16	23.2	37.4
Incomplete Higher	14	3.61	2.9	9.9
Complete Higher	16	4.11	5.3	16.6

Source: Barro and Lee (2010).

Weil (2013), p. 186

Average hourly wages by education, FH2007–FH2017 (FH2017 dollars)

	Less than high school	High school	Some college	College	Advanced degree
All					
FH2007	$13.53	$17.72	$20.03	$31.04	$39.28
FH2016	$13.30	$17.53	$19.50	$32.46	$41.45
FH2017	$13.55	$17.83	$19.41	$32.40	$41.58
Annualized percent change					
2016-2017	1.9%	1.7%	-0.4%	-0.2%	0.3%
2007-2017	0.0%	0.1%	-0.3%	0.4%	0.6%

Source: EPI analysis of Current Population Survey Outgoing Rotation Group microdata

Change in average hourly wages by education, FH2007–FH2017

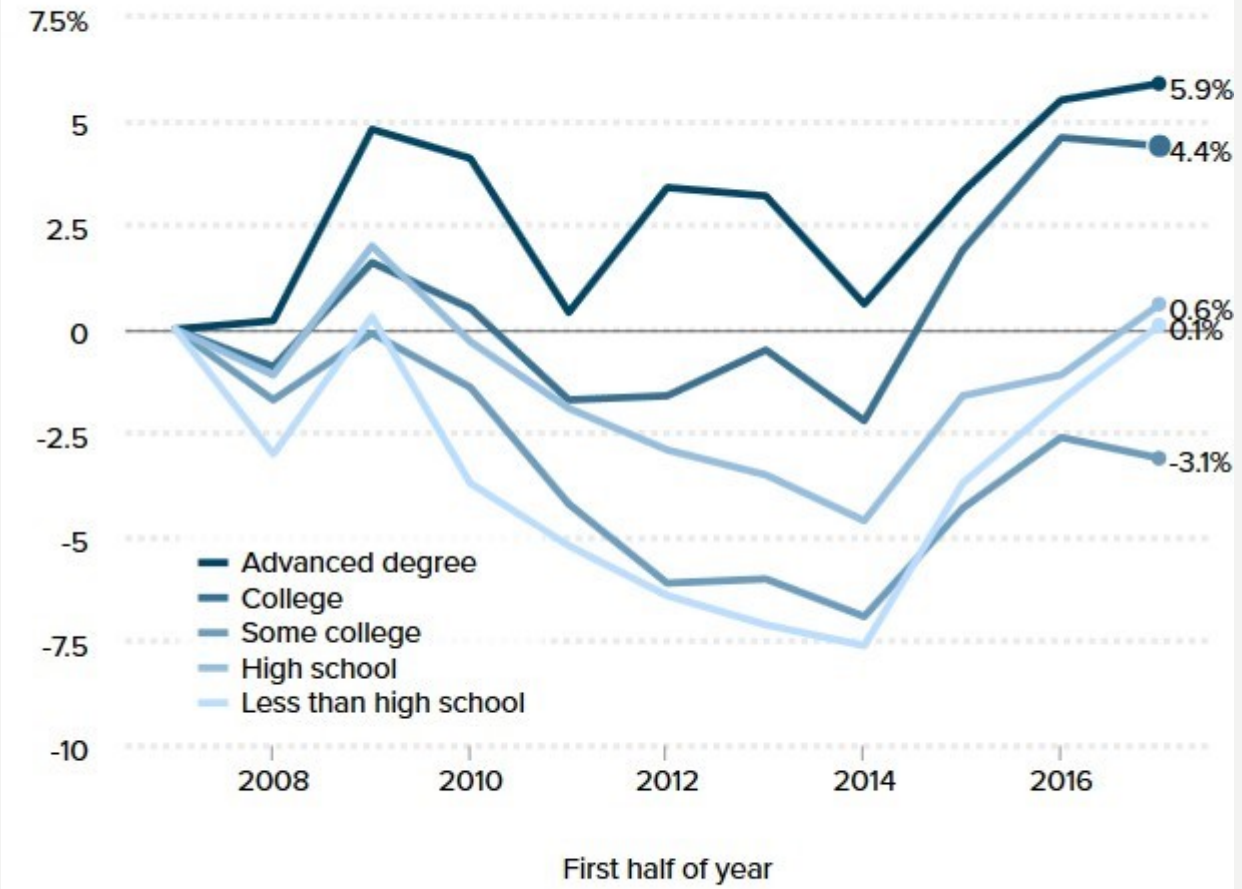


Chart Data

Source: EPI analysis of Current Population Survey Outgoing Rotation Group microdata

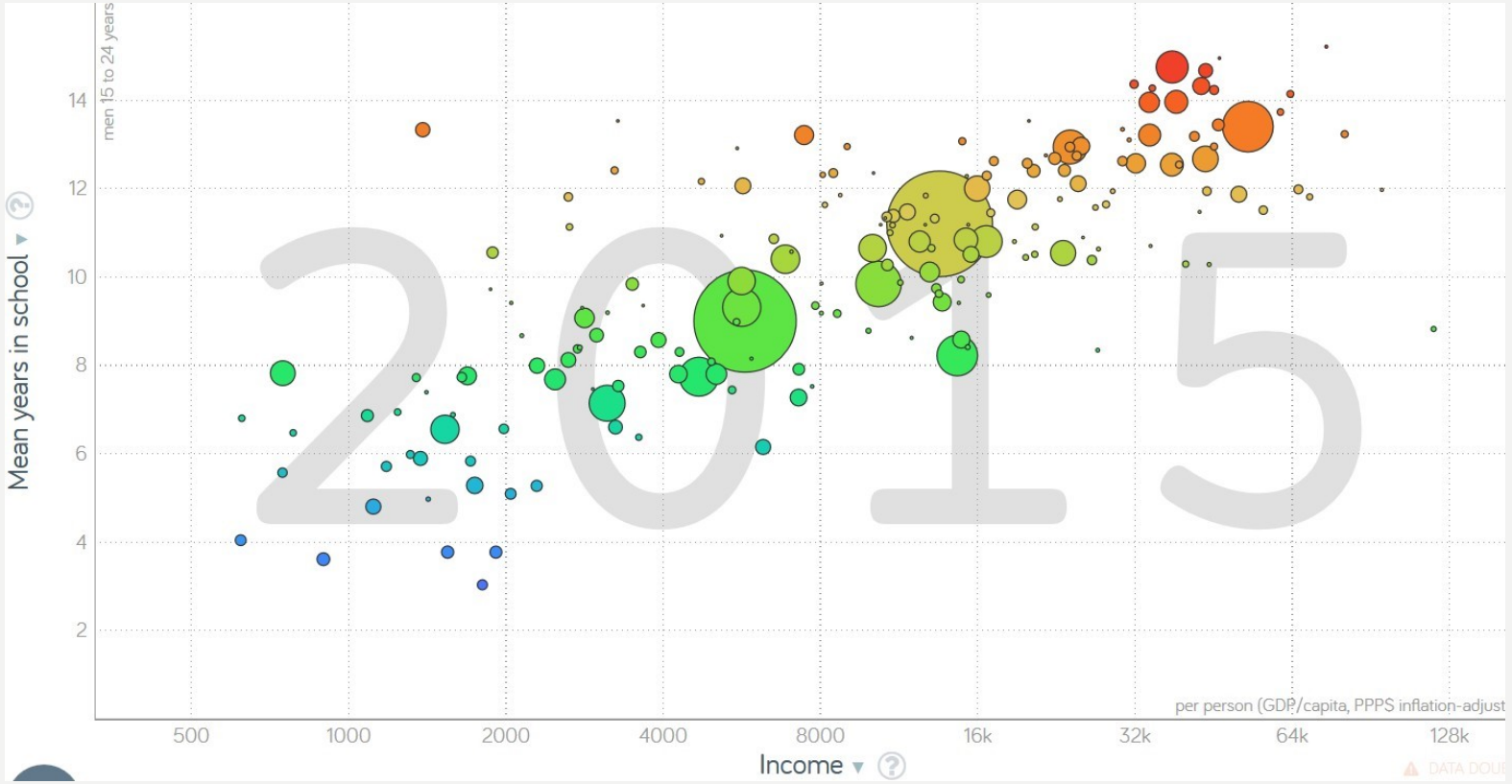
Hourly wages by wage percentile, 2000–2016 (2016 dollars)

	Wage by percentile										Wage ratio		
	10th	20th	30th	40th	50th	60th	70th	80th	90th	95th	50th/10th	95th/50th	95th/10th
2000	$8.90	$10.93	$12.64	$14.64	$17.04	$20.20	$23.82	$29.13	$37.92	$48.32	1.9	2.8	5.4
2007	$9.11	$10.95	$12.77	$15.00	$17.43	$20.76	$24.63	$30.30	$40.55	$52.48	1.9	3.0	5.8
2015	$9.08	$10.25	$12.56	$15.08	$17.33	$20.40	$25.21	$31.29	$42.87	$56.88	1.9	3.3	6.3
2016	$9.35	$10.91	$12.91	$15.03	$17.86	$21.00	$25.08	$31.89	$43.86	$57.86	1.9	3.2	6.2
Annualized percent changes											**Wage ratio change**		
2000–2016	0.3%	0.0%	0.1%	0.2%	0.3%	0.2%	0.3%	0.6%	0.9%	1.1%	0.0	0.4	0.8
2000–2007	0.3%	0.0%	0.1%	0.3%	0.3%	0.4%	0.5%	0.6%	1.0%	1.2%	0.0	0.2	0.3
2007–2016	0.3%	0.0%	0.1%	0.0%	0.3%	0.1%	0.2%	0.6%	0.9%	1.1%	0.0	0.2	0.4
2015–2016	2.9%	6.4%	2.8%	-0.3%	3.1%	3.0%	-0.5%	1.9%	2.3%	1.7%	0.0	0.0	-0.1

Note: Sample based on all workers age 18–64. The *x*th-percentile wage is the wage at which *x*% of wage earners earn less and (100 - *x*)% earn more.

Source: EPI analysis of Current Population Survey Outgoing Rotation Group microdata

AVERAGE YEARS OF SCHOOLING VERSUS GDP PER CAPITA



Source: https://www.gapminder.org/

INCORPORATING HUMAN CAPITAL INTO THE SOLOW MODEL

$$Y = AF(K, L, H)$$

h-human capital per worker

$$y = \frac{Y(t)}{L(t)} = \frac{AF(K, H, L)}{L} = AF(\frac{K}{L}, \frac{H}{L}, \frac{L}{L}) = A\left(\frac{K}{L}\right)^{\alpha}\left(\frac{H}{L}\right)^{1-\alpha} = Ak^{\alpha}h^{1-\alpha}$$

$$y^{ss} = Ak^{\alpha}h^{1-\alpha} = hA^{1/1-\alpha}\left(\frac{s}{\delta + n + g}\right)^{\alpha/1-\alpha}$$

KEY TERMS

- Technology
- Human capital
- productivity

Sources:

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