



**GDAŃSK UNIVERSITY  
OF TECHNOLOGY**

FACULTY OF MANAGEMENT AND ECONOMICS

# ESSENTIALS OF STATISTICS

## SKEWNESS, KURTOSIS

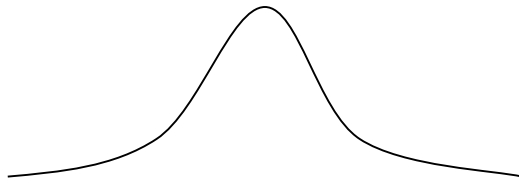
KAROLINA TURA, PHD

DEPARTMENT OF ECONOMIC SCIENCE

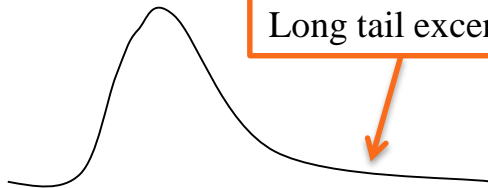
# AGENDA

1. **Skewness**
2. **Kurtosis**
3. **Practice**

# SKEWNESS



Symmetrical distribution  $\bar{x} = Me = D$

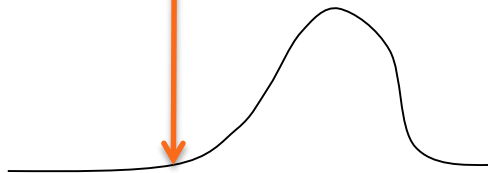


Long tail extending to the right

The distribution is skewed to the right

$$\bar{x} > Me > D$$

Long tail extending to the left



The distribution is skewed to the left

$$\bar{x} < Me < D$$

# SKEWNESS

## Moment coefficient of skewness

$$\mu_3 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3 \quad \text{Detailed form}$$

$$\mu_3 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3 n_i \quad \text{Frequency table}$$

$$\mu_3 = \frac{1}{n} \sum_{i=1}^n (\dot{x}_i - \bar{x})^3 n_i \quad \text{Frequency table with intervals}$$

$$\alpha_3 = \frac{\mu_3}{S^3}; \alpha_3 \in \langle -2, 2 \rangle$$

## Standard coefficient of skewness

$$A(\bar{x}) = \frac{\bar{x} - D}{S}; A(\bar{x}) \in \langle -1, 1 \rangle$$

$\alpha_3$	Interpretation
$\alpha_3 = 0$	Symmetrical distribution
$\alpha_3 > 0$	The distribution is skewed to the right
$\alpha_3 < 0$	The distribution is skewed to the left

# TASK 1. DETAILED FORM

15 students scored the points on the exam of Statistics:

1,2,3,4,4,5,5,5,6,6,6,6,7,8,8.

Find and interpret:

- a) *skewness*,
- b) kurtosis and excess kurtosis.

# HINT

	$x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^3$	$(x_i - \bar{x})^4$
	1,00	-4,07	16,54	-67,25	273,50
	2,00	-3,07	9,40	-28,84	88,44
	3,00	-2,07	4,27	-8,83	18,24
	4,00	-1,07	1,14	-1,21	1,29
	4,00	-1,07	1,14	-1,21	1,29
	5,00	-0,07	0,00	0,00	0,00
	5,00	-0,07	0,00	0,00	0,00
	5,00	-0,07	0,00	0,00	0,00
	6,00	0,93	0,87	0,81	0,76
	6,00	0,93	0,87	0,81	0,76
	6,00	0,93	0,87	0,81	0,76
	6,00	0,93	0,87	0,81	0,76
	7,00	1,93	3,74	7,23	13,97
	8,00	2,93	8,60	25,24	74,04
	8,00	2,93	8,60	25,24	74,04
Summary	76,00	0	56,93	-46,39	547,85

$$\bar{x} = \frac{\sum_{i=1}^{n_i} x_i}{n} = \frac{76}{15} \approx 5.07$$

$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{56.93}{15} \approx 3.795$$

$$\sqrt{S^2} = S = \sqrt{3.795} \approx 1.95$$

$$\mu_3 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3 =$$

$$= \frac{-46.93}{15} \approx -3.13$$

$$\alpha_3 = \frac{-3.13}{1.95^3} \approx -0.42$$

The distribution is skewed to the left

# TASK 2. FREQUENCY TABLE

The number of hours (per week) which students spend on learning Statistics in 2014

is given in table.

Find and interpret:

- a) skewness (moment),
- b) kurtosis and excess kurtosis.

$x$ Hours (per week)	$n$ Frequency
0	25
1	54
2	11
3	9
4	1

# HINT

$$\bar{x} = \frac{\sum_{i=1}^k x_i n_i}{N} = \frac{107}{100} = 1.07$$

$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 n_i = \frac{80.51}{100} = 0.8051$$

$$\sqrt{S^2} = \sqrt{0.8051} \approx 0.897$$

	$x_i$	$n$	$x_i n$	$n_{sk}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^2 n$	$(x_i - \bar{x})^3$	$(x_i - \bar{x})^3 n$	$(x_i - \bar{x})^4$	$(x_i - \bar{x})^4 n$
	0,00	25,00	0,00	25,00	-1,07	1,14	28,62	-1,23	-30,63	1,31	32,77
	1,00	54,00	54,00	79,00	-0,07	0,00	0,26	0,00	-0,02	0,00	0,00
	2,00	11,00	22,00	90,00	0,93	0,86	9,51	0,80	8,85	0,75	8,23
	3,00	9,00	27,00	99,00	1,93	3,72	33,52	7,19	64,70	13,87	124,87
	4,00	1,00	4,00	100,00	2,93	8,58	8,58	25,15	25,15	73,70	73,70
Sum		100,00	107,00	393,00	4,65	21,62	80,51	31,92	68,06	89,63	239,57

$$\mu_3 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3 n_i = \frac{68.06}{100} \approx 0.68$$

$$\alpha_3 = \frac{0.68}{0.9^3} = 0.93$$

The distribution is skewed to the right



# TASK 3. FREQUENCY TABLE WITH INTERVALS

Observations of consumer credit borrowers were collected at the Bank X in 2008.

Results were grouped in a series of observations given in the table below.

Find and interpret:

- a) skewness (moment),
- b) kurtosis and excess kurtosis.

Credit [zł] $x$	Credit borrowers $n$
0-5	300
5-10	470
10-15	693
15-20	328
20-25	120
25-30	35

# HINT

$$\bar{x} = \frac{\sum_{i=1}^k \dot{x}_i n_i}{N} = \frac{22340}{1946} \approx 11.5$$

$$S^2 = \frac{1}{n} \sum_{i=1}^n (\dot{x}_i - \bar{x})^2 n_i = \frac{67800.22}{1946} \approx 34.84$$

$$S = \sqrt{S^2} = \sqrt{34.84} \approx 5.9$$

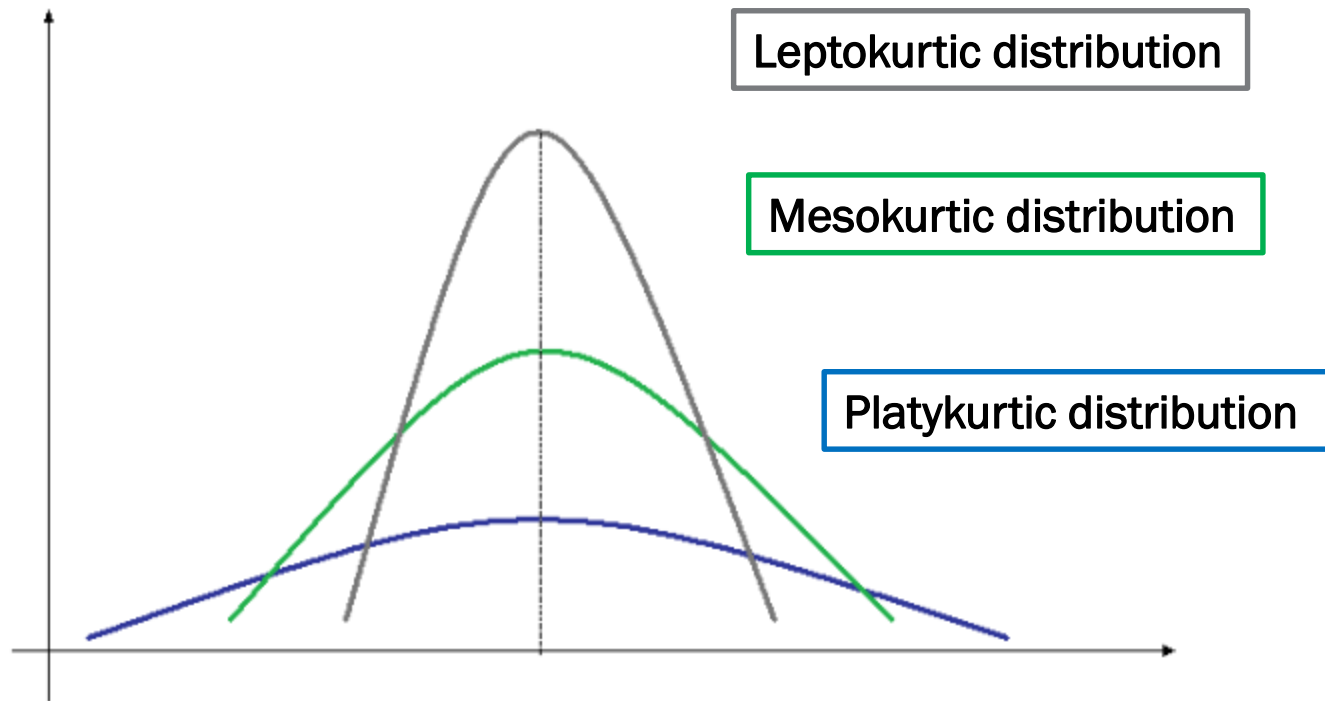
	$x_i$	$n$	$n_{sk}$	$\dot{x}$	$\dot{x}_i n$	$\dot{x}_i - \bar{x}$	$(\dot{x}_i - \bar{x})^2$	$(\dot{x}_i - \bar{x})^3$	$(\dot{x}_i - \bar{x})^3 n$	$(\dot{x}_i - \bar{x})^2 n$	$(\dot{x}_i - \bar{x})^4$	$(\dot{x}_i - \bar{x})^4 n$
	0-5	300	300	2,5	750	-8,98	80,64	-724,151	-217245	24192,12	6502,87	1950862,23
	5-10	470	770	7,5	3525	-3,98	15,84	-63,0448	-29631,1	7444,99	250,92	117931,59
	10-15	693	1463	12,5	8662	1,02	1,04	1,061208	735,4171	721,00	1,08	750,13
	15-20	328	1791	17,5	5740	6,02	36,24	218,1672	71558,84	11886,85	1313,37	430784,24
	20-25	120	1911	22,5	2700	11,02	121,44	1338,273	160592,8	14572,85	14747,77	1769732,49
	25-30	35	1946	27,5	962,5	16,02	256,64	4111,379	143898,3	8982,41	65864,29	2305250,32
Sum		1946	8181	90	22340	21,12	511,84	4881,685	129909	67800,22	88680,31	6575311,00

$$\mu_3 = \frac{1}{n} \sum_{i=1}^n (\dot{x}_i - \bar{x})^3 n_i = \frac{129909}{1946} \approx 66.76$$

$$\alpha_3 = \frac{66.76}{5.9^3} = 0.33$$

The distribution is skewed to the right

# KURTOSIS



Measure of the "tailedness" of the distribution  
Descriptor of the shape of a probability distribution

# KURTOSIS

$$\mu_4 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4$$

Detailed form

$$\mu_4 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4 n_i$$

Frequency table

$$\mu_4 = \frac{1}{n} \sum_{i=1}^n (\dot{x}_i - \bar{x})^4 n_i$$

Frequency table with intervals

Kurtosis

$$\alpha_4 = \frac{\mu_4}{S^4}$$

Excess kurtosis

$$E = \alpha_4 - 3$$

$\alpha_4$	E	Interpretation
$\alpha_4 = 3$	E=0	Mezokurtic
$\alpha_4 > 3$	E>0	Leptokutic
$\alpha_4 < 3$	E<0	Platykurtic

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# HINT

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Summary	76,00	0	56,93	-46,39	547,85

$$\bar{x} = \frac{\sum_{i=1}^{n_i} x_i}{n} = \frac{76}{15} \approx 5.07$$

$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{56.93}{15} \approx 3.795$$

$$\sqrt{S^2} = S = \sqrt{3.795} \approx 1.95$$

$$\mu_4 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4 =$$

$$= \frac{547.85}{15} = 36.52$$

$$\alpha_4 = \frac{\mu_4}{S^4} = \frac{36.52}{1.95^4} \approx 2.53$$

Platykurtic

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$$\mu_4 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4 n_i = \frac{239.57}{100} \approx 2.4$$

$$\alpha_4 = \frac{\mu_4}{S^4} = \frac{2.4}{0.9^4} \approx 3.66$$

Leptokutic



# TASK 3. FREQUENCY TABLE WITH INTERVALS

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$$S^2 = \frac{1}{n} \sum_{i=1}^n (\dot{x}_i - \bar{x})^2 n_i = \frac{67800.22}{1946} \approx 34.84$$

$$S = \sqrt{S^2} = \sqrt{34.84} \approx 5.9$$

	$x_i$	$n$	$n_{sk}$	$\dot{x}$	$\dot{x}_i n$	$\dot{x}_i - \bar{x}$	$(\dot{x}_i - \bar{x})^2$	$(\dot{x}_i - \bar{x})^3$	$(\dot{x}_i - \bar{x})^3 n$	$(\dot{x}_i - \bar{x})^2 n$	$(\dot{x}_i - \bar{x})^4$	$(\dot{x}_i - \bar{x})^4 n$
	0-5	300	300	2,5	750	-8,98	80,64	-724,151	-217245	24192,12	6502,87	1950862,23
	5-10	470	770	7,5	3525	-3,98	15,84	-63,0448	-29631,1	7444,99	250,92	117931,59
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	25-30	35	1946	27,5	962,5	16,02	256,64	4111,379	143898,3	8982,41	65864,29	2305250,32
Sum		1946	8181	90	22340	21,12	511,84	4881,685	129909	67800,22	88680,31	6575311,00

$$\mu_4 = \frac{1}{n} \sum_{i=1}^n (\dot{x}_i - \bar{x})^4 n_i = \frac{6575311}{1946} = 3378.89$$

$$\alpha_4 = \frac{\mu_4}{S^4} = \frac{3378.89}{5.9^4} \approx 3.66$$

Leptokutic

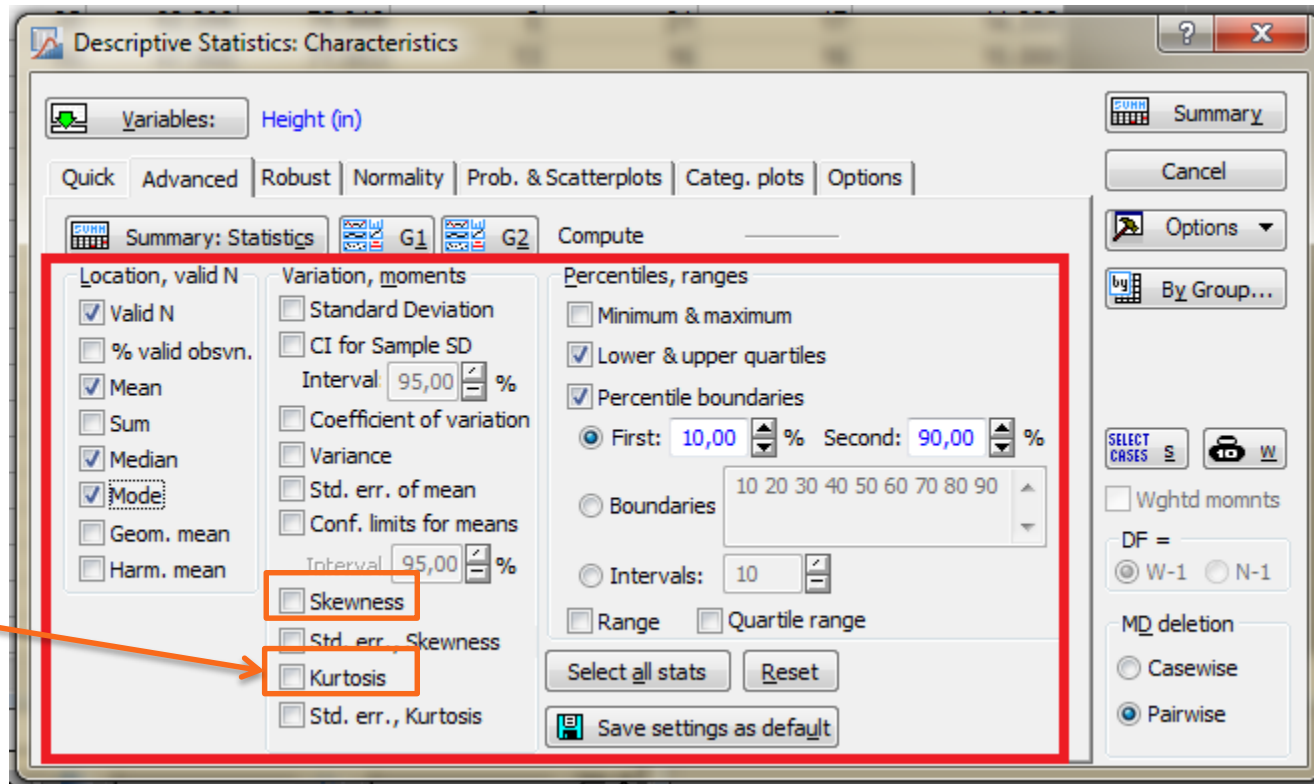
**STATISTICA**

## TASK 6.

Scientists examined height of randomly selected men from the city of Gdynia. On the basis of the data contained in the file CharacteristicsHeight.sta perform an analysis. Find and interpret:

- a) skewness,
- b) kurtosis and excess kurtosis.

# HINT



Excess

# TASK 7.

Scientists examined height of randomly selected women from the city of Gdynia. On the basis of the data contained in the file CharacteristicsHeight.sta perform an analysis.

Find and interpret:

- a) skewness,
- b) kurtosis and excess kurtosis.

# PREPARATION FOR THE NEXT CLASSES

McClave, J. T., Benson, P. G., Sincich, T. (2008) , *Statistics for Business & Economics*, Pearson Education Inc., New Jersey.

**Thank you for your  
attention**



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