Karolina Tura-Gawron, PhD

NONLINEAR RECORDESSION

NONLINEAR FUNCTIONS

- I. Quadratic function (parabola)
- II. Hyperbolic function (hyperbola)
- **III.** Polynomial function
- IV. Logarythmic function
- V. Exponential function
- VI. Power function

QUADRATIC FUNCTION (PARABOLA)

 $y=a+bx+cx^2$



HYPERBOLA



$$y=1/x$$

LOGARYTHMIC FUNCTION





POLYNOMIAL FUNCTION



TASK. 1.

A real estate agent would like to predict the **selling price** of single-family homes. After careful consideration, he concludes that the variables likely to be most closely related to selling price is **the size of the house** (in 100s ft²). As an experiment, he takes a random sample of fifteen recently sold houses and records the selling price (in \$ 1,000s). These are shown in the accompanying table. Find and interpret the:

- I. linear regression model,
- II. qudratic nonlinear regression model,
- III. hyperbolic nonlinear regression model,
- IV. logarythmic nonlinear regression model.



TASK. 1.

House size	Selling Price
20	89.5
14.8	79.9
20.5	83.1
12.5	56.9
18	66.6
14.3	82.5
27.5	126.3
16.5	79.3
24.3	119.9
20.2	87.6
22	112.6
19	120.8
12.3	78.5
14	74.3
16.7	74.8

LINEAR REGRESSION

	Regression Summary for Dependent Variable: Selling Price (Zzza) R= ,80470742 R2= ,64755403 Adjusted R2= ,62044280 F(1,13)=23,885 p<.00030 Std.Error of estimate: 12,997						
	b* Std.Err. b Std.Err. t(13) p-value						
N=15		of b*		of b			
Intercept			18,35380	14,80774	1,239473	0,237080	
House size	0,804707	0,164655	3,87855	0,79361	4,887237	0,000297	

FIXED NONLINEAR REGRESSION

St	atistics	Dat	ta Min	ning	Graphs	Tools	Data	
R	5		.	dvance	d Models *	Neural	Nets	
bution	More		11	Genera	al Linear	•		
ting	Distributi	ions		Genera	alized Linear/	Nonlinear		
			凶	Stepwi	Stepwise Model Builder			
				Genera	al Regression		- 1	
			PLS deptd	Genera	al Partial Leas	t Squares		
1			ß	NIPAL	s		- H	
op\ES	\Homes	xlsx :	1	Varian	ce Compone	nts	_ L	
ce	Age (ye	ars)	35	Surviv	al		-	
89,5			4 3	Cox Pr	oportional H	azards	- 1	
79,9		1	\wedge	Nonlin	ear Estimatio	n		
83,1			1	Fixed I	Nonlinear Reg	gression	귀나	
56,9			∎x ²	Log-Li	near		-	
66,6			<u>~?</u>	Time S	Series/Foreca	sting		
82,5		1	왎	Struct	ural Equation	_		
120,3							_	

FIXED NONLINEAR REGRESSION

🚈 Nonlinear C	omponents Regression:	Zzza	? <mark>×</mark>
Quick			ОК
Nonlinear Tran	sformation Functions	Valid Range	Cancel
∏ X**2	(X to the power of 2)	-5.0E+08 to 5.0E+08	Ontions
□ X**3	(X to the power of 3)	-5.0E+05 to 5.0E+05	Jee options .
□ X**4	(X to the power of 4)	-5.0E+04 to 5.0E+04	
□ X**5	(X to the power of 5)	-5.0E+03 to 5.0E+03	
	(square root)	X greater or equal to 0	
	(natural log)	X greater than 0	
LOG(X)	(log 10)	X greater than 0	
∏ e ** X		-40 to +40	
□ 10 ** X		-18 to +18	
🗌 1/X		X not equal 0	

PARABOLA

Mon 🔀	linear C	omponents Regressio	n: Zzza	? ×
Quick				ОК
Nonlin	near Tran	sformation Functions	Valid Range	Cancel
<mark>⊠ x</mark>	 2	(X to the power of 2)	-5.0E+08 to 5.0E+08	Poptions ▼
□ X	" 3	(X to the power of 3)	-5.0E+05 to 5.0E+05	
□ X	" 4	(X to the power of 4)	-5.0E+04 to 5.0E+04	
□ X	** 5	(X to the power of 5)	-5.0E+03 to 5.0E+03	
E SC	QRT(X)	(square root)	X greater or equal to 0	
	V(X)	(natural log)	X greater than 0	
	DG(X)	(log 10)	X greater than 0	
🗌 e'	- X		-40 to +40	
□ 10)**X		-18 to +18	
□ 1/	x		X not equal 0	
				4

Select dependent and independent variable lists:: Zzza					
Dependent variables:	Independent variables:	ОК			
1-House size 2-Selling Price 3-V1**2 4-V2**2	1-House size 2-Selling Price 3-V1**2 4-V2**2	Cancel			

PARABOLA

	Regression Summary for Dependent Variable: Selling Price (Zzza) R= ,81226759 R2= ,65977864 Adjusted R2= ,60307508 F(2,12)=11,636 p<,00155 Std.Error of estimate: 13,291					
	b*	Std.Err.	b	Std.Err.	t(12)	p-value
N=15		of b*		of b		
Intercept			57,30011	61,21403	0,936062	0,367698
House size	-0,081556	1,360157	-0,39309	6,55572	-0,059961	0,953174
V1**2	0,893133	1,360157	0,11111	0,16921	0,656640	0,523805

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🜃 Nonlinear Components Regression: Zzza

8 23

YPERBOLA	Quick			С ОК
	Nonlinear Trar	nsformation Functions	Valid Range	Cancel
	🗆 X**2	(X to the power of 2)	-5.0E+08 to 5.0E+08	Dotions -
	🗆 X**3	(X to the power of 3)	-5.0E+05 to 5.0E+05	
	🗆 X**4	(X to the power of 4)	-5.0E+04 to 5.0E+04	
	🗖 X**5	(X to the power of 5)	-5.0E+03 to 5.0E+03	
	SQRT(X)	(square root)	X greater or equal to 0	
	LN(X)	(natural log)	X greater than 0	
	LOG(X)	(log 10)	X greater than 0	
	□ e X		-40 to +40	
	□ 10 ** X		-18 to +18	
	✓ 1/X		X not equal 0	

🜃 Select dependent and independent variable lists:: Zzza 🛛 🔗 🔜						
Dependent variables: 1-House size 2-Selling Price 3-1/V1	Independent variables: 1-House size 2-Selling Price 3-1/V1	OK Cancel				
4-1/V2	4-1/V2					

HYPERBOLA

a province of a continue of the continue of th					
b* Std.Err. b Std.Err. t(13) p-value					
of b* of b	=15				
156,28 16,7410 9,335	itercept				
0,182360 -1162,58 281,3841 -4,131	′V1 -				
Std.Err. of b* b Std.Err. of b t(13) 0 f b* 156,28 16,7410 9,335 0,182360 -1162,58 281,3841 -4,131	=15 Itercept				

LOGARITHMIC FUNCTION

Quick			ОК
Nonlinear Tran	sformation Functions	Valid Range	Cancel
∏ X**2	(X to the power of 2)	-5.0E+08 to 5.0E+08	Dotions
∏ X**3	(X to the power of 3)	-5.0E+05 to 5.0E+05	
□ X**4	(X to the power of 4)	-5.0E+04 to 5.0E+04	
□ X**5	(X to the power of 5)	-5.0E+03 to 5.0E+03	
SQRT(X)	(square root)	X greater or equal to 0	
🔽 LN(X)	(natural log)	X greater than 0	
LOG(X)	(log 10)	X greater than 0	
⊟ e [⊷] X		-40 to +40	
□ 10 ** X		-18 to +18	
□ 1/X		X not equal 0	

🔀 Select dependent and indep	? <mark>×</mark>	
Dependent variables:	Independent variables:	ок
1-House size 2-Selling Price 3-LN-V1 4-LN-V2	1-House size 2-Selling Price 3-LN-V1 4-LN-V2	Cancel

LOGARITHMIC FUNCTION

	Regression Summary for Dependent Variable: Selling Price (Zzza)					
	R= ,78417346 R2= ,61492802 Adjusted R2= ,58530710					
	F(1,13)=20,760 p<,00054 Std.Error of estimate: 13,585					
	b*	Std.Err.	b	Std.Err.	t(13)	p-value
N=15		of b*		of b		
Intercept			-110,575	43,90718	-2,51838	0,025682
LN-V1	0,784173	0,172107	69,401	15,23194	4,55631	0,000539

MODELS' COMPARISON

Function	R square	Models' significance	Coefficients , significance	Standrad error of estimate
Linear	0.65	p<0.0003	No	12.997
Parabola	0.66	p<0.002	No	13.291
Hyperbola	0.57	p<0.0018	Yes	14.39
Logarithmic function	0.61	p<0.0005	Yes	13.585

LINEARIZATION

- 1. Exponential function
- 2. Power function



EXPONENTIAL FUNCTION





TASK. 2.

A real estate agent would like to predict the **selling price** of single-family homes. After careful consideration, he concludes that the variables likely to be most closely related to selling price is **the size of the house** (in 100s ft²). As an experiment, he takes a random sample of fifteen recently sold houses and records the selling price (in \$ 1,000s). These are shown in the accompanying table. Find and interpret the:

- I. linear regression model,
- II. exponential function model,
- III. power function model .

TASK. 2.

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20.2	87.6
22	112.6
19	120.8
12.3	78.5
14	74.3
16.7	74.8

EXPONENTIAL FUNCTION

🕍 Nonlinear C	omponents Regression	: Zzza	? 🗙
Quick			📰 ОК
Nonlinear Tran	sformation Functions	Valid Range	Cancel
□ X**2	(X to the power of 2)	-5.0E+08 to 5.0E+08	> Options -
∏ X**3	(X to the power of 3)	-5.0E+05 to 5.0E+05	· · · · · · · · ·
□ X**4	(X to the power of 4)	-5.0E+04 to 5.0E+04	
□ X**5	(X to the power of 5)	-5.0E+03 to 5.0E+03	
SQRT(X)	SQRT(X) (square root) X greater or equal to		
🗹 LN(X) (natural log) X greater than 0		X greater than 0	
🗆 LOG(X) (log 10)		X greater than 0	
□ e**X		-40 to +40	
□ 10 ** X		-18 to +18	
□ 1/X		X not equal 0	
🔀 Sele	ct dependent and indep	endent variable lists:: Spre	adsh 📪 💌
Depend	ent variables:	Independent variables:	ОК
1-x		1-x 2-x	Cancel
2-9 3-LN-V	1	3-LN-V1	Cancer
4-LN-V	2	4-LN-VZ	

EXPONENTIAL FUNCTION



If the size of the house increases by 1 unit, the selling price increases by (1.009-1)*100%=0.9%, on average.

POWER FUNCTION

-	🕍 Nonlinear C	omponents Reg	gression: Zzza	? 💌			
	Quick	Quick					
	Nonlinear Tran	sformation Function	ons Valid Range	Cancel			
4	□ X**2	(X to the power	of 2) -5.0E+08 to 5.0E+08	> Options -			
	□ X**3	(X to the power	of 3) -5.0E+05 to 5.0E+05				
1	□ X**4	(X to the power	of 4) -5.0E+04 to 5.0E+04				
	□ X**5	(X to the power	of 5) -5.0E+03 to 5.0E+03				
	SQRT(X)	(square root)	X greater or equal to 0				
	IN(X)	(natural log)	X greater than 0				
	LOG(X)	(log 10)	X greater than 0				
	∏ e [⊷] X		-40 to +40				
	□ 10 ** X		-18 to +18				
	🗌 1/X		X not equal 0				
🜠 Se	🖉 Select dependent and independent variable lists:: Zzza 🛛 😨 💌						
Dependent variables: Independent variables:							
1-Hou	1-House size						
2-Selling Price Cancel							
4-LN-	4-LN-V2 4-LN-V2						

POWER FUNCTION

Regression Summary for Dependent Variable: LN-V2 (Zzza)						
R= ,78122672 R2= ,61031519 Adjusted R2= ,58033943						
F(1,13)=20,360 p<,00058 Std.Error of estimate: ,14968						
b* Std.Err. b Std.Err. t(13) p-value						
	of b*		of b			
		2,285599	0,483779	4,724473	0,000397	
0,781227	0,173135	0,757283	0,167829	4,512238	0,000584	
	Regression R= ,7812267 F(1,13)=20,3 b* 0,781227	Begression Summary for R= ,78122672 R2= ,6103 F(1,13)=20,360 p<,00058 b* Std.Err. of b* 0,781227 0,173135	Begression Summary for Dependent \ R= ,78122672 R2= ,61031519 Adjuster F(1,13)=20,360 p<,00058 Std.Error of b* Std.Err. b* Std.Err. of b* 2,285599 0,781227 0,173135 0,757283	Regression Summary for Dependent Variable: LN-V R= ,78122672 R2= ,61031519 Adjusted R2= ,5803 F(1,13)=20,360 p<,00058 Std.Error of estimate: ,14 b* Std.Err. b Std.Err. of b* of b of b 0,781227 0,173135 0,757283 0,167829	Regression Summary for Dependent Variable: LN-V2 (Zzza) R= ,78122672 R2= ,61031519 Adjusted R2= ,58033943 F(1,13)=20,360 p<,00058 Std.Error of estimate: ,14968 b* Std.Err. b Std.Err. t(13) of b* 0 f b 0 f b 1000000000000000000000000000000000000	

$$\ln y = 2.285 + 0.757 x$$

$$\ln a = 2.285 \rightarrow a = e^{2.285} \rightarrow a = 9.823$$

$$y = 9.823 x^{0.757}$$

If the size of the house increases by 1%, the selling price increases by 0.757%, on average.

MODELS' COMPARISON

Function	R square	Models' significance	Coefficients significance	Standrad error of estimate
Exponential	0.63	p<0.0004	Yes	0.145
Power	0.61	p<0.0005	Yes	0.15

