## TIME SERIES ANALYSIS

Classical time series model (addictive):

$$
Y_{t}=T_{t}+C_{t}+S_{t}+I_{t}
$$

$Y_{t}$ - time series;
$T_{t}$-trend; a long term relatively smooth pattern or direction that the series exhibits;
$C_{t}$ - cycle, wavelike or oscillatory pattern about a long term trend that is generally apparent over a numer of years; it has duration of more than one year;
$S_{t}$ - seasonal variations, are like cycles but they occur over short repetitive calendar periods and have durations of less than one year;
$I_{t}$ - residual or irregular variation; the random movement that a series exhibits after the trend, cycle and seasonal variation are removed.


## Mechanical method

The moving average (MA), takes away the short term seasonal and irregular variation, leaving a combined trend and cyclical movement. Moving averages are widely used to to remove seasonal variation, irregular variation or both.

Task 1. Monthly sales figures for gasoline were recorded at all the gas stations in a particular town, as shown in table. Calculate the three-month and five-month moving averages. Create a graph. (A moving average is a simple arithmetic average computed over any numer of time periods. For a three period moving average we would take the first three months ( 1,2 and 3 ) and average them. Then we would move to the next three month grouping $(2,3$ and 4) and average them; and so on.)

Hint:

|  |  | 3-Month MA |  | 5- Month MA |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Month | Gasoline <br> sales | Moving <br> Total | Moving <br> Average=Moving <br> total/3 | Moving <br> Total | Moving <br> Average=Moving total/5 |
| 1 | 37 | - | - | - | - |
| 2 | 70 |  |  | - | - |
| 3 | 45 |  |  |  |  |
| 4 | 26 |  |  |  |  |
| 5 | 60 |  |  |  |  |
| 6 | 45 |  |  |  |  |
| 7 | 31 |  |  |  |  |
| 8 | 79 |  |  | - | - |
| 9 | 24 |  |  |  |  |
| 10 | 61 |  |  |  |  |
| 11 | 25 |  |  |  |  |
| 12 | 44 | - | - |  |  |

Task 2. The data concerned the newborn in Poland in the years 1974-1978 is available in the file Newborn.sta.
a) Create a time series plot.

Hint:
<Graphs> <2D Graphs> <Line plots (Variables)>

b) Use $k=12$ month moving average and find the trend.

Hint: <Statistics> <Advanced linear/nonlinear models> <Times series/Forecasting> <Exponential smoothing and forecasting>

Part <Advanced> <Other transfomations \& plots>
Part <Smothing>
<OK>

c) Compare this two plots.


Task 3. The data concerned working hours (in milions) in the McDonalds Restaurants in Exeter in years 19911995 is available in the file Workinghours.sta.
a) Create a time series plot.
b) Use $k=4$ moving average and find the trend.
c) Compare this two plots.

## Analytical method

$$
\begin{gathered}
\qquad \begin{array}{c}
\hat{y}=a+b t \\
a=\bar{y}-b \bar{t} \\
\text { Trend line: } b=\frac{S S_{t y}}{S S_{t}} \\
S S_{t y}=\sum t_{i} y_{i}-\frac{\left(\sum t_{i}\right)\left(\sum y_{i}\right)}{n} \\
S S_{t}=\sum t_{i}^{2}-\frac{\left(\sum t_{i}\right)^{2}}{n}
\end{array}, l
\end{gathered}
$$

## Note!

Task 4. Annual sales figures for a pharmaceutical company have been recorded over the past 10 years; they are shown in the table. Calculate the linear trend line of the data. Predict the annual sales in 1985.

Hint:

| Year | Sales; $\boldsymbol{y}$ | Time; $\boldsymbol{t}$ | $\boldsymbol{t}^{\boldsymbol{\wedge}} \mathbf{2}$ | $\boldsymbol{t y}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 9 7 5}$ | 18 | 1 | 1 | 18 |
| $\mathbf{1 9 7 6}$ | 19,4 | 2 | 4 | 38,8 |
| $\mathbf{1 9 7 7}$ | 18 | 3 | 9 | 54 |
| $\mathbf{1 9 7 8}$ | 19,9 | 4 | 16 | 79,6 |
| $\mathbf{1 9 7 9}$ | 19,3 | 5 | 25 | 96,5 |
| $\mathbf{1 9 8 0}$ | 21,1 | 6 | 36 | 126,6 |
| $\mathbf{1 9 8 1}$ | 23,5 | 7 | 49 | 164,5 |
| $\mathbf{1 9 8 2}$ | 23,2 | 8 | 64 | 185,6 |
| $\mathbf{1 9 8 3}$ | 20,4 | 9 | 81 | 183,6 |
| $\mathbf{1 9 8 4}$ | 24,4 | 10 | 100 | 244 |
| Sum | 207,2 | 55 | 385 | 1191,2 |

Task.5. The data concerned students (in thousands) in Poland in the years 1964-1994 is shown in the file Students.sta.
a) Create a time series plot.
b) Find and interpret simple linear trend model.
c) Predict the numer of students in the year 1995

Task 6. The data concerned buses (in thousands) in Poland in the years 1990-2004 is shown in the file Buses.sta.
a) Create a time series plot.
b) Find and interpret simple linear trend model.
c) Predict the numer of buses in the year 2005

Task 7. The data concerned the numer of insurances (in thousands) sold in Poland in the years 1990-1996 is shown in the file Insurance.sta.
a) Create a time series plot.
b) Find and interpret parabolic trend model.
c) Predict the numer of insurances in the year 1997.

