**Introduction to Laboratory 4**

Laboratory 4 concerns the implementation of the **False-Position method** and of the **Secant method** to find the root of a function $f(x)$. Both methods use a linear function to calculate a new approximation to the root, but contrary to the **Newton-Raphson method** they do not require the derivative of the function ($f'(x)$).

The **False-Position method** works in a similar way as the **Bisection method**, but instead of calculating the new approximation of the root from the middle of the interval $x_M = (x_L + x_R)/2$ (see Laboratory 1), we estimate a new value from the straight line passing through the points $(x_L, f(x_L))$ and $(x_R, f(x_R))$ (see Laboratory 4 and Figure 1). The new approximation of the root is calculated from:

$$x_0 = \frac{x_L f(x_R) - x_R f(x_L)}{f(x_R) - f(x_L)}$$

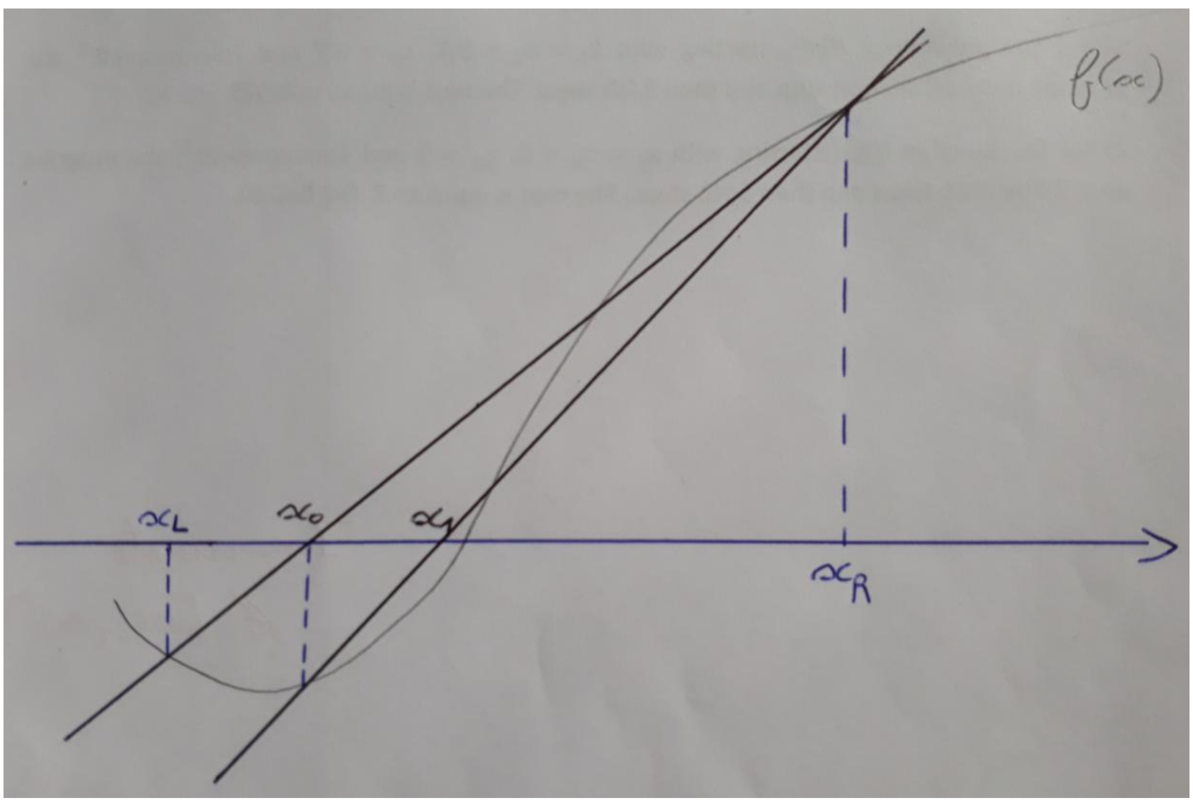


Figure 1: Example illustrating the **False-Position method** for two iterations (i.e. calculation of $x_0$ and $x_1$). In the first step the interval is $[x_L, x_R]$, in the second step the interval is $[x_0, x_R]$.

The **Secant method** resembles the **Newton-Raphson method**, the differences are that (i) we need to start from two initial values $x_0$ and $x_1$ (instead of one initial value with Newton-Raphson) and, (ii) the derivative the function at the point $x_i$ is approximated by $f'(x_i) \approx (f(x_i) - f(x_{i-1}))/(x_i - x_{i-1})$. Then, using the Newton-Raphson formula (see Laboratory 2), the new approximation of the root is calculated from:

$$x_{i+1} = x_i - f(x_i)\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} = \frac{x_{i-1}f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Equivalently, this means that the new approximation of the root is obtained from the straight line passing through the points $(x_{i-1}, f(x_{i-1}))$ and $(x_i, f(x_i))$ (see Figure 2).

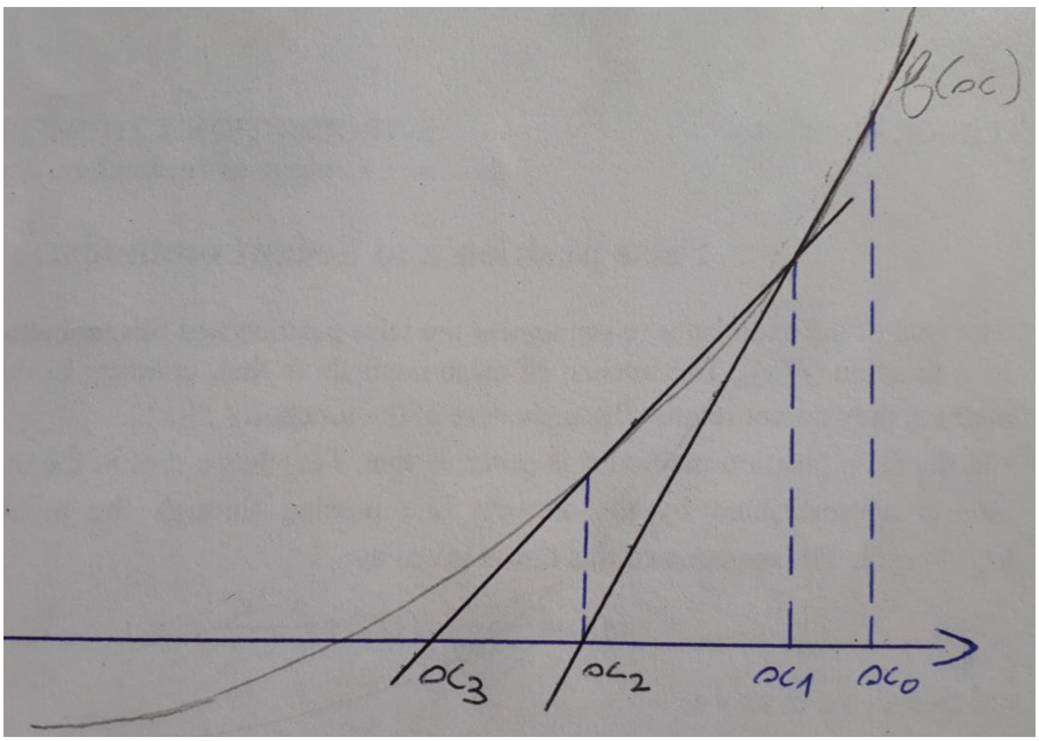


Figure 2: Example illustrating the **Secant method** for two iterations (i.e. calculation of $x_2$ and $x_3$).

**Solutions:**

The root of the function $f(x) = \cos x - x$ is equal to 0.739 085 13.

1) With the **False-Position method**, starting with $x_L = 0$, $x_R = 1$ and *Tolerance*=$10^{-8}$, the program required 8 iterations.

2) With the **Secant method**, starting with $x_0 = 0$, $x_1 = 0.2$ and *Tolerance*=$10^{-8}$, the program required 6 iterations.