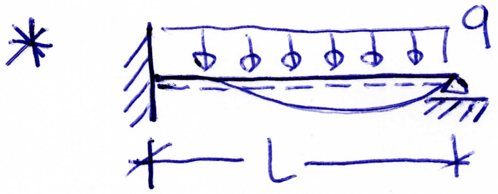
METODA SIŁ

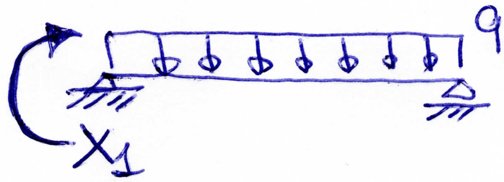
ĆW. 6 /1

$EI = const$ $\quad n_s = 1$

*

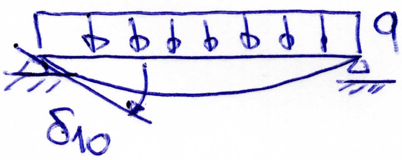


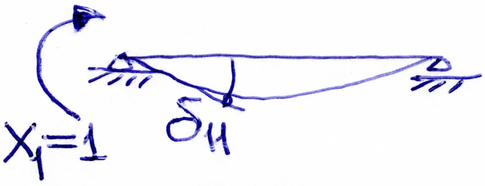
UKŁAD PODSTAWOWY METODY SIŁ



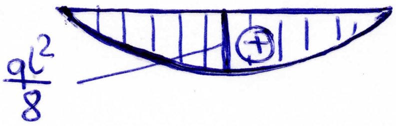
obciążenie zadane – „0"

stan $X_1 = 1$

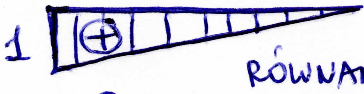




M_0



M_1



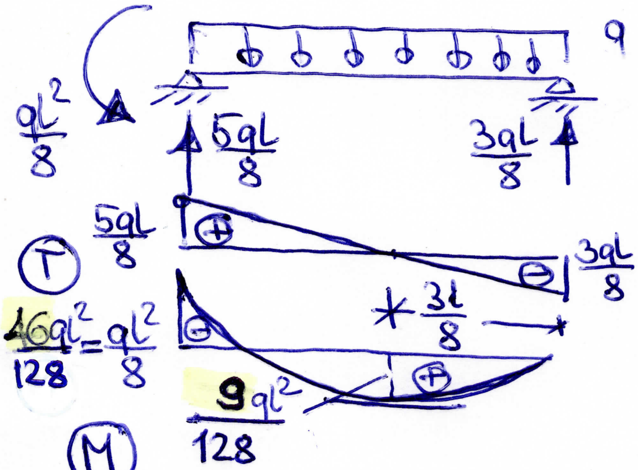
RÓWNANIE ZGODNOŚCI PRZEMIESZCZEŃ

$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = \frac{1}{EI} \cdot \frac{2}{3} \cdot L \cdot \frac{qL^2}{8} \cdot \frac{1}{2} \cdot 1 = \frac{qL^3}{24EI}$$

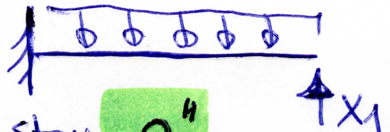
$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{1}{EI} \cdot \frac{1}{2} \cdot L \cdot 1 \cdot \frac{2}{3} \cdot 1 = \frac{L}{3EI}$$

$$\delta_{10} + \delta_{11} X_1 = 0$$

$$X_1 = -\frac{qL^2}{8}$$

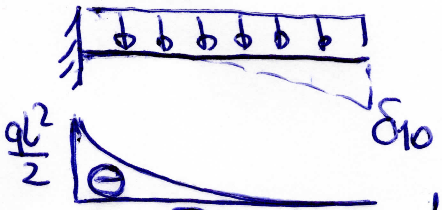


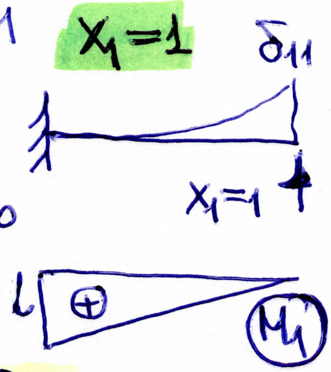
INNE PRZYJĘCIE UKŁADU PODSTAWOWEGO METODY SIŁ:



stan „0"

$X_1 = 1$

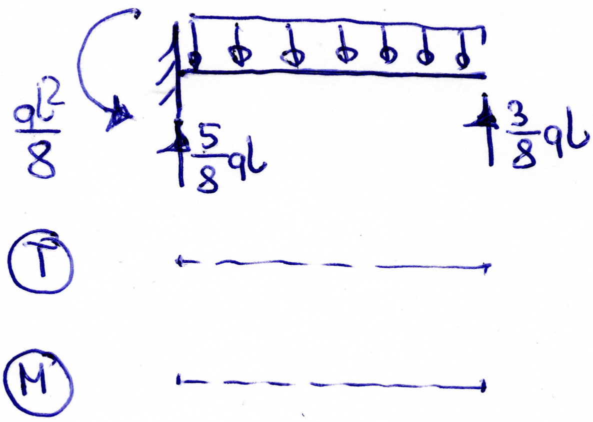




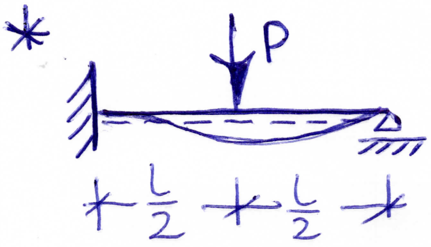
$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = \frac{-1}{EI} \cdot \frac{1}{3} \cdot L \cdot \frac{qL^2}{2} \cdot \frac{3}{4} \cdot L = -\frac{qL^4}{8EI}$$

$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{1}{EI} \cdot \frac{1}{2} \cdot L \cdot L \cdot \frac{2}{3} \cdot L = \frac{L^3}{3EI}$$

$$\Rightarrow X_1 = \frac{3}{8} qL$$



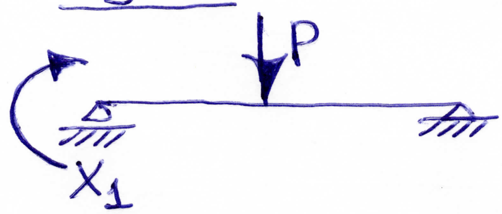
Spostrzeżenie: TEN SAM UKŁAD SIŁ (traktowanych łącznie: obciążeń zewnętrznych i reakcji podporowych), są to zatem schematy statycznie równoważne.

* 

EI = const

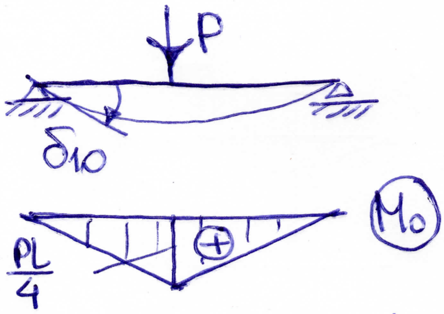
$m_s = 1$

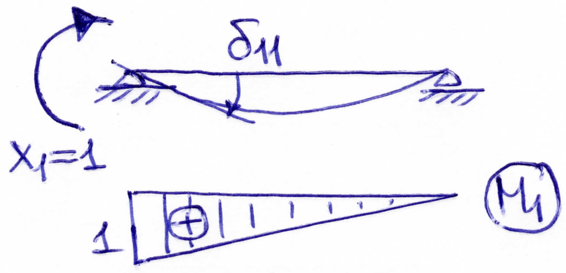
UKŁAD PODSTAWOWY METODY SIŁ



obciążenie zadane — „0"

stan $X_1 = 1$





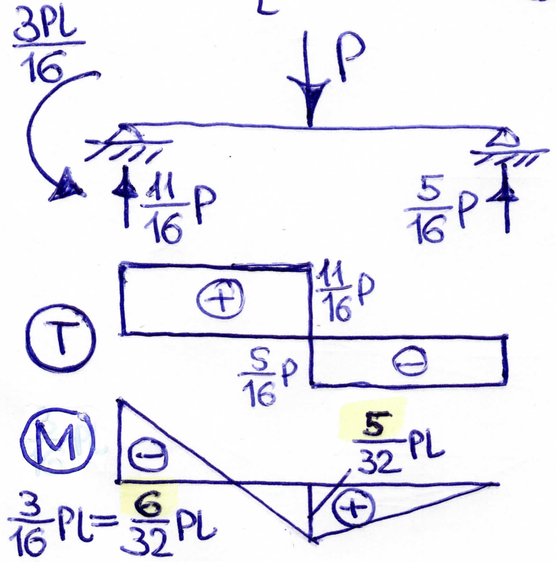
$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = \frac{1}{EI} \cdot \frac{1}{2} \cdot L \cdot \frac{PL}{4} \cdot \frac{1}{2} \cdot 1 = \frac{PL^2}{16EI}$$

$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{L}{3EI}$$

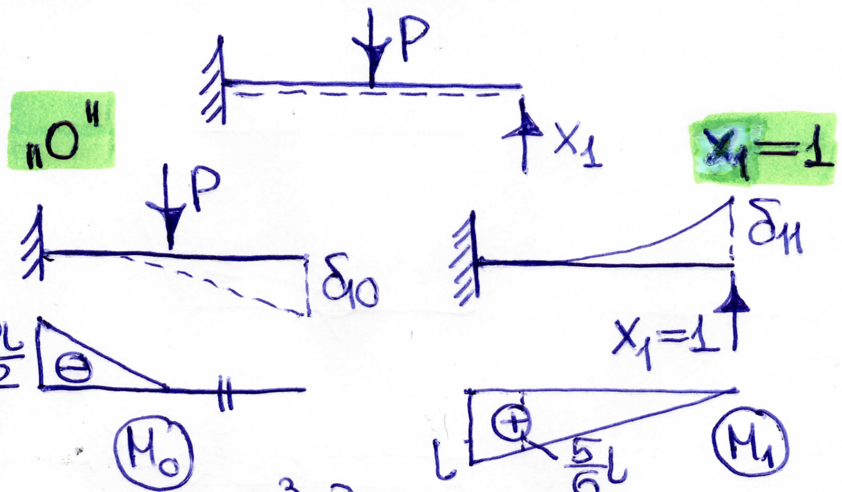
RÓWNANIE ZGODNOŚCI PRZEMIESZCZEŃ

$$\delta_{10} + \delta_{11} X_1 = 0$$

$$X_1 = -\frac{3}{16} PL$$



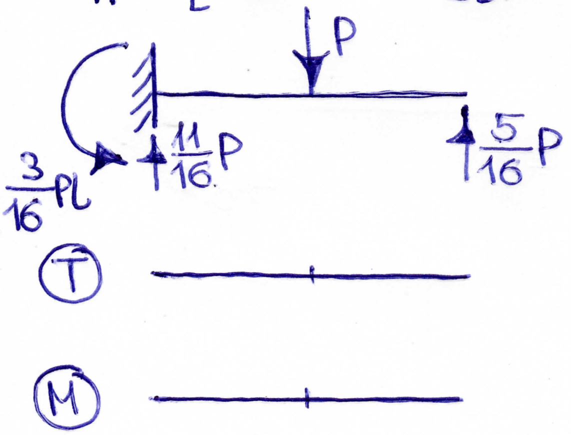
INNY UKŁAD PODSTAWOWY METODY SIŁ:



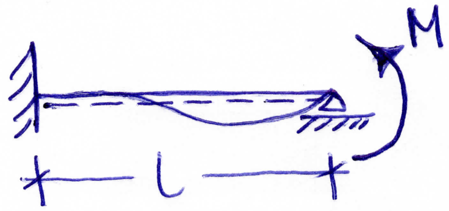
$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = -\frac{1}{EI} \cdot \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{PL}{2} \cdot \frac{5}{6} L = -\frac{5}{48} \frac{PL^3}{EI}$$

$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{L^3}{3EI}$$

$$X_1 = \frac{5}{16} P$$

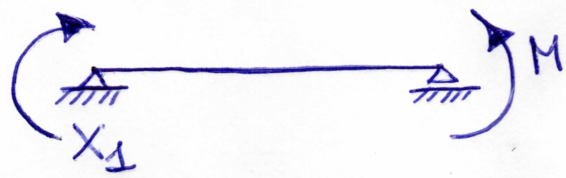


spostrzeżenie: TEN SAM UKŁAD SIŁ (traktowanych łącznie: obciążeń zewnętrznych i reakcji podporowych) są to zatem schematy statycznie równoważne

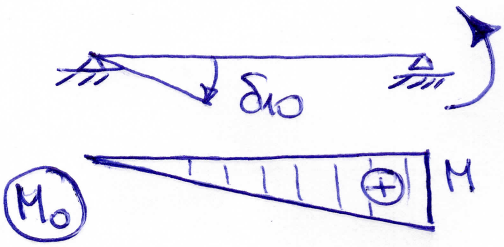
* 

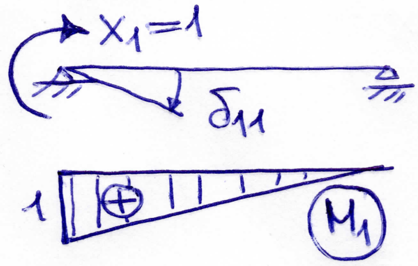
$EI = \text{const}$ $n_s = 1$

UKŁAD PODSTAWOWY METODY SIŁ:



obciążenie zadane „O” $X_1 = 1$

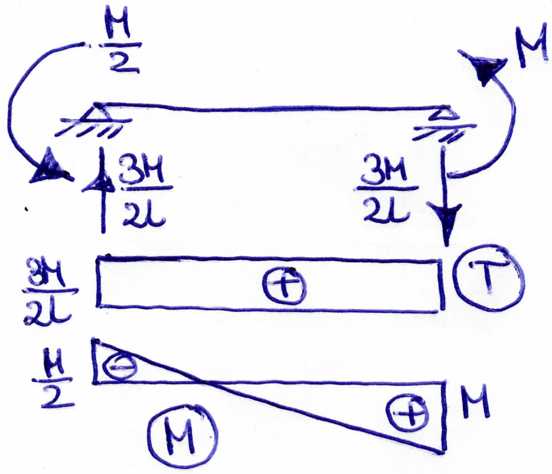




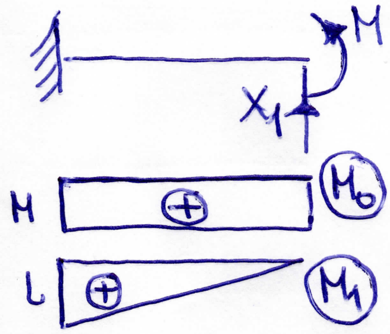
$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = \frac{1}{EI}\cdot\frac{1}{2} L \cdot M \cdot \frac{1}{3}\cdot 1 = \frac{ML}{6EI}$$

$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{L}{3EI}$$

$$\delta_{10} + \delta_{11} X_1 = 0 \Rightarrow X_1 = -\frac{M}{2}$$



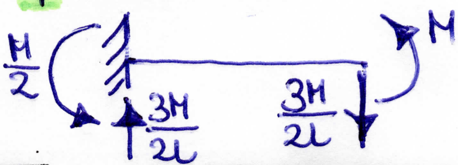
INNY UKŁAD PODSTAWOWY



$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = \frac{1}{EI}\cdot M \cdot L \cdot \frac{L}{2} = \frac{ML^2}{2EI}$$

$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{L^3}{3EI}$$

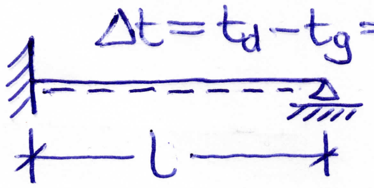
$$\delta_{10} + \delta_{11} X_1 = 0 \Rightarrow X_1 = -\frac{3M}{2L}$$



schemat statycznie równoważny

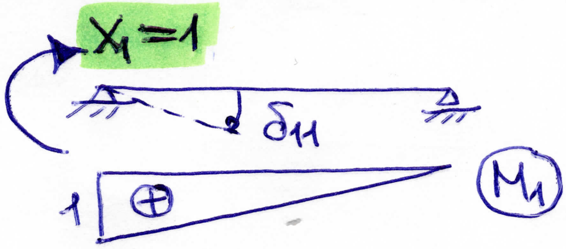
---

* $\Delta t = t_d - t_g = \text{const}$



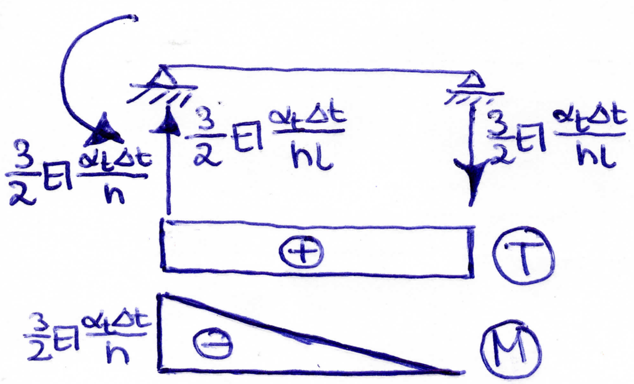
DANE: $\alpha_t, \Delta t, h, EI = \text{const}$

UKŁAD PODSTAWOWY METODY SIŁ, STAN $X_1 = 1$



$$\left.\begin{aligned} \delta_{10} &= \frac{\alpha_t \Delta t}{h}\int_L M_1 ds = \frac{\alpha_t \Delta t}{h}\cdot\frac{L}{2} \\ \delta_{11} &= \int_L \frac{M_1 M_1}{EI} ds = \frac{L}{3EI} \end{aligned}\right\}\quad \begin{aligned} &\delta_{10} + \delta_{11} X_1 = 0 \\ &X_1 = -\frac{3}{2} EI \frac{\alpha_t \Delta t}{h} \end{aligned}$$

!

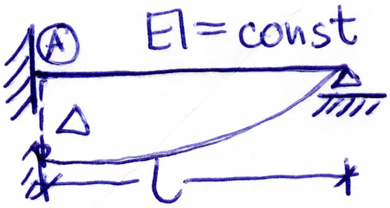


Reakcje i siły wewnętrzne pod działaniem pozastatycznym są niezerowe, gdyż UKŁAD STATYCZNIE NIEWYZNACZALNY

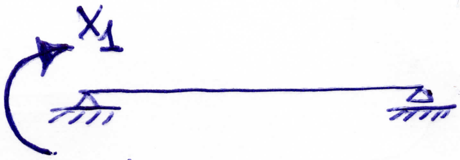
Reakcje i siły wewnętrzne są proporcjonalne do sztywności giętnej EI.

*

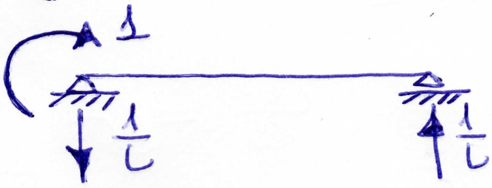
$n_s = 1$



UKŁAD PODSTAWOWY METODY SIŁ

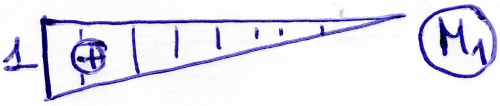


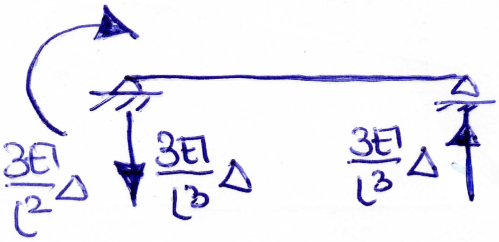
stan $X_1 = 1$

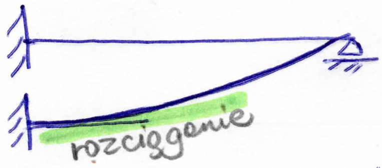


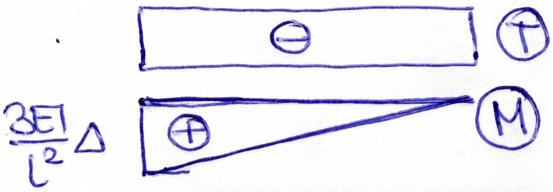
$$\delta_{10} = -\Delta \cdot \frac{1}{L}$$

$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{L}{3EI} \Rightarrow X_1 = \frac{3EI}{L^2}\Delta$$









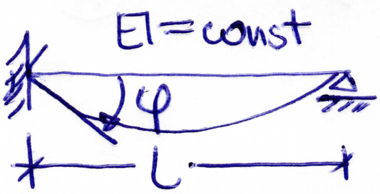
UKŁAD STATYCZNIE NIEWYZNACZALNY

także i w tym przypadku działanie pozastatyczne (osiadanie podpory) wywołuje reakcje i siły wewnętrzne, proporcjonalne do EI

---

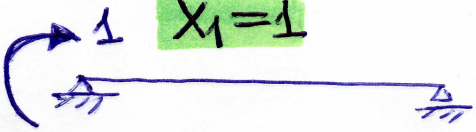
*

$n_s = 1$



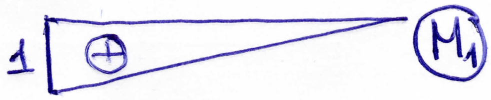
UKŁAD PODSTAWOWY METODY SIŁ

$X_1 = 1$

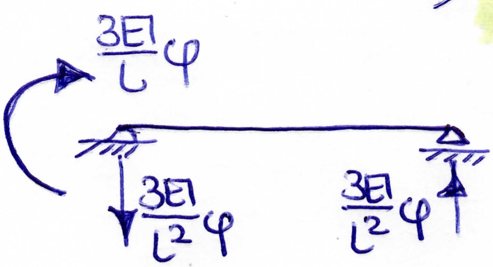


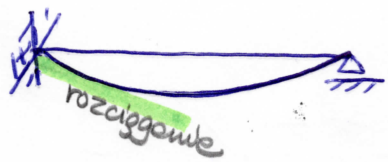
$$\delta_{10} = -\varphi \cdot 1$$ (ODPOWIEDNIKIEM MOMENTU UTWIERDZENIA JEST $X_1 = 1$)

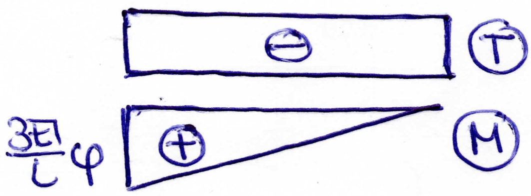
$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{L}{3EI}$$



$$\Rightarrow X_1 = \frac{3EI}{L}\varphi$$

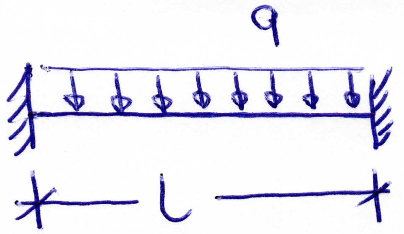






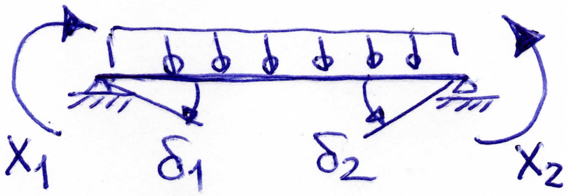
UKŁAD STATYCZNIE NIEWYZNACZALNY, działanie pozastatyczne (wychylenie kątowe podpory) – powstają reakcje i siły wewnętrzne proporcjonalne do sztywności giętnej EI

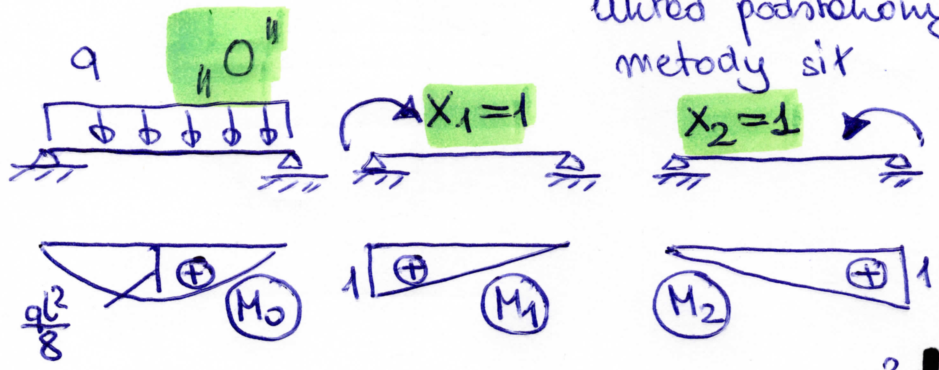
*



$n_s = 3$, jednak ze względu na brak sił osiowych zostają one pominięte – efektywnie $n_s = 2$

układ podstawowy metody sił





$$\delta_{10} = \delta_{20} = \frac{1}{EI}\cdot\frac{2}{3}\cdot L\cdot\frac{qL^2}{8}\cdot\frac{1}{2} = \frac{qL^3}{24EI}$$

$$\delta_1 = \delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \Rightarrow 2X_1 + X_2 = -\frac{qL^2}{4}$$

$$\delta_2 = \delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \Rightarrow X_1 + 2X_2 = -\frac{qL^2}{4}$$

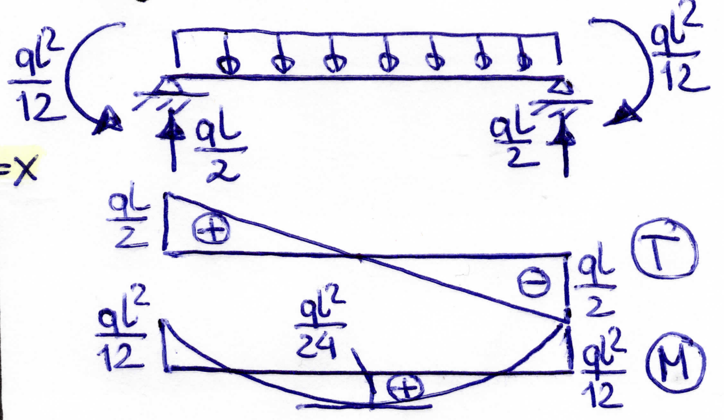
$$\delta_{11} = \delta_{22} = \frac{L}{3EI}$$

$$\delta_{12} = \delta_{21} = \frac{L}{6EI}$$

ROZWIĄZANIE: $X_1 = X_2 = -\frac{qL^2}{12}$

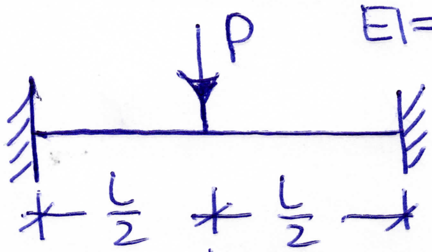
symetria $\rightarrow X_1 = X_2$

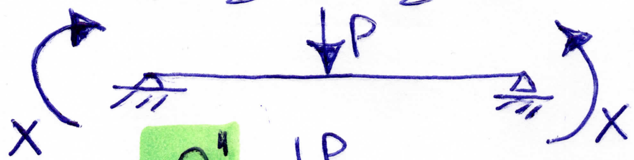
można przyjąć tzw. grupowe nadliczbowe $X_1 = X_2 = X$



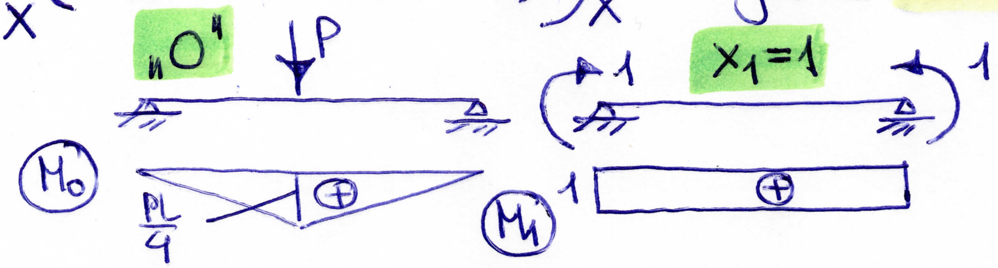
*

$EI = \mathrm{const}$





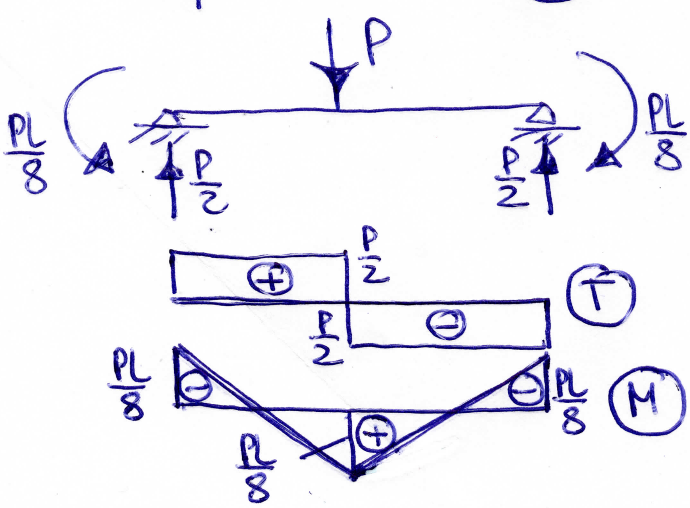
$n_s = 2$ (realnie), symetria $\rightarrow$ jedna nadliczbowa grupowa $X_1 = X_2 = X$



$$\delta_{10} = \frac{1}{EI}\cdot\frac{1}{2}\cdot L\cdot\frac{PL}{4}\cdot 1 = \frac{PL^2}{8EI}$$

$$\delta_{11} = \frac{1}{EI}\cdot 1\cdot L\cdot 1 = \frac{L}{EI}$$

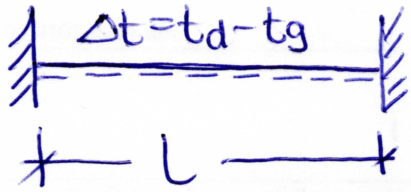
$$X_1 = -\frac{PL}{8}$$



W obu powyższych przykładach symetria układu i obciążenia:

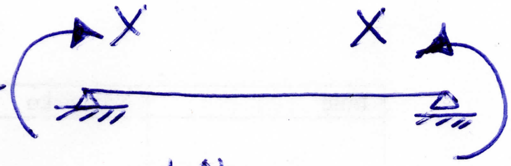
- symetryczne układy reakcji i wykresy (M)
- antysymetryczne wykresy (T)

*

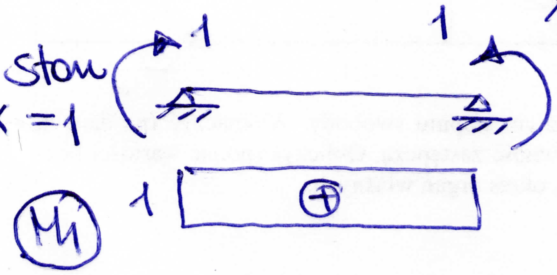


dane: $EI, \alpha_t, \Delta t, h, L$

układ podstawowy, grupowe nadliczbowe $X_1 = X_2 = X$



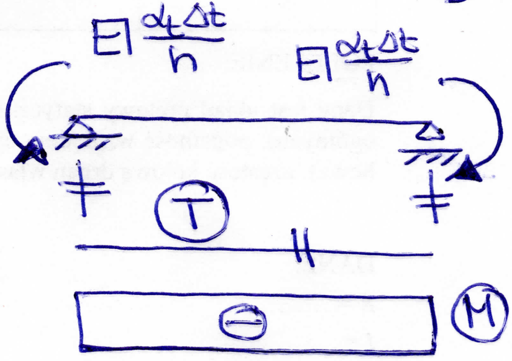
stan $X=1$



$$\delta_{10} = \frac{\alpha_t \Delta t}{h} \cdot 1 \cdot L$$

$$\delta_{11} = \frac{L}{EI}$$

stąd $X = -EI\frac{\alpha_t \Delta t}{h}$



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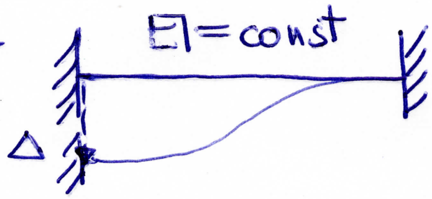
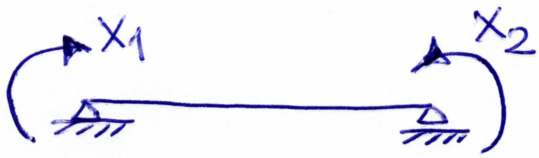
*



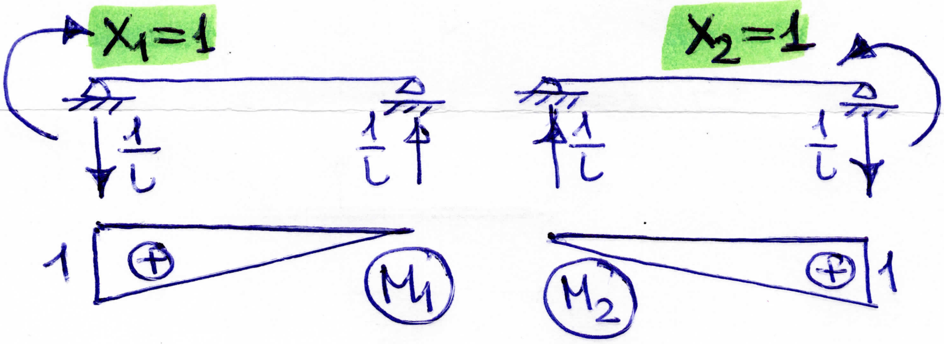
$n_S = 2$ (realnie)

układ podstawowy metody sił



brak symetrii

$X_1 = 1$ $X_2 = 1$



$$\delta_{10} = -\frac{\Delta}{L}, \quad \delta_{20} = \frac{\Delta}{L}$$

$$\delta_{11} = \delta_{22} = \frac{L}{3EI}$$

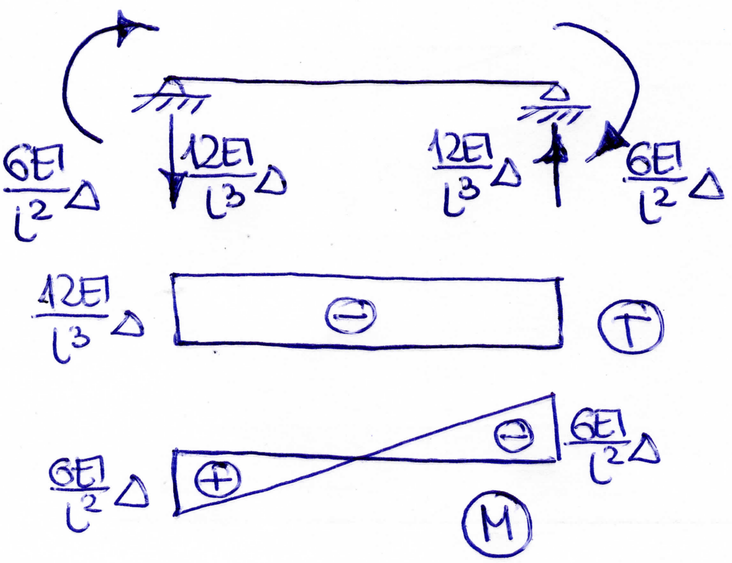
$$\delta_{12} = \delta_{21} = \frac{L}{6EI}$$

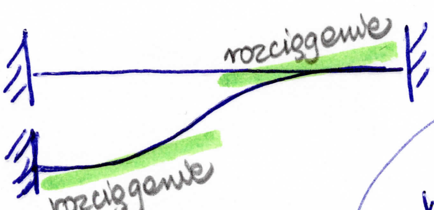
$$\delta_1 = \delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \Rightarrow 2X_1 + X_2 = \frac{6EI}{L^2}\Delta$$

$$\delta_2 = \delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \Rightarrow X_1 + 2X_2 = -\frac{6EI}{L^2}\Delta$$

ROZWIĄZANIE: $X_1 = \frac{6EI}{L^2}\Delta$

$X_2 = -\frac{6EI}{L^2}\Delta$





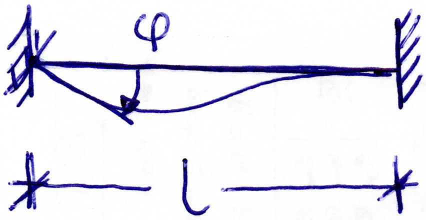
$W = 4 - 1 = 3$

$$W_1 = \frac{6EI}{L^2}\Delta \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = \frac{18EI}{L^2}\Delta$$

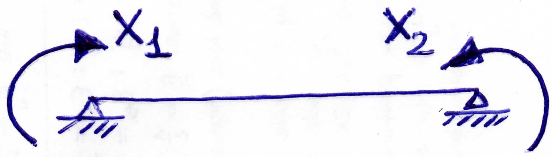
$$W_2 = \frac{6EI}{L^2}\Delta \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -\frac{18EI}{L^2}\Delta$$

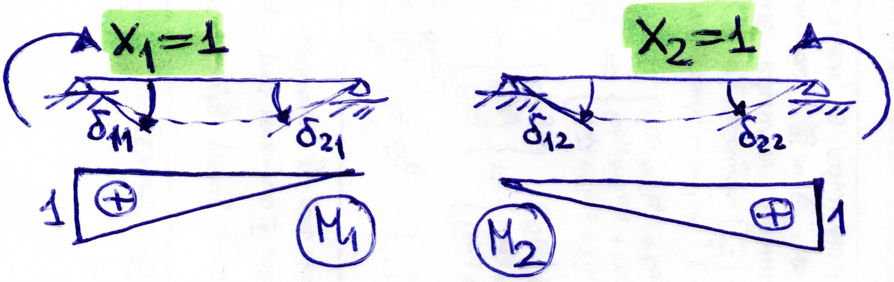
$X_1 = \frac{6EI}{L^2}\Delta \qquad X_2 = -\frac{6EI}{L^2}\Delta$

* $EI = const$ $n_s = 2$ (ołtyme) ĆW. 6/7



układ podstawowy metody sił





$\delta_{10} = -\varphi \cdot 1$ (nadliczbowa $X_1 = 1$)

$\delta_{20} = 0$

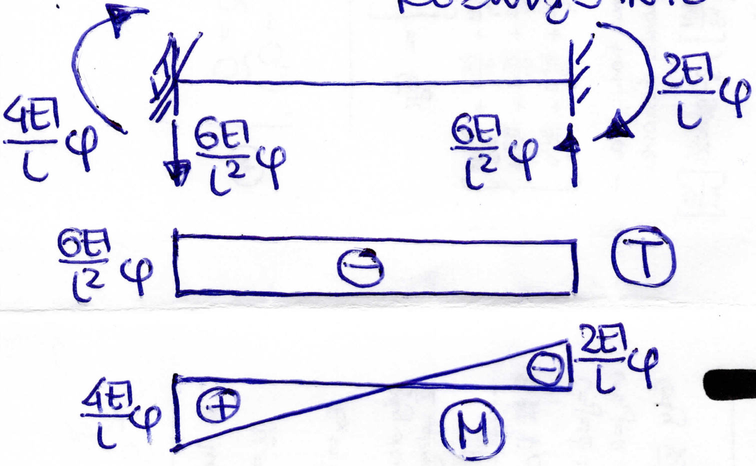
$\delta_{11} = \delta_{22} = \frac{L}{3EI}$

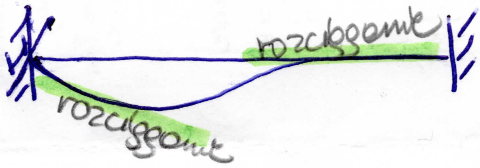
$\delta_{12} = \delta_{21} = \frac{L}{6EI}$

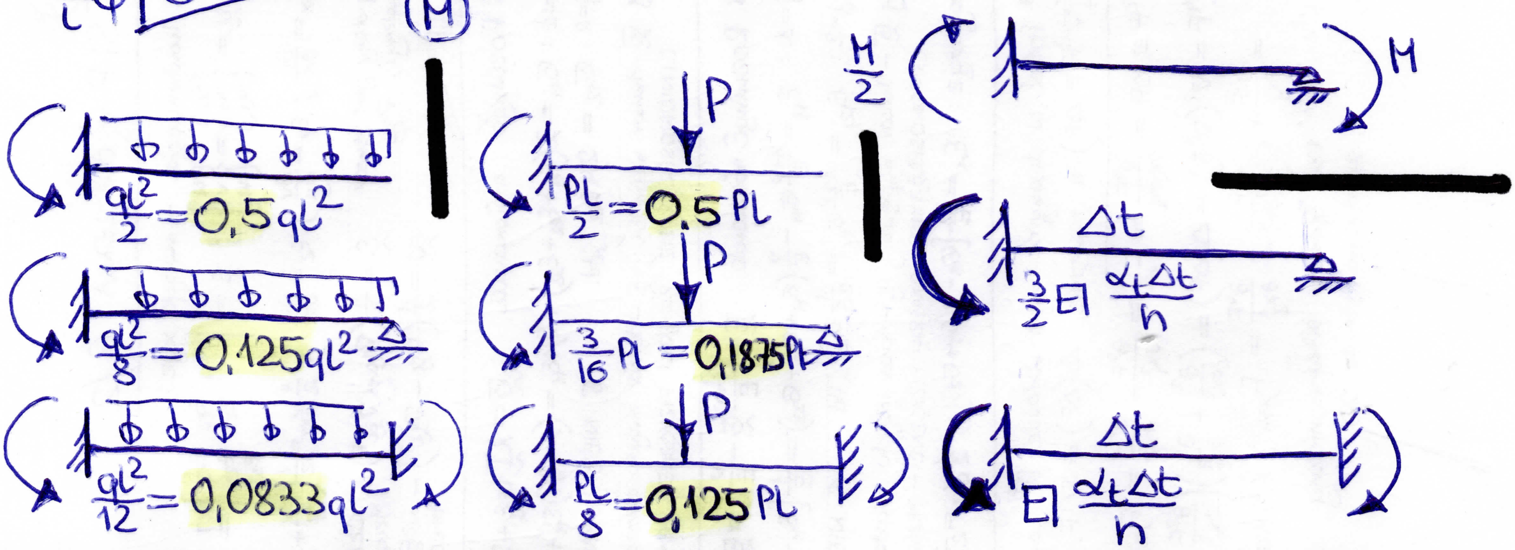
$\delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0 \Rightarrow 2X_1 + X_2 = \frac{6EI}{L}\varphi$

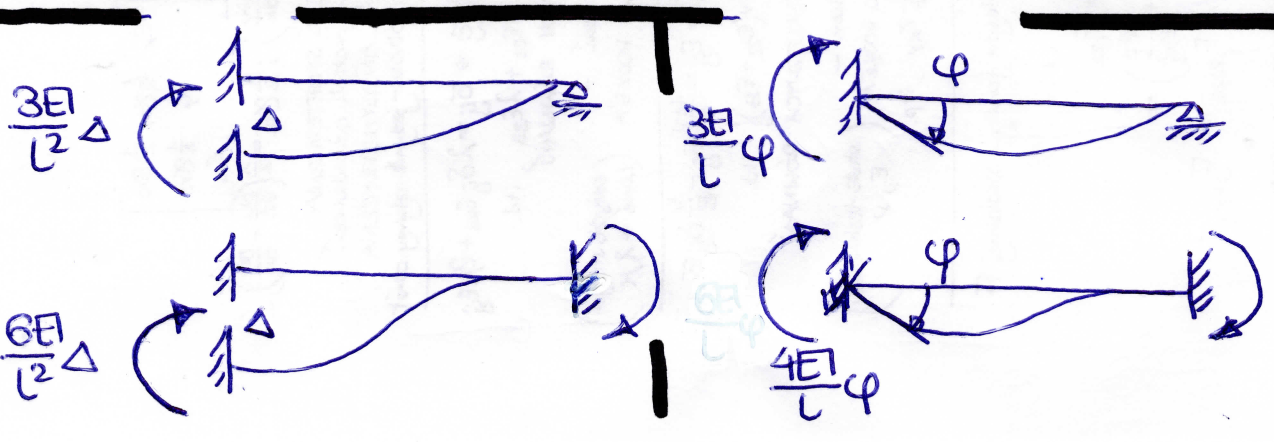
$\delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0 \Rightarrow X_1 + 2X_2 = 0$

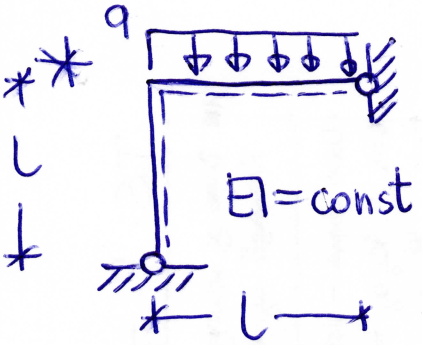
ROZWIĄZANIE: $X_1 = \frac{4EI}{L}\varphi$, $X_2 = -\frac{2EI}{L}\varphi$

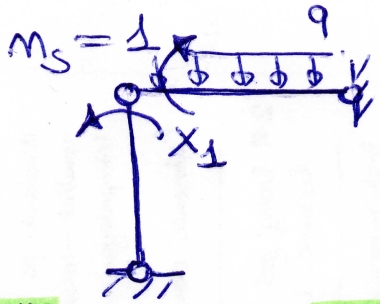






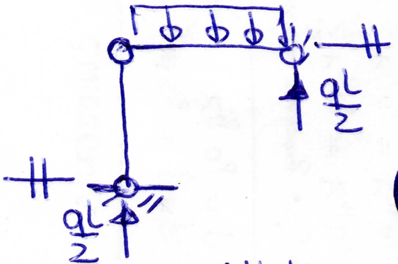


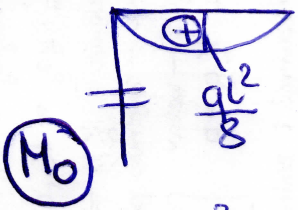




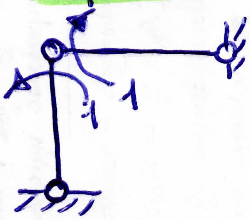
układ podstawowy metody sił

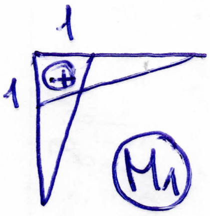
„0"





$X_1 = 1$

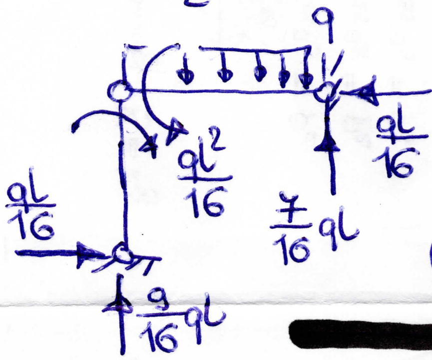


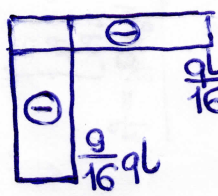


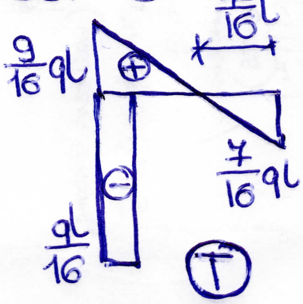
$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = \frac{1}{EI} \cdot \frac{2}{3} \cdot L \cdot \frac{qL^2}{8} \cdot \frac{1}{2} \cdot 1 = \frac{qL^3}{24EI}$$

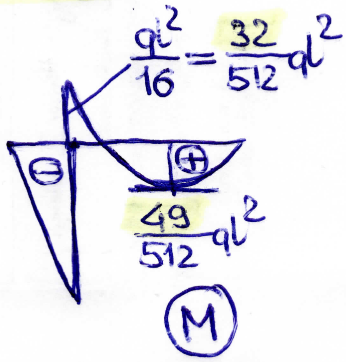
$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = 2 \cdot \frac{1}{EI} \cdot \frac{1}{2} \cdot L \cdot 1 \cdot \frac{2}{3} \cdot 1 = \frac{2L}{3EI}$$

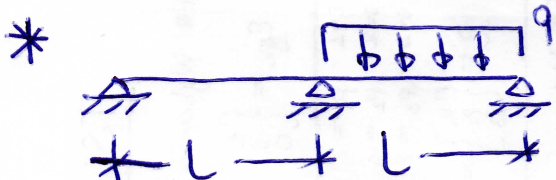
$$X_1 = -\frac{qL^2}{16}$$



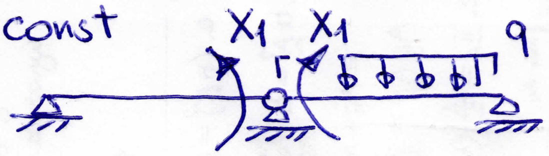






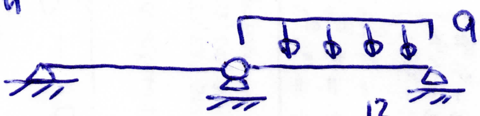


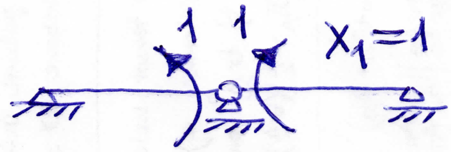
EI=const



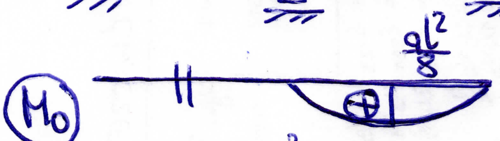
układ podstawowy metody sił

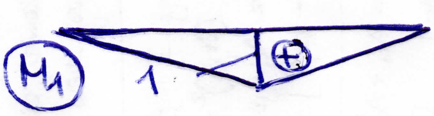
„0"



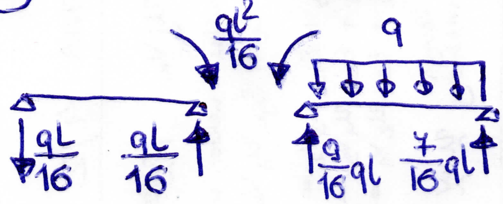


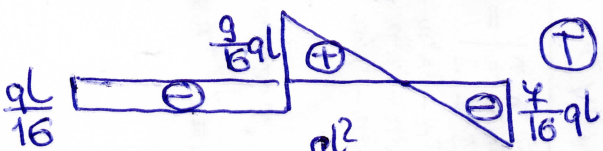
$$\delta_{10} = \frac{qL^3}{24EI} \qquad \delta_{11} = \frac{2L}{3EI}$$

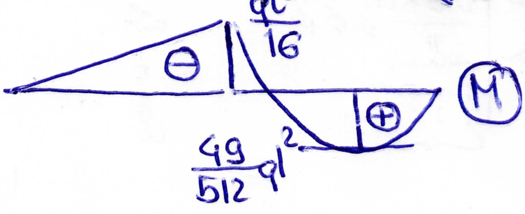




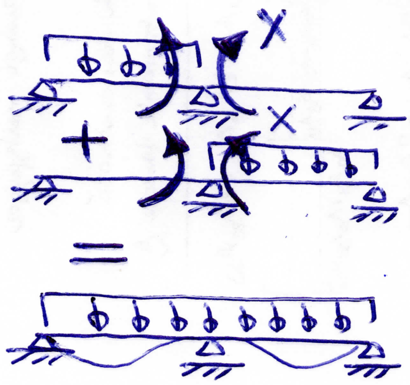
$$X_1 = -\frac{qL^2}{16}$$







SPOSTRZEŻENIE: poza wykresem N rozwiązania ramy i belki identyczne



$$X = -\frac{qL^2}{16}$$