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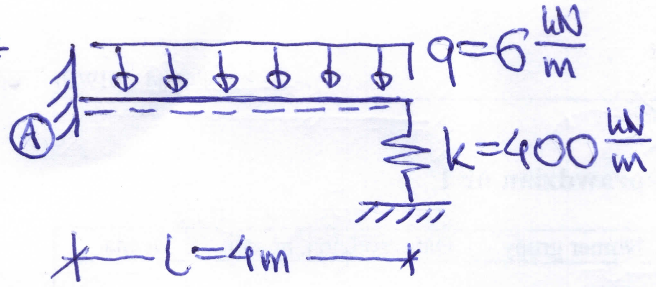


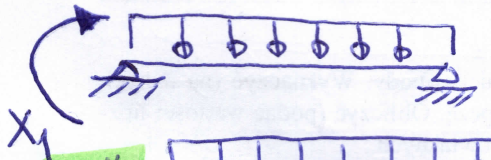
$EI=10^3\,kNm^2=const$ Ćw. 7/1

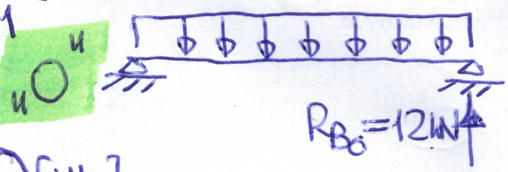
porównać wartość M_A z wariantami:

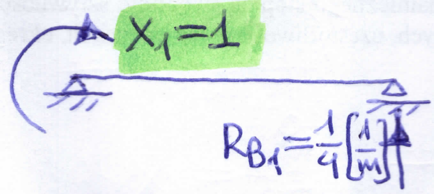
a. $k\to\infty$ (podpora stała)

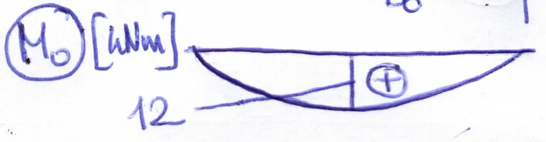
b. $k=0$ (brak podpory – belka wspornikowa)

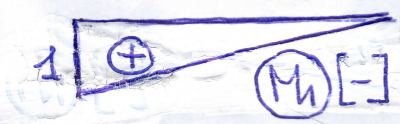
$n_s=1$ układ podstawowy metody sił (z podporą stałą)







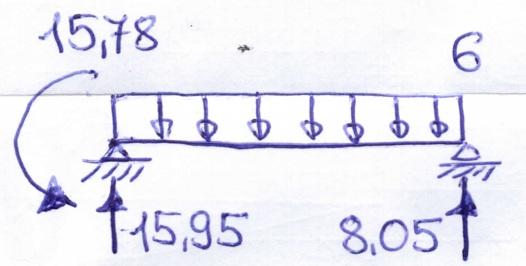


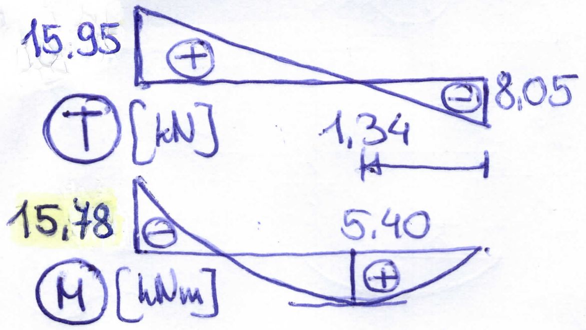


$$\delta_{10}=\int_L\frac{M_0M_1}{EI}ds+\frac{1}{k}R_{B_0}R_{B_1}=\frac{1}{10^3}\cdot\frac{2}{3}\cdot4\cdot12\cdot\frac{1}{2}\cdot1+\frac{12\cdot0{,}25}{400}=0{,}0235\,[-]$$

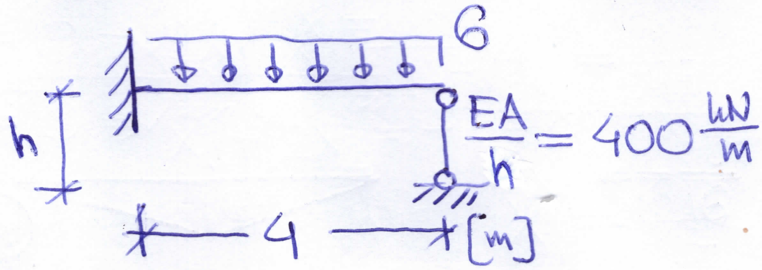
$$\delta_{11}=\int_L\frac{M_1M_1}{EI}ds+\frac{1}{k}R_{B_1}R_{B_1}=\frac{1}{10^3}\cdot\frac{1}{3}\cdot4\cdot1\cdot1+\frac{0{,}25\cdot0{,}25}{400}=\frac{1}{671{,}33}\left[\frac{1}{kNm}\right]$$

$$\delta_{10}+\delta_{11}X_1=0\Rightarrow X_1=-15{,}78\,kNm$$





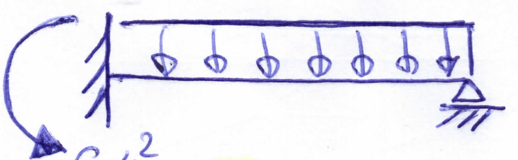
Powyższe zadanie można rozwiązać przyjmując w odniesieniu do podpory sprężystej model pręta kratowego



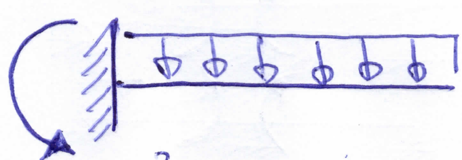
Przypadki ekstremalne:

a) $k\to\infty$ – podpora stała

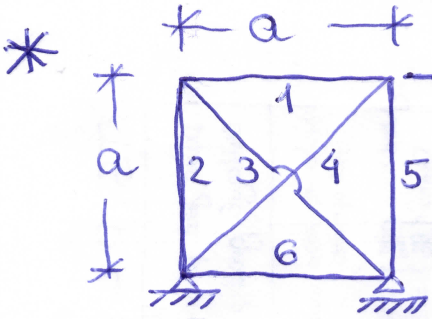
b) $k=0$ – brak podpory



$$\frac{6\cdot4^2}{8}=12\,kNm$$

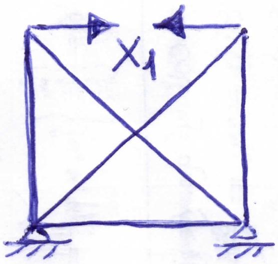


$$\frac{6\cdot4^2}{2}=48\,kNm$$

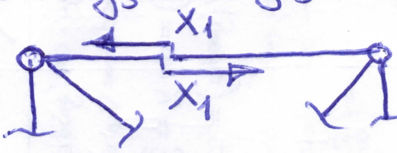


$EA = \text{const}$ $\quad n_s = 1$

układ podstawowy metody sił

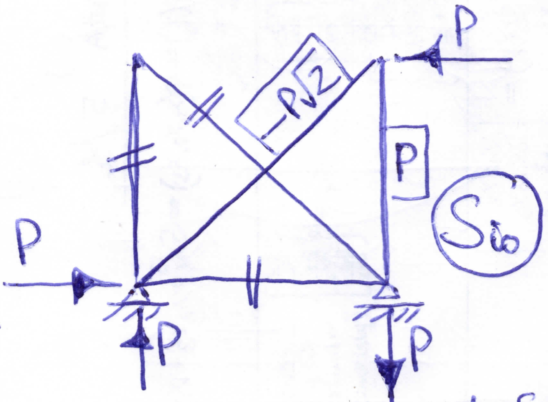


faktycznie – pręt rozcięty, pozostają obie jego części

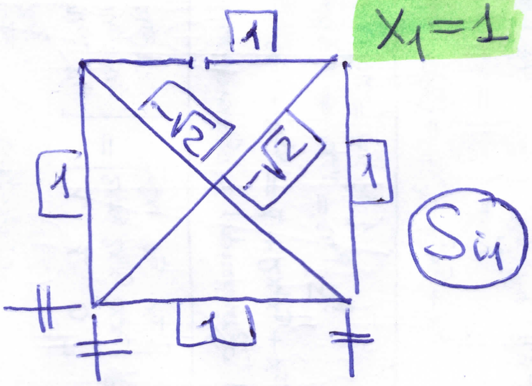


stan „0”

układ wewnętrznie statycznie niewyznaczalny



$X_1 = 1$

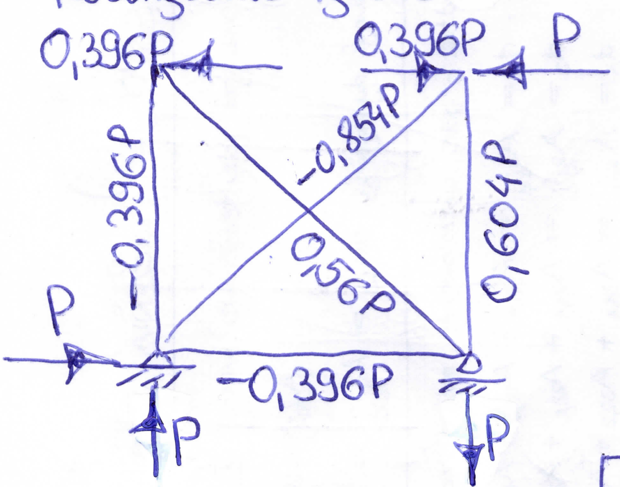


$$\delta_{10} = \sum \frac{S_{i0} S_{i1}}{EA_i} l_i = \frac{1}{EA}\left[1 \cdot P \cdot a + P\sqrt{2} \cdot \sqrt{2} \cdot a\sqrt{2}\right] = \frac{Pa}{EA}\left(1 + 2\sqrt{2}\right) \quad 3{,}8284$$

$$\delta_{11} = \sum \frac{S_{i1} S_{i1}}{EA_i} l_i = \frac{1}{EA}\left[4 \cdot 1 \cdot 1 \cdot a + 2 \cdot \sqrt{2} \cdot \sqrt{2} \cdot a\sqrt{2}\right] = \frac{a}{EA}\left(4 + 4\sqrt{2}\right) \quad 9{,}6568$$

stąd $X_1 = -0{,}396P$

Rozwiązanie końcowe



można drogą superpozycji: $S_i = S_{i0} + S_{i1} X_1$

$S_1 = X_1 = -0{,}396P$ (na wys. obok niewidoczne)

$S_2 = -0{,}396P$

$S_3 = 0{,}56P$

$S_4 = -0{,}854P$

$S_5 = 0{,}604P$

$S_6 = -0{,}396P$

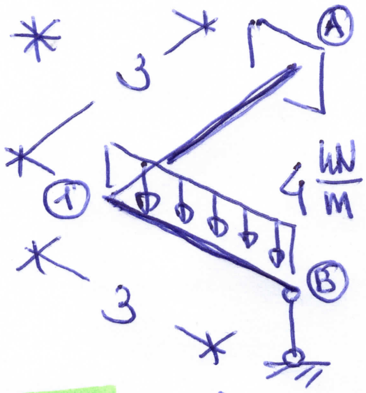
lub w formie tabeli:

i	S_{i0}	S_{i1}	$S_{i0} S_{i1} l_i$	$S_{i1} S_{i1} l_i$	S_i
1					
2					
3					
4					
5					
6					
			δ_{10}	δ_{11}	(z dzielnikiem EA)

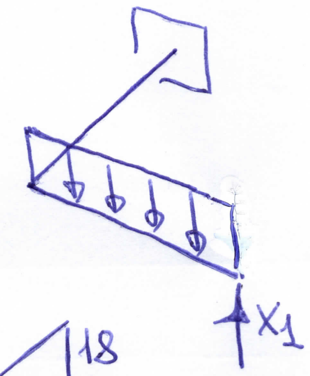
projektowo – ze względu na nośność na ściskanie i stateczność → PRĘT NR 4

ze względu na nośność na rozciąganie → PRĘT NR 5

$EI = 8000\ \text{kNm}^2 \qquad GI_0 = 6000\ \text{kNm}^2$ Ćw. 7/3

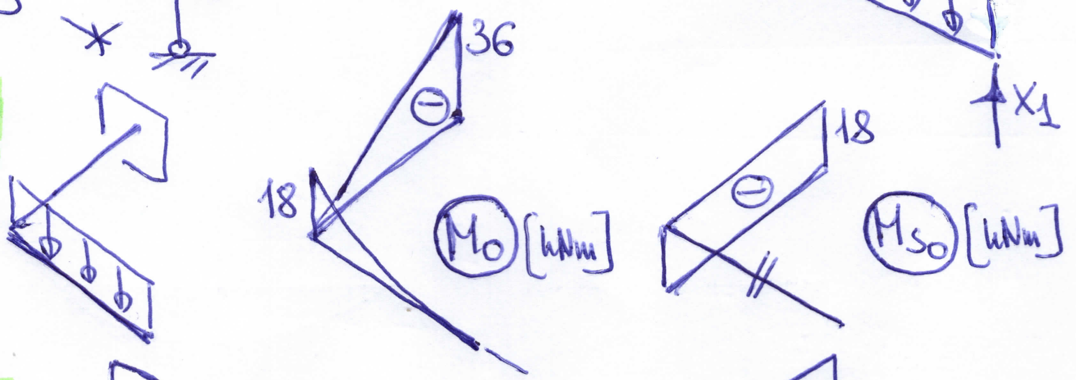


układ podstawowy metody sił
$n_s = 1$

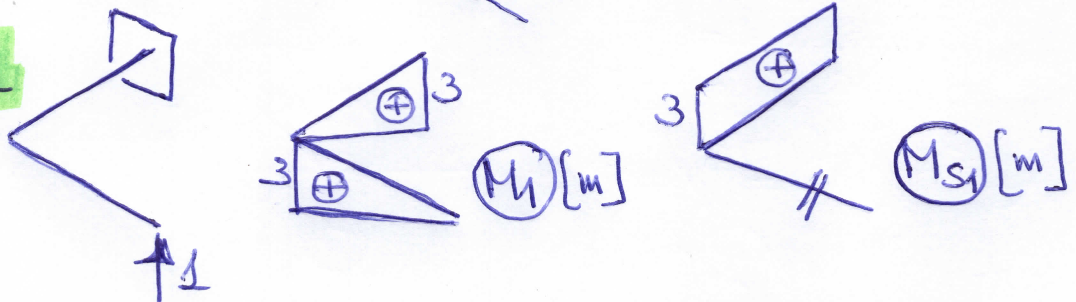


DŹWIGAR ZAŁAMANY W PLANIE

„0"



$X_1 = 1$

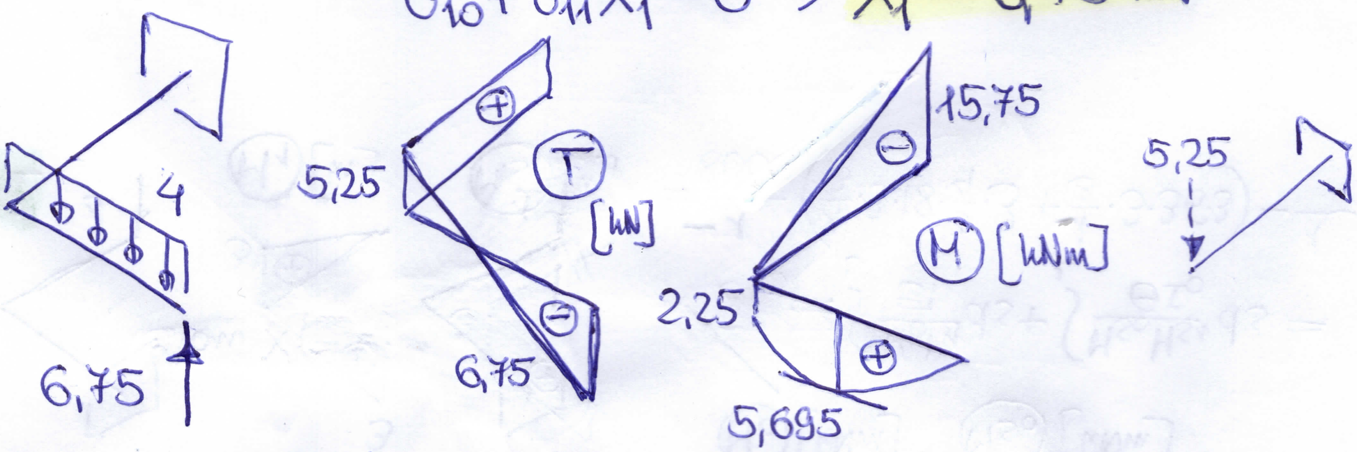


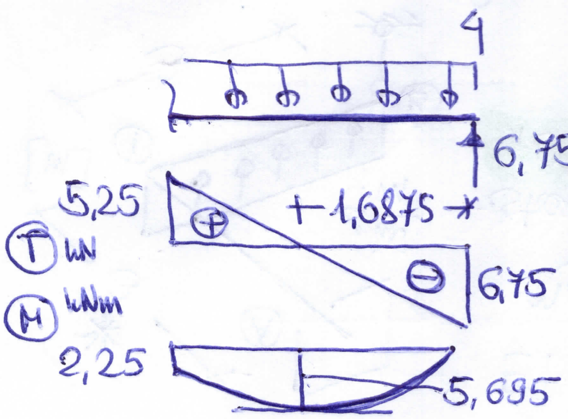
$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds + \int_L \frac{M_{s0} M_{s1}}{GI_0} ds = \frac{-1}{8000}\left(\frac{1}{3}\cdot 3\cdot 18\cdot \frac{3}{4}\cdot 3 + \frac{1}{2}\cdot 3\cdot 36\cdot \frac{2}{3}\cdot 3\right) - \frac{1}{6000}\cdot 3\cdot 18\cdot 3 =$$

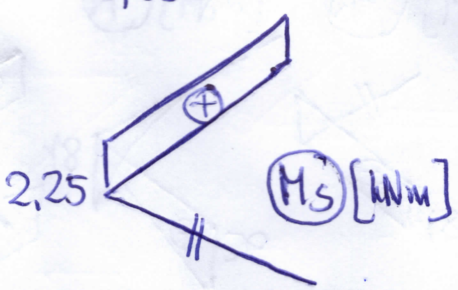
$$= -0{,}04556\ \text{m}$$

$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds + \int_L \frac{M_{s1} M_{s1}}{GI_0} ds = \frac{1}{8000}\cdot 2\cdot \frac{1}{3}\cdot 3\cdot 3\cdot 3 + \frac{1}{6000}\cdot 3\cdot 3\cdot 3 = 0{,}00675\ \frac{\text{m}}{\text{kN}}$$

$$\delta_{10} + \delta_{11} X_1 = 0 \Rightarrow X_1 = 6{,}75\ \text{kN}$$







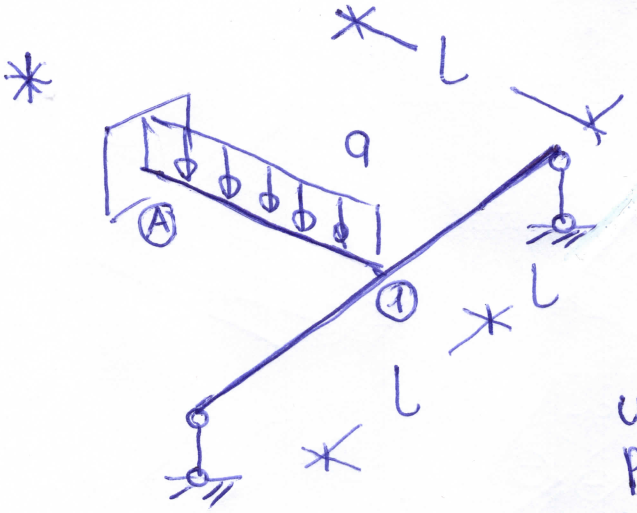
A-1 $M_s = 6{,}75\cdot 3 - 12\cdot \frac{3}{2} = 2{,}25\ \text{kNm}$

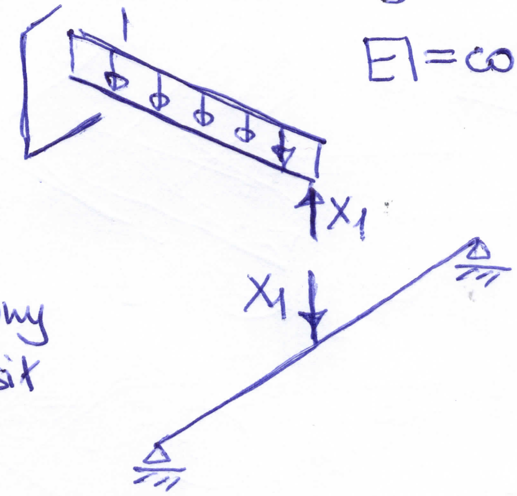
RUSZT BELKOWY PRZEGUBOWY

ĆW. 7/4

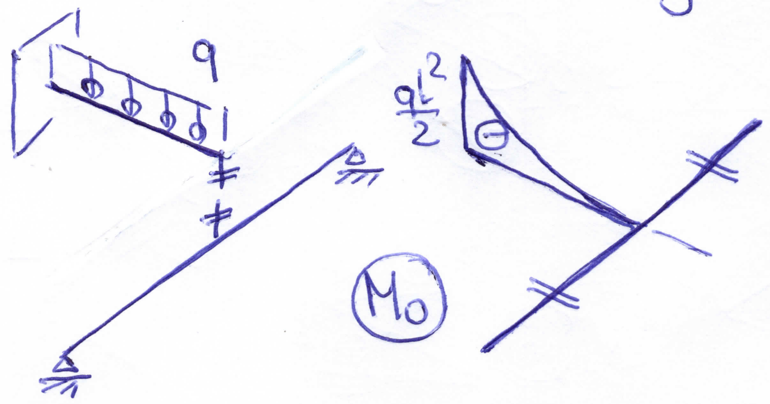
$n_s = 1$

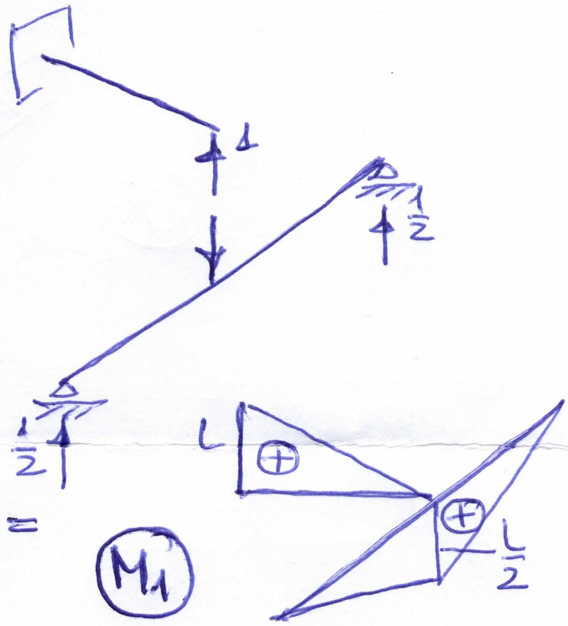
$EI = \mathrm{const}$





układ podstawowy metody sił

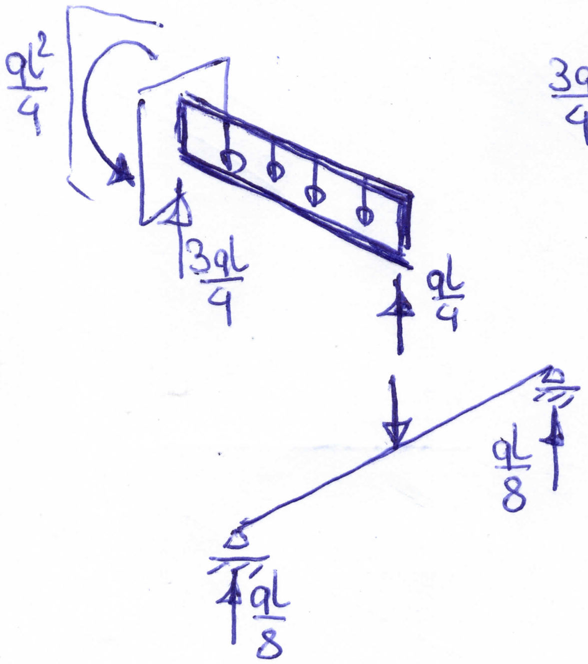


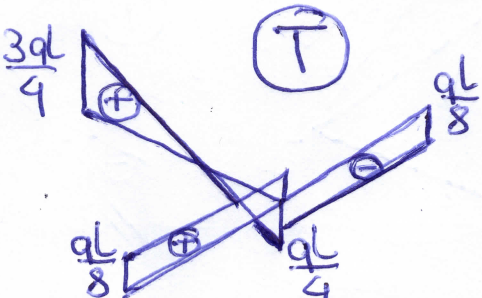


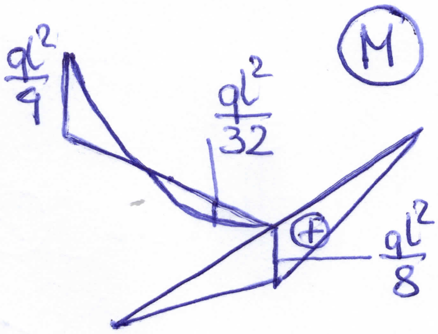
$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = \frac{-1}{EI} \cdot \frac{1}{3} \cdot L \cdot \frac{qL^2}{2} \cdot \frac{3}{4} \cdot L = -\frac{qL^4}{8EI}$$

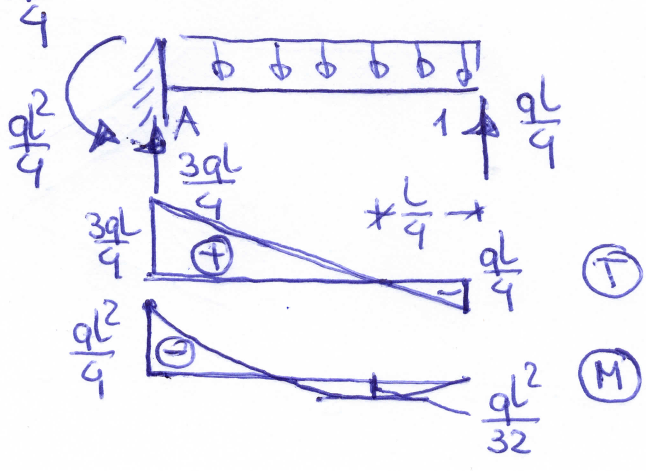
$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{1}{EI}\left(\frac{1}{3} \cdot L \cdot L \cdot L + 2 \cdot \frac{1}{3} \cdot L \cdot \frac{L}{2} \cdot \frac{L}{2}\right) = \frac{L^3}{2EI}$$

$$\delta_{10} + \delta_{11} X_1 = 0 \Rightarrow X_1 = \frac{qL}{4}$$

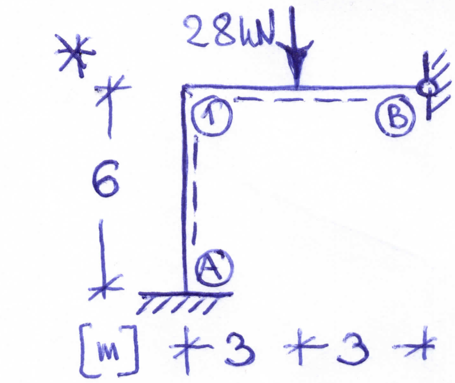






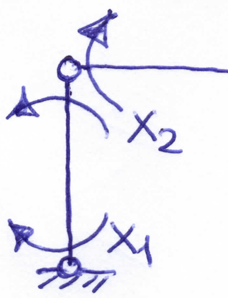


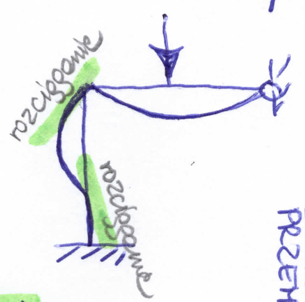
ćw. 7/5



$n_s = 2$ $\quad EI = \text{const}$

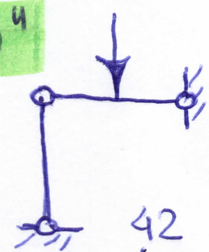
układ podstawowy metody sił

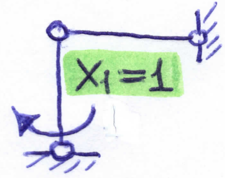


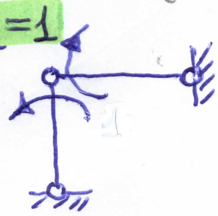


PRZEWIDYWANY STAN PRZEMIESZCZEŃ

"0"

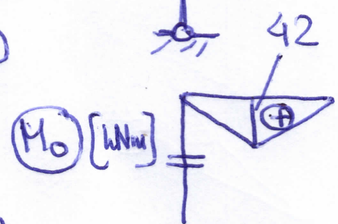


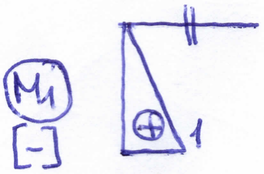


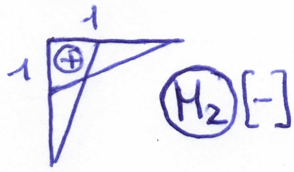


poszczególne stany w układzie podstawowym

$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds = 0$$







$$\delta_{20} = \int_L \frac{M_0 M_2}{EI} ds = \frac{1}{EI} \cdot \frac{1}{2} \cdot 6 \cdot 42 \cdot \frac{1}{2} = \frac{63}{EI}$$

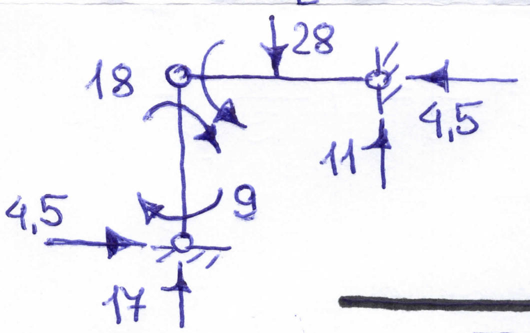
$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds = \frac{1}{EI} \cdot \frac{1}{3} \cdot 6 \cdot 1 \cdot 1 = \frac{2}{EI}$$

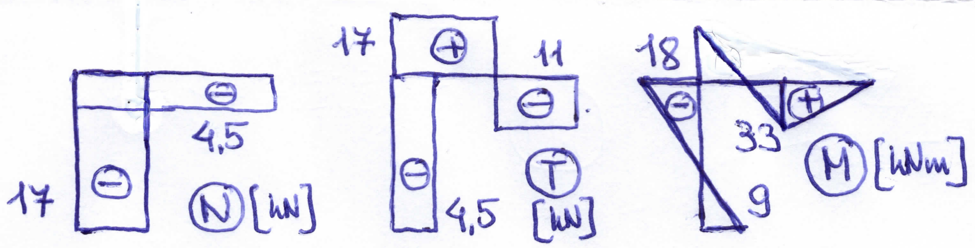
$$\delta_{22} = \int_L \frac{M_2 M_2}{EI} ds = \frac{4}{EI}$$

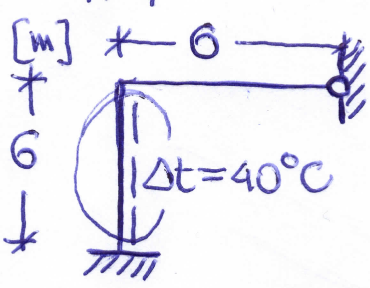
$$\delta_{12} = \delta_{21} = \int_L \frac{M_1 M_2}{EI} ds = \frac{1}{EI}$$

$$\begin{cases} \delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0 \Rightarrow 2X_1 + X_2 = 0 \\ \delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0 \Rightarrow X_1 + 4X_2 = -63 \end{cases}$$

Rozwiązanie: $X_1 = 9\,\text{kNm}$, $X_2 = -18\,\text{kNm}$







$EI = 10^4\,\text{kNm}^2 \qquad \alpha_t = 10^{-5} \frac{1}{°C} \qquad h = 0{,}4\,\text{m}$

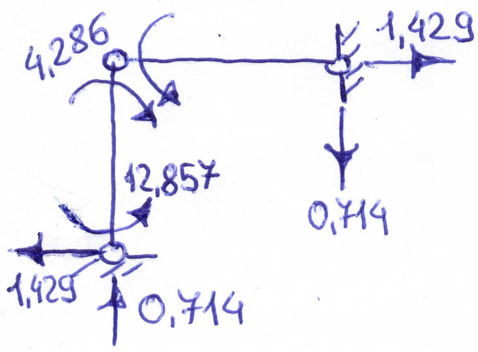
ten sam układ podstawowy → $\delta_{11}, \delta_{22}, \delta_{12} = \delta_{21}$ bez zmian

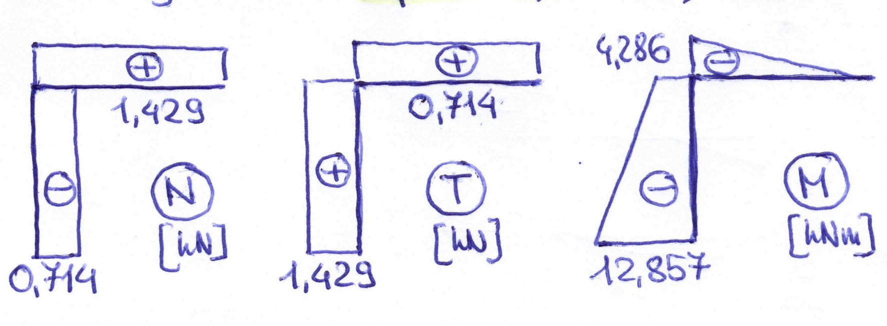
$$\delta_{10} = \int_L M_1 \frac{\alpha_t \Delta t}{h} ds = \frac{10^{-5} \cdot 40}{0{,}4} \cdot \frac{1}{2} \cdot 6 \cdot 1 = 0{,}003\ [\text{rad}]$$

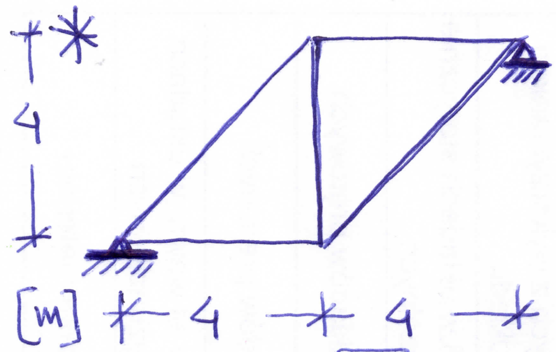
$$\delta_{20} = \int_L M_2 \frac{\alpha_t \Delta t}{h} ds = 0{,}003\ [\text{rad}]$$

$$\begin{cases} \delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0 \Rightarrow 2X_1 + X_2 = -0{,}003 EI = -30 \\ \delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0 \Rightarrow X_1 + 4X_2 = -0{,}003 EI = -30 \end{cases}$$

rozwiązanie: $X_1 = -12{,}857\,\text{kN}$, $X_2 = -4{,}286\,\text{kNm}$







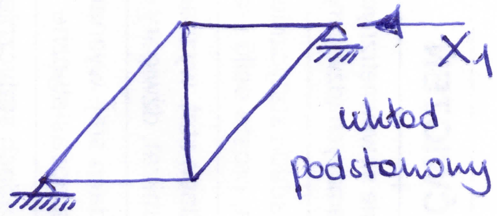
obwodowe pręty → $t_0 = 40K$ ĆW. 7/6

względem temperatury montażu

dane: $\alpha_t = 10^{-5}\ \frac{1}{K}$, $EA = 10^5\ kN = const$

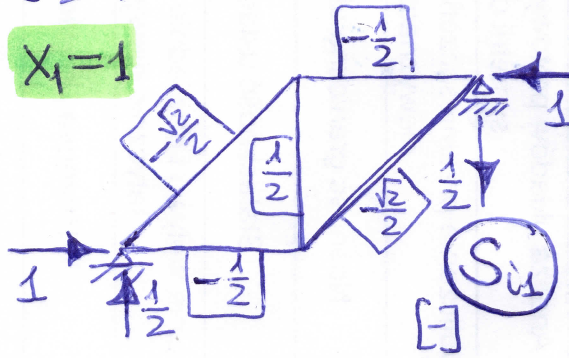
$n_s = 1$

układ zewnętrznie statycznie niewyznaczalny



układ podstawowy

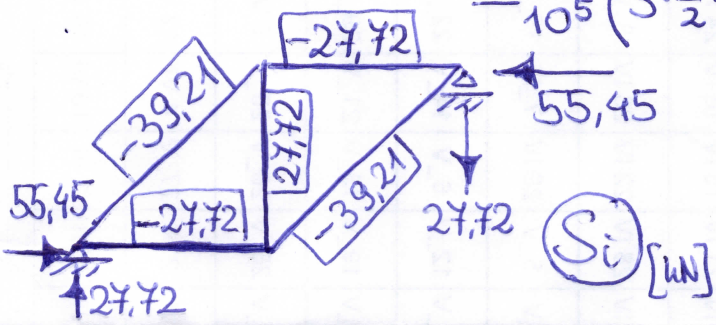
$X_1 = 1$



$$\delta_{10} = \sum S_{i1}\alpha_t t_{0i} l_i =$$
$$= -10^{-5}\cdot 40\cdot 2\left(\frac{1}{2}\cdot 4 + \frac{\sqrt{2}}{2}\cdot 4\sqrt{2}\right) = -4{,}8\cdot 10^{-3}\,m$$

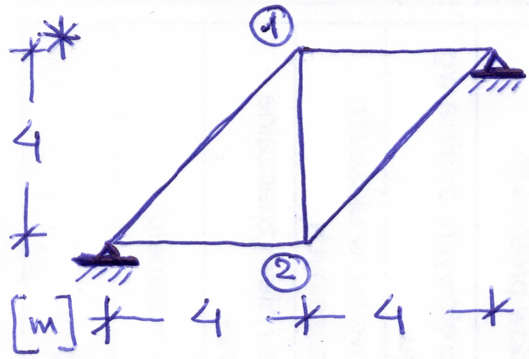
$$\delta_{11} = \sum \frac{S_{i1}S_{i1}}{EA_i} l_i =$$
$$= \frac{1}{10^5}\left(3\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 4 + 2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\cdot 4\sqrt{2}\right) = 8{,}657\cdot 10^{-5}\left[\frac{m}{kN}\right]$$

stąd $X_1 = 55{,}45\ kN$



superpozycja $S_i = \cancel{S_{i0}} + S_{i1}X_1$ →

przy oddziaływaniu pozastatycznym proporcjonalność $S_i = S_{i1}X_1$

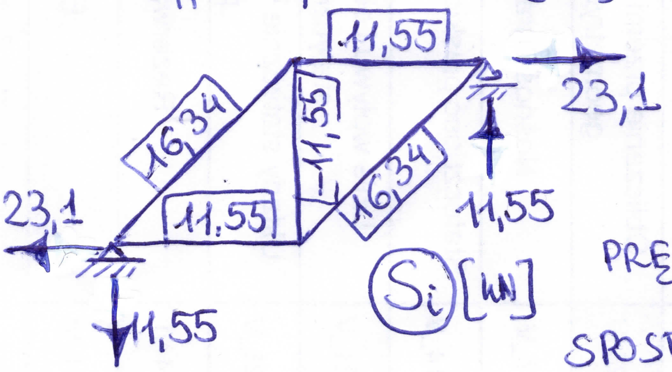


wmontowanie pręta pionowego 1-2 dłuższego o $\Delta l_{1-2} = 4\ mm$

Przyjmując układ podstawowy metody sił wg poprzedniego przykładu siły S_{i1} pozostają te same (stan $X_1 = 1$), jest $S^{(1)}_{1-2} = \frac{1}{2}$ [-]

W przypadku imperfekcji $\delta_{10} = \Delta l_{1-2}\cdot S^{(1)}_{1-2} = 0{,}002$ [m]

$\delta_{11} = 8{,}657\cdot 10^{-5}\left[\frac{m}{kN}\right]$ (bez zmian), zatem $X_1 = -23{,}1\ kN$



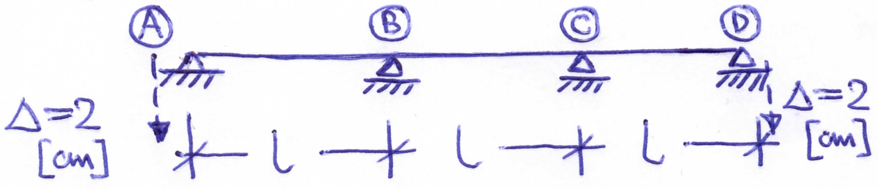
podobnie, jak w poprzednim przypadku (działanie pozastatyczne) jest

$S_i = S_{i1}\cdot X_i$ (proporcjonalność)

PRĘT WMONTOWANY JAKO DŁUŻSZY JEST ŚCISKANY, POZOSTAŁE SĄ ROZCIĄGANE

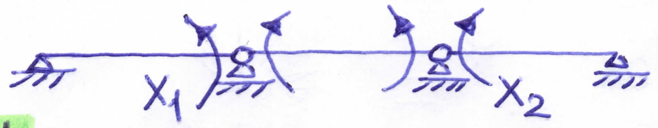
SPOSTRZEŻENIE: identyczny stan można uzyskać RÓWNOMIERNYM OGRZANIEM pręta 1-2 tak, aby swobodne wydłużenie termiczne $\Delta l_t = \alpha_t t_0 l = 4mm$ (IMPERFEKCJA)

* $L=4\,m$, $EI=8000\,kNm^2$, $\Delta=2\,cm$



$m_s=2$

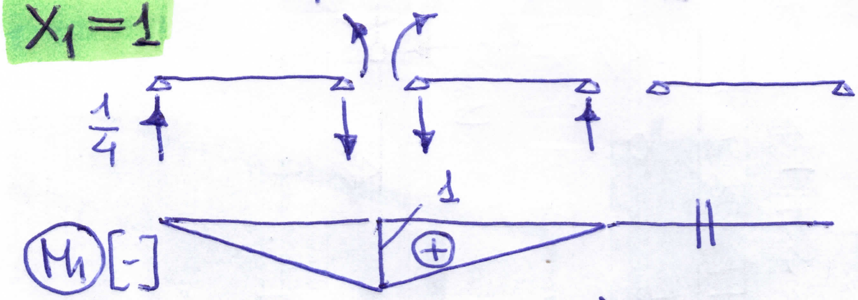
$\Delta_A=\Delta_D=-0{,}02\,m$



$\underline{X_1=1}$ $R_A^{(1)}=\frac{1}{4}\left[\frac{1}{m}\right]$, $R_D^{(1)}=0$

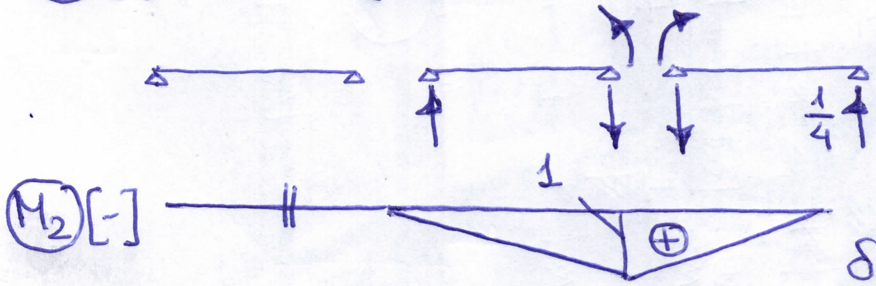
$\underline{X_2=1}$ $R_A^{(2)}=0$, $R_D^{(2)}=\frac{1}{4}\left[\frac{1}{m}\right]$

$X_1=1$



$\delta_{10}=-R_A^{(1)}\Delta_A-R_D^{(1)}\Delta_D=0{,}005\,[-]$

$\delta_{20}=-R_A^{(2)}\Delta_A-R_D^{(2)}\Delta_D=0{,}005\,[-]$



$\delta_{11}=\frac{1}{EI}\cdot 2\cdot\frac{1}{3}\cdot 4\cdot 1\cdot 1=\frac{8}{3EI}$

$\delta_{22}=\frac{1}{EI}\cdot 2\cdot\frac{1}{3}\cdot 4\cdot 1\cdot 1=\frac{8}{3EI}$

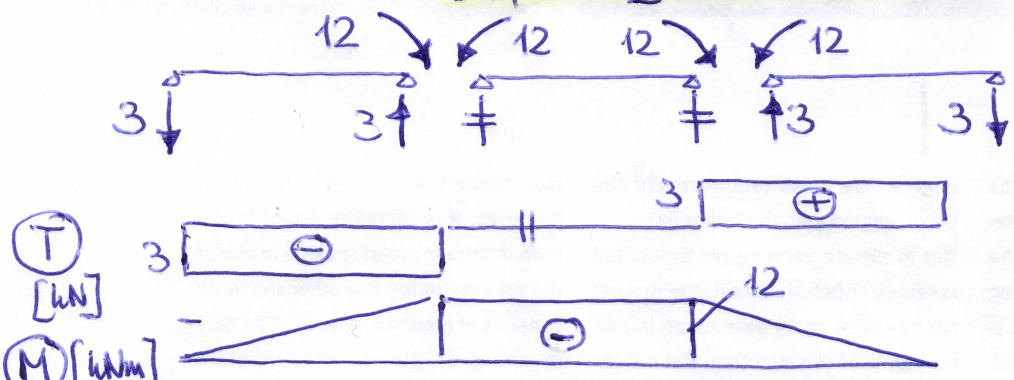
$\delta_{12}=\delta_{21}=\frac{1}{EI}\cdot\frac{1}{6}\cdot 4\cdot 1\cdot 1=\frac{2}{3EI}$

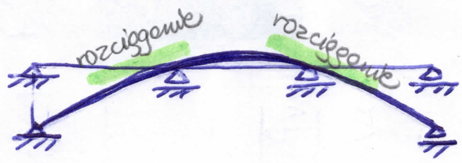
$\delta_{10}+\delta_{11}X_1+\delta_{12}X_2=0 \Rightarrow 8X_1+2X_2=-120$ $(4X_1+X_2=-60)$

$\delta_{20}+\delta_{21}X_1+\delta_{22}X_2=0 \Rightarrow 2X_1+8X_2=-120$ $(X_2+4X_1=-60)$

$W=15$ $W_1=-60\begin{vmatrix}1 & 1\\ 1 & 4\end{vmatrix}=-180$ $W_2=-60\begin{vmatrix}4 & 1\\ 1 & 1\end{vmatrix}=-180$

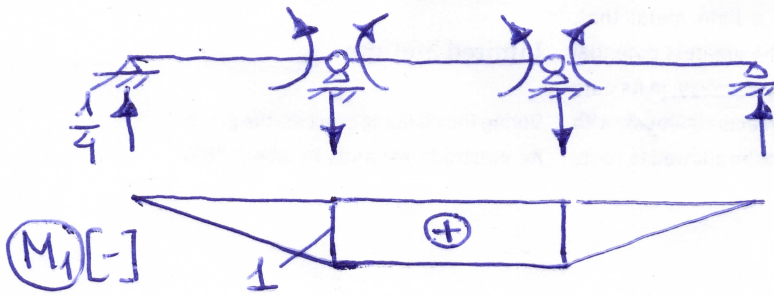
$X_1=X_2=-12\,kNm$





szkic przemieszczeń

Wykorzystanie symetrii układu i obciążenia – symetryczny wykres momentów zginających → nadliczbowa grupowa $X_1=X_2=X$



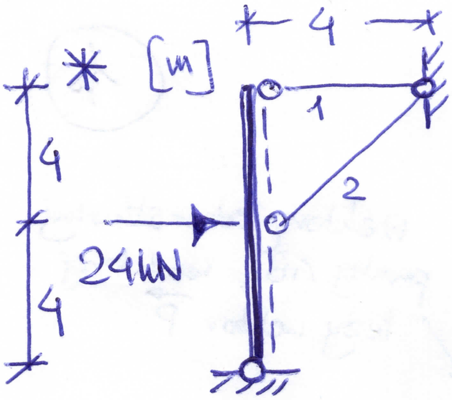
$\delta_{10}=-2(-0{,}02)\cdot\frac{1}{4}=0{,}01\,[-]$

$\delta_{11}=\frac{1}{8000}\left(2\cdot\frac{1}{3}\cdot 4\cdot 1\cdot 1+4\cdot 1\cdot 1\right)=\frac{1}{1200}$

$X_1=-12\,kNm$

dalsze rozwiązanie bez zmian

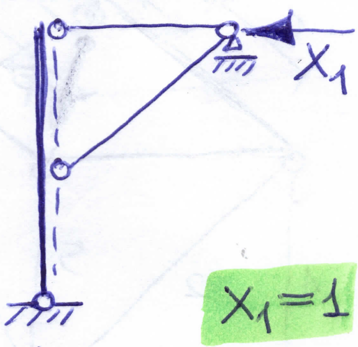
ĆW. 7/8



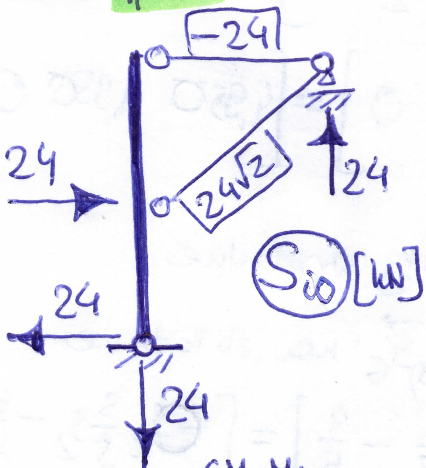
pręt ramowy $EI = 3\cdot 10^4$ kNm^2

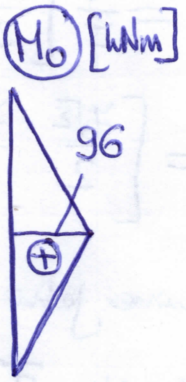
pręty kratowe $EA = 10^5$ kN

układ podstawowy metody sił:

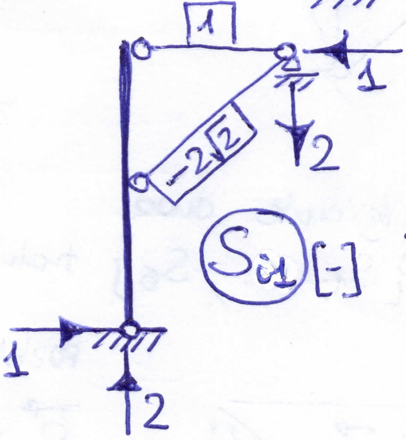


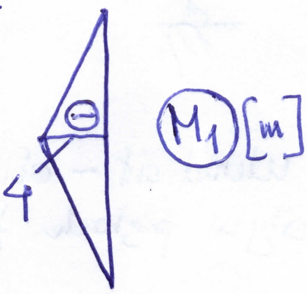
stan „0”





$X_1 = 1$





$$\delta_{10} = \int_L \frac{M_0 M_1}{EI} ds + \sum \frac{S_{i0} S_{i1}}{EA_i} l_i = -\frac{2}{30000}\cdot\frac{1}{3}\cdot 4\cdot 96\cdot 4 +$$

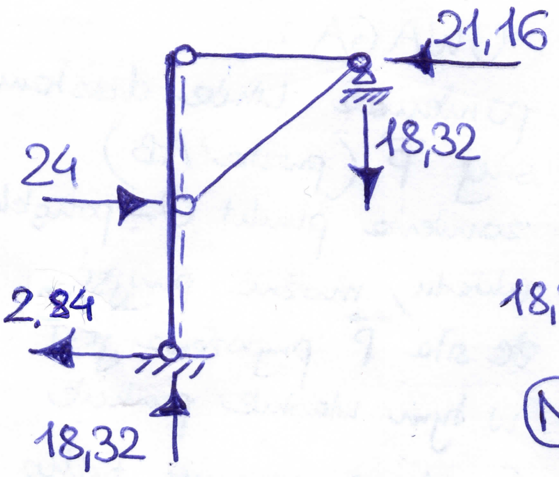
$$-\frac{1}{10^5}\left(24\cdot 1\cdot 4 + 24\sqrt{2}\cdot 2\sqrt{2}\cdot 4\sqrt{2}\right) = -0{,}03413 - 0{,}006391 =$$

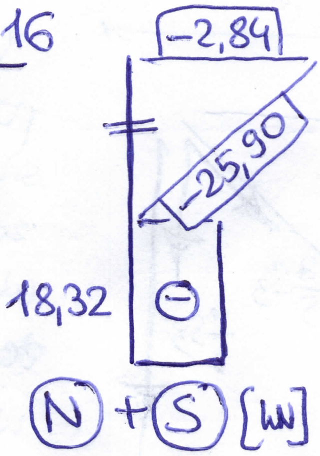
$$= -0{,}040524 \text{ m}$$

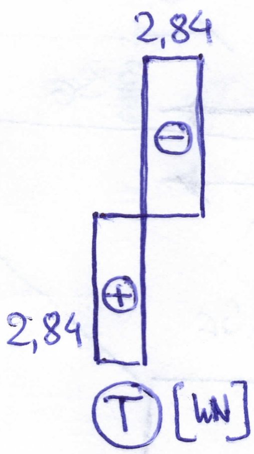
$$\delta_{11} = \int_L \frac{M_1 M_1}{EI} ds + \sum \frac{S_{i1} S_{i1}}{EA_i} l_i = \frac{2}{30000}\cdot\frac{1}{3}\cdot 4\cdot 4\cdot 4 + \frac{1}{10^5}\left(4\cdot 1\cdot 1 + 2\sqrt{2}\cdot 2\sqrt{2}\cdot 4\sqrt{2}\right) =$$

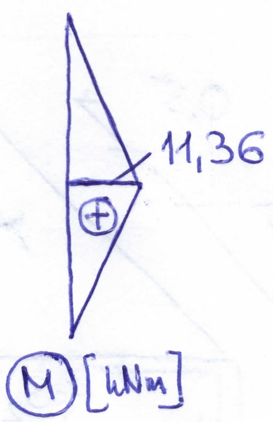
$$= 0{,}001422 + 0{,}0004925 = 0{,}0019148 \ \frac{\text{m}}{\text{kN}}$$

$X_1 = 21{,}16$ kN





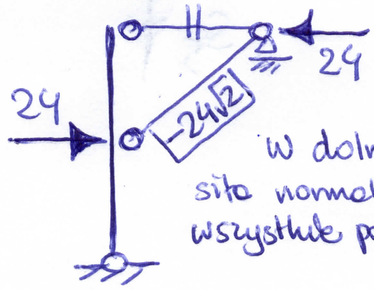




przypadek $EA \to \infty$

$\delta_{10} = -0{,}03413$

$\delta_{11} = 0{,}0014222$ $\Rightarrow$ $X_1 = 24$ kN



siła w pręcie kratowym $S = -24\sqrt{2}$ kN

W dolnej części słupa siła normalna równa -24 kN

wszystkie pozostałe siły zerowe