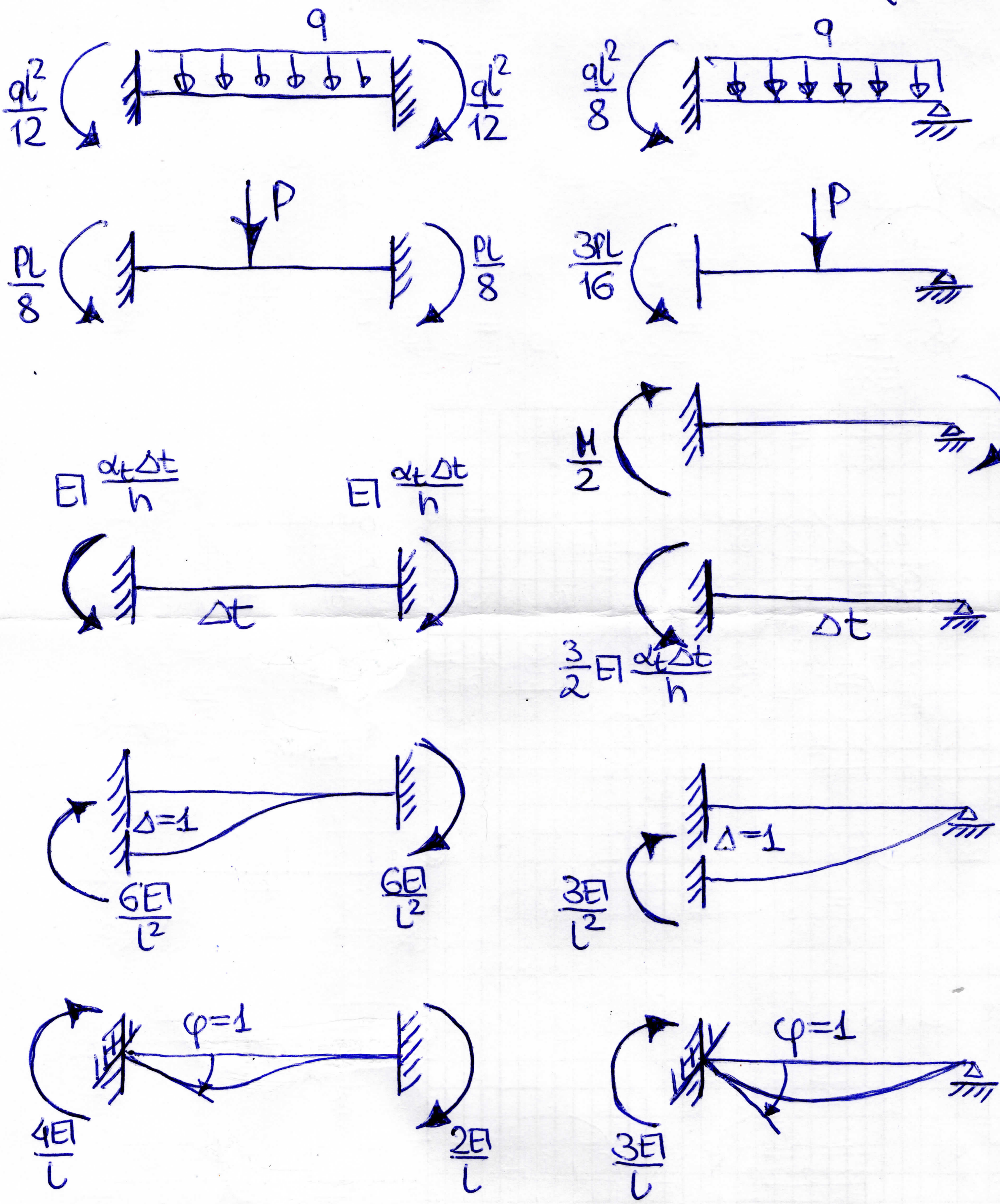
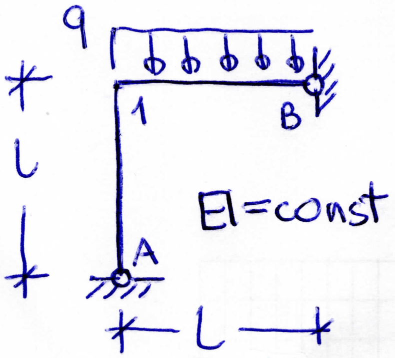
METODA PRZEMIESZCZEŃ

CW. 8/1

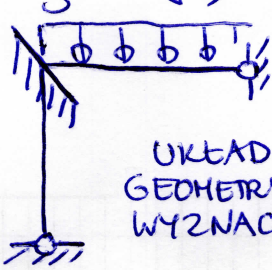
momenty wyjściowe w belkach obustronnie utwierdzonych (11) i jednostronnie utwierdzonych (10) pod działaniem najczęstszych obciążeń i oddziaływań pozastatycznych (dane: l, EI)



METODA PRZEMIESZCZEŃ

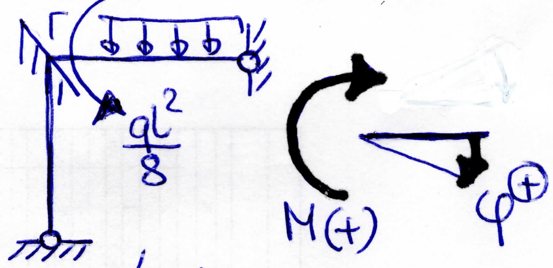


$n_g = 1 \, (\varphi)$



UKŁAD GEOMETRYCZNIE WYZNACZALNY

stan „0" – moment wyjściowy

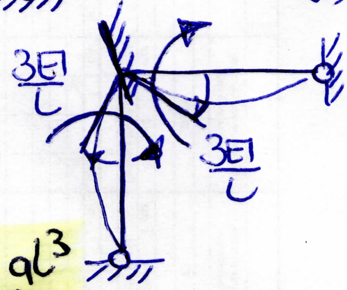


sumaryczne momenty przywęzłowe:

$$M_{1A} = \frac{3EI}{L}\varphi$$

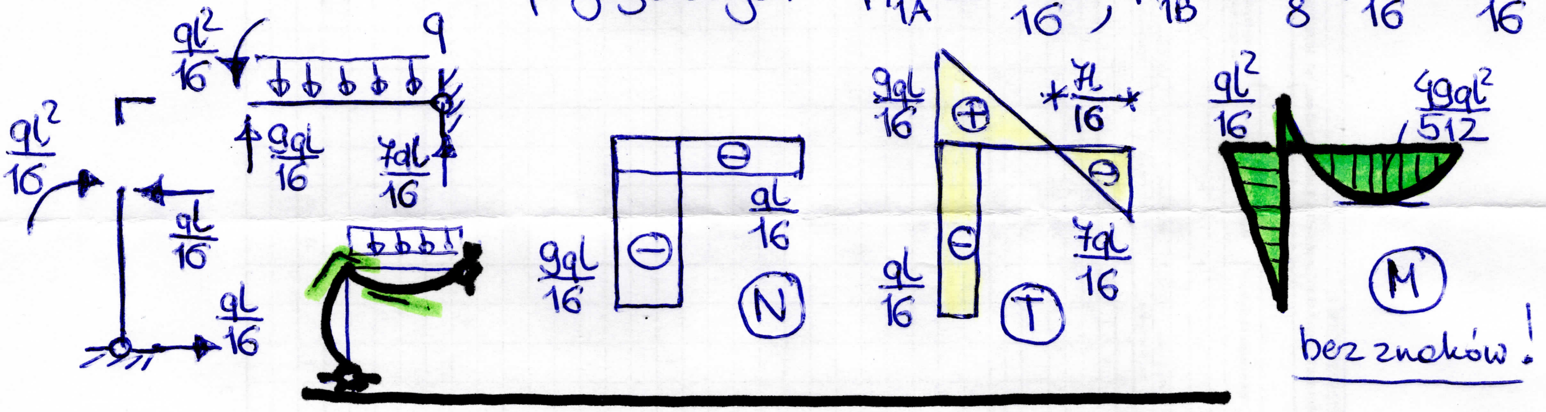
$$M_{1B} = -\frac{ql^2}{8} + \frac{3EI}{L}\varphi$$

stan $\varphi = 1$ (wymuszenie)



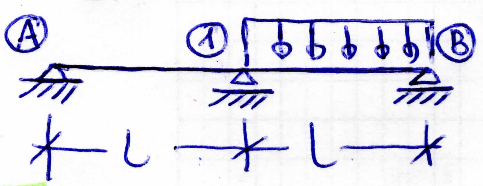
równanie równowagi: $\Sigma M_1 = M_{1A} + M_{1B} = 0 \Rightarrow \varphi = \frac{ql^3}{48EI}$

wartości momentów przywęzłowych: $M_{1A} = \frac{ql^2}{16}$, $M_{1B} = -\frac{ql^2}{8} + \frac{ql^2}{16} = -\frac{ql^2}{16}$



bez znaków!

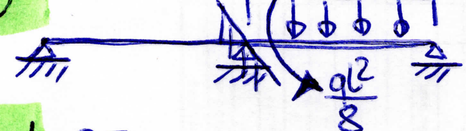
EI = const



$n_g = 1 \, (\varphi)$. Sumaryczne momenty przywęzłowe:

$$M_{1A} = \frac{3EI}{L}\varphi, \quad M_{1B} = -\frac{ql^2}{8} + \frac{3EI}{L}\varphi$$

„0"

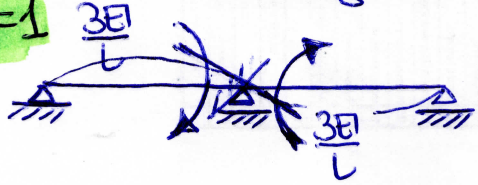


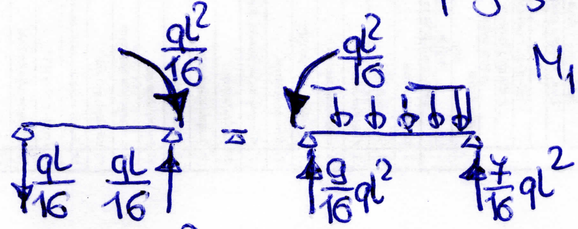
Równanie równowagi: $\Sigma M_1 = M_{1A} + M_{1B} = 0 \Rightarrow \varphi = \frac{ql^3}{48EI}$

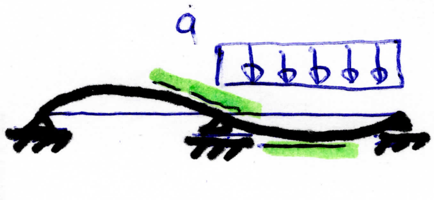
wartości momentów przywęzłowych: $M_{1A} = \frac{ql^2}{16}$

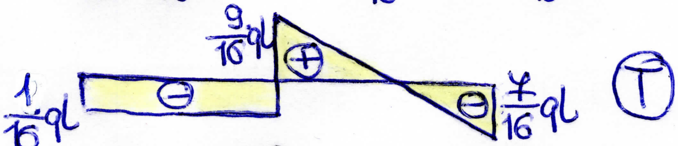
$$M_{1B} = -\frac{ql^2}{8} + \frac{ql^2}{16} = -\frac{ql^2}{16}$$

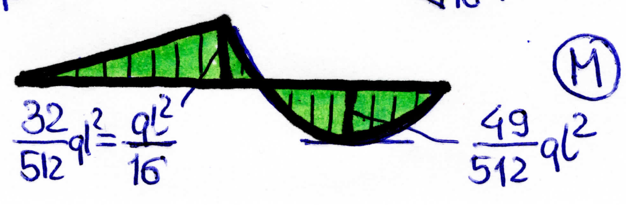
$\varphi = 1$



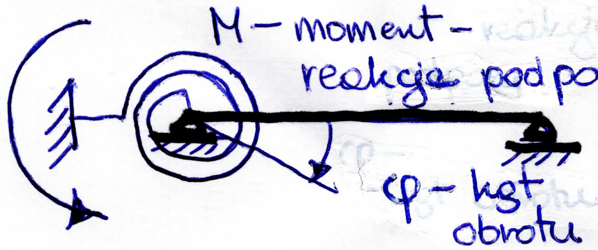








OGÓLNY PRZYPADEK: podpora sprężysta rotacyjna — Ćw. 8/3

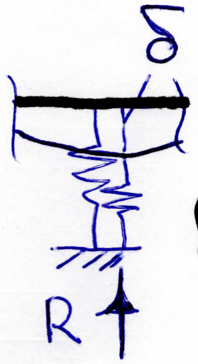


M – moment – reakcja podpory

φ – kąt obrotu

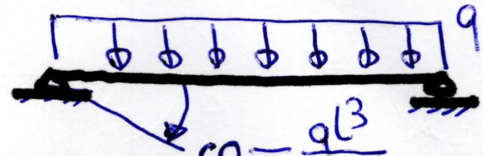
$$M = k_\varphi \cdot \varphi$$

analogia do podpory translacyjnej

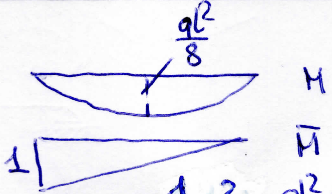


$$R = k_s \cdot \delta$$

(A) przypadek: Swobodny przegub



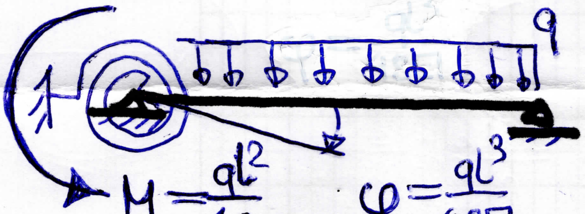
$M = 0$ $\quad \varphi = \dfrac{ql^3}{24EI}$



$$\varphi = \frac{1}{EI}\cdot\frac{2}{3}l\cdot\frac{ql^2}{8}\cdot\frac{1}{2} = \frac{ql^3}{24EI}$$

$$M = k_\varphi \cdot \varphi \Rightarrow 0 = k_\varphi \cdot \frac{ql^3}{24EI} \Rightarrow k_\varphi = 0$$

(B) dane zadanie

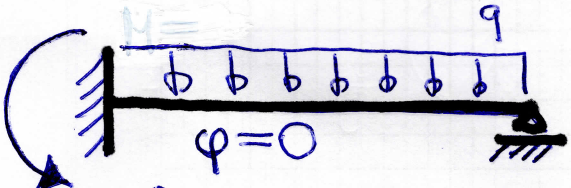


$M = \dfrac{ql^2}{16}$ $\quad \varphi = \dfrac{ql^3}{48EI}$

$$M = k_\varphi \cdot \varphi \Rightarrow \frac{ql^2}{16} = k_\varphi \cdot \frac{ql^3}{48EI}$$

$$k_\varphi = 3\frac{EI}{l}$$

(C) przypadek: belka z podporą utwierdzoną



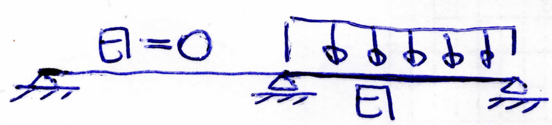
$\varphi = 0$

$M = \dfrac{ql^2}{8}$

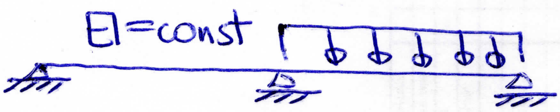
$$M = k_\varphi \cdot \varphi \Rightarrow \frac{ql^2}{8} = k_\varphi \cdot 0$$

$$k_\varphi \to \infty$$

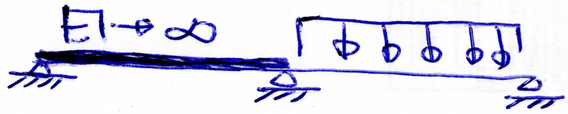
(A)



(B)



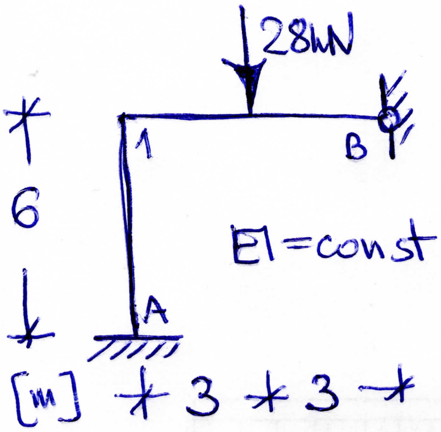
(C)

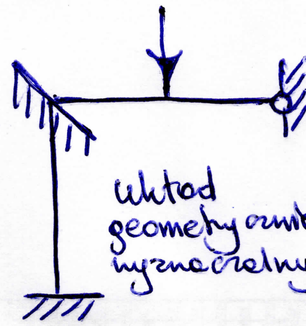


INNA INTERPRETACJA

Ćw. 8/4

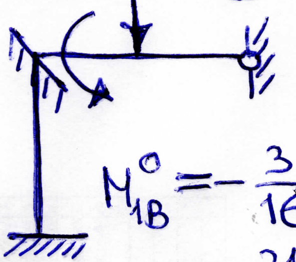
$n_g = 1\ (\varphi)$





układ geometrycznie wyznaczalny

stan „0" – moment wyjściowy



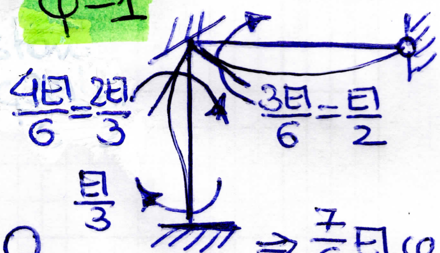
$M^0_{1B} = -\frac{3}{16}\cdot 6\cdot 28 =$
$= -31{,}5\ kNm$

Sumaryczne momenty przywęzłowe:

$M_{1A} = \frac{2}{3}EI\varphi$, $M_{A1} = \frac{1}{3}EI\varphi$

$M_{1B} = -31{,}5 + \frac{EI}{2}\varphi$

$\varphi = 1$

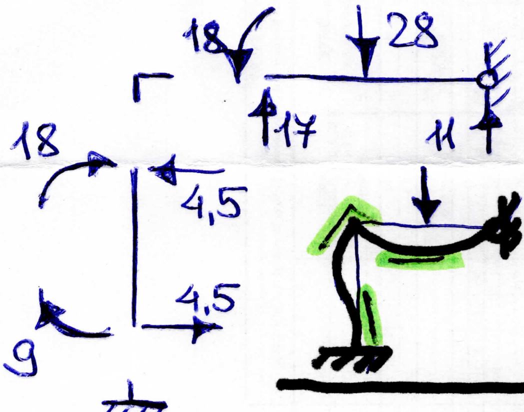


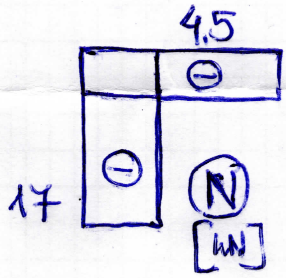
równanie równowagi: $\sum M_1 = M_{1A} + M_{1B} = 0$ $\Rightarrow \frac{7}{6}EI\varphi = 31{,}5$

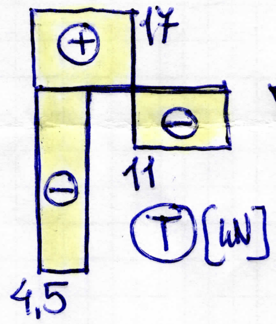
$\varphi = \frac{27}{EI}$

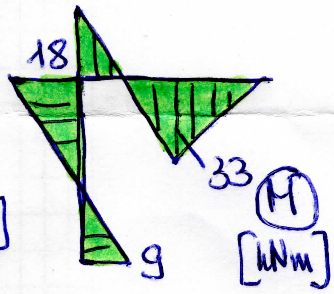
Wartości momentów przywęzłowych:

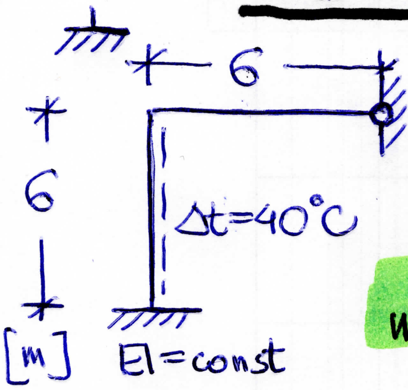
$M_{1A} = 18\ kNm$ $M_{1B} = -18\ kNm$ $M_{A1} = 9\ kNm$





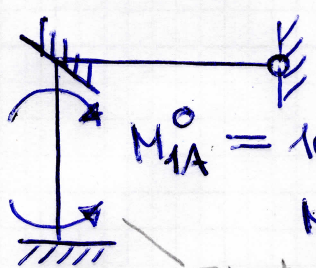






$\alpha_t = 10^{-5}\frac{1}{°C}$ $h = 0{,}4m$ $EI = 10^4\ kNm^2$

„0"



$M^0_{1A} = 10^4 \cdot \frac{10^{-5}\cdot 40}{0{,}4} = 10\ kNm$

$M^0_{A1} = -10\ kNm$

zwrot rzeczywisty

Sumaryczne momenty przywęzłowe:

$M_{1A} = 10 + \frac{2}{3}EI\varphi$

$M_{1B} = \frac{EI}{2}$

$M_{A1} = -10 + \frac{1}{3}EI\varphi$

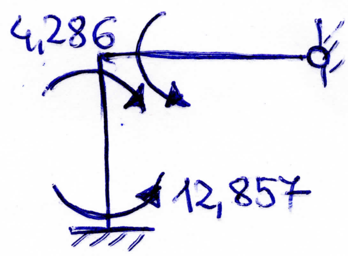
stan $\varphi = 1$ i związane z nim momenty bez zmian

równanie równowagi: $\sum M_1 = 0 \Rightarrow \varphi = \frac{-8{,}5714}{EI}$

$M_{1A} = 4{,}286\ kNm$

$M_{1B} = -4{,}286\ kNm$

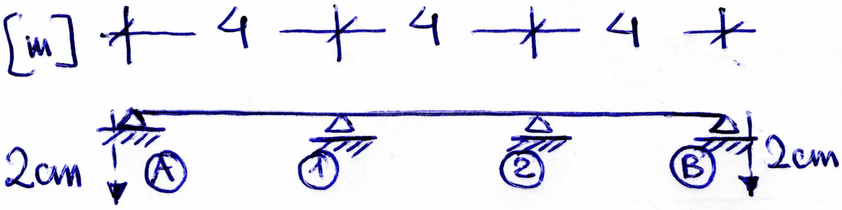
$M_{A1} = -12{,}857\ kNm$

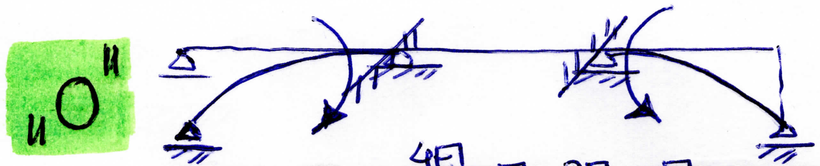


ćw. 8/5

$EI = 8000\ kNm^2$

$n_g = 2\ (\varphi_1, \varphi_2)$

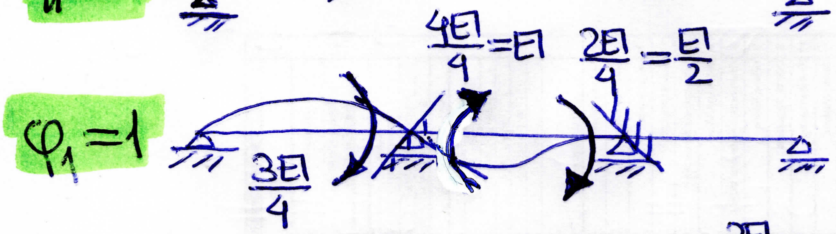


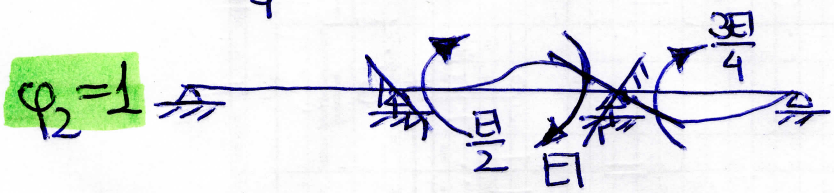


momenty wyjściowe:

$$M^0_{1A} = \frac{3\cdot 8000}{16}\cdot 0{,}02 = 30\ kNm$$

$$M^0_{2B} = \quad = -30\ kNm$$





sumaryczne momenty przywęzłowe:

$$M_{1A} = 30 + \frac{3}{4}EI\varphi_1$$

$$M_{12} = EI\varphi_1 + \frac{EI}{2}\varphi_2$$

$$M_{21} = \frac{EI}{2}\varphi_1 + EI\varphi_2$$

$$M_{2B} = -30 + \frac{3}{4}EI\varphi_2$$

równowagi:

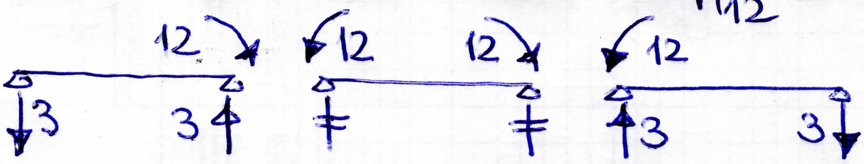
$$\Sigma M_1 = M_{1A} + M_{12} = 0 \Rightarrow \frac{7}{4}EI\varphi_1 + \frac{EI}{2}\varphi_2 + 30 = 0 \Rightarrow 7\varphi_1 + 2\varphi_2 = \frac{-120}{EI}$$

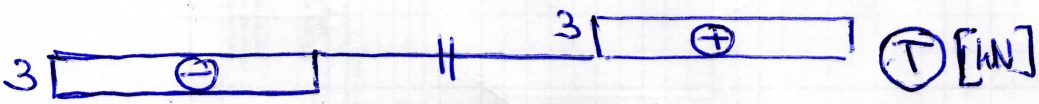
$$\Sigma M_2 = M_{21} + M_{2B} = 0 \Rightarrow \frac{EI}{2}\varphi_1 + \frac{7}{4}EI\varphi_2 - 30 = 0 \Rightarrow 2\varphi_1 + 7\varphi_2 = \frac{120}{EI}$$

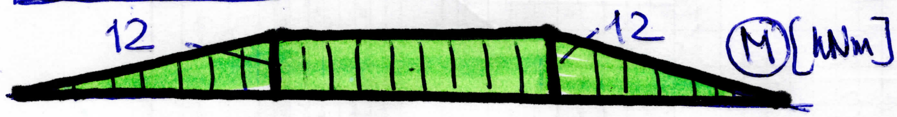
$$W = 45 \quad W_1 = \frac{120}{EI}\begin{vmatrix} -1 & 2 \\ 1 & 7 \end{vmatrix} = \frac{-1080}{EI} \quad W_2 = \frac{120}{EI}\begin{vmatrix} 7 & -1 \\ 2 & 1 \end{vmatrix} = \frac{1080}{EI}$$

$$\varphi_1 = -\frac{24}{EI}, \quad \varphi_2 = \frac{24}{EI} \Rightarrow \quad M_{1A} = 12\ kNm \quad M_{21} = 12\ kNm$$

$$M_{12} = -12\ kNm \quad M_{2B} = -12\ kNm$$







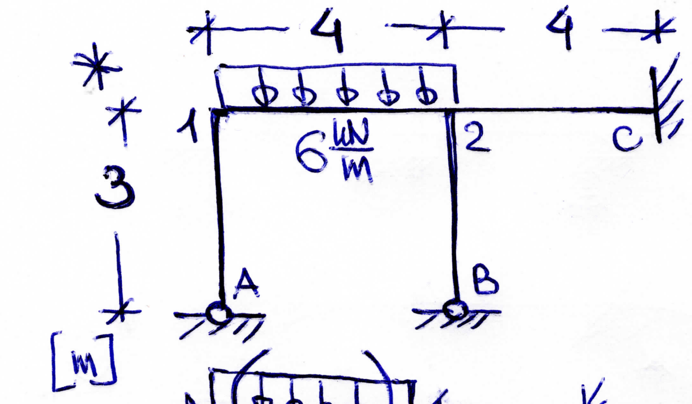
Ze względu na symetrię układu i obciążenia (oddziaływania zewnętrznego - pozastatycznego) można przyjąć $\varphi_2 = -\varphi_1$ i od razu rozpatrywać równowagę TYLKO JEDNEGO WĘZŁA

$$M_{1A} = 30 + \frac{3}{4}EI\varphi_1$$

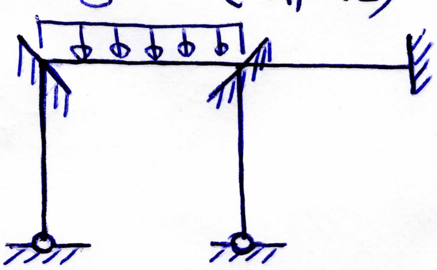
$$M_{12} = EI\varphi_1 + \frac{EI}{2}\varphi_2 \underset{sym.}{=} \frac{EI}{2}\varphi_1$$

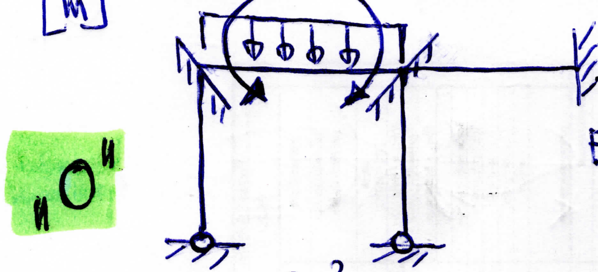
równanie równowagi: $\Sigma M_1 = M_{1A} + M_{12} = 30 + \frac{5}{4}EI\varphi_1 = 0 \Rightarrow \varphi_1 = \frac{-24}{EI}$

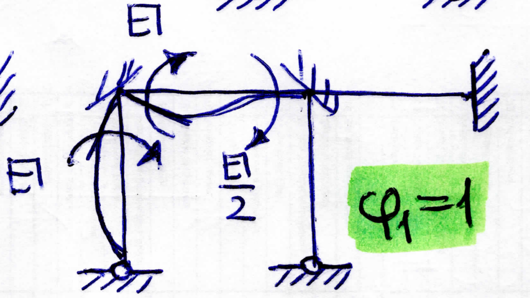
stąd $M_{1A} = 12\ kNm$, $M_{12} = -12\ kNm$, pozostałe symetrycznie

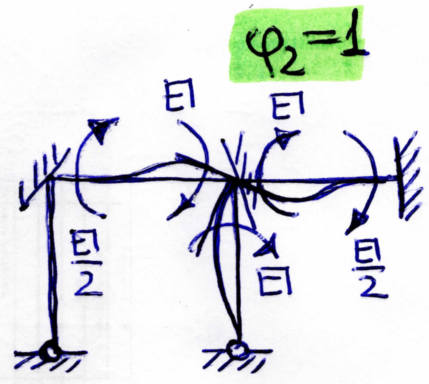


$n_g = 2(\varphi_1, \varphi_2)$









$M^{0}_{12} = -\frac{6 \cdot 4^2}{12} = -8$ kNm

$M^{0}_{21} = 8$ kNm

Sumaryczne momenty przywęzłowe:

$M_{1A} = EI\varphi_1$

$M_{12} = -8 + EI\varphi_1 + \frac{EI}{2}\varphi_2$

$M_{21} = 8 + \frac{EI}{2}\varphi_1 + EI\varphi_2$

$M_{2B} = M_{2C} = EI\varphi_2$

$M_{C2} = \frac{EI}{2}\varphi_2$

Równania równowagi:

$\Sigma M_1 = M_{1A} + M_{12} = -8 + 2EI\varphi_1 + \frac{EI}{2}\varphi_2 = 0 \Rightarrow 4\varphi_1 + \varphi_2 = \frac{16}{EI}$

$\Sigma M_2 = M_{21} + M_{2B} + M_{2C} = 8 + \frac{EI}{2}\varphi_1 + 3EI\varphi_2 = 0 \Rightarrow \varphi_1 + 6\varphi_2 = -\frac{16}{EI}$

$W = 23 \quad W_1 = \frac{16}{EI}\begin{vmatrix} 1 & 1 \\ -1 & 6 \end{vmatrix} = \frac{112}{EI} \quad W_2 = \frac{16}{EI}\begin{vmatrix} 4 & 1 \\ 1 & -1 \end{vmatrix} = -\frac{80}{EI}$

$\varphi_1 = 4{,}87\frac{1}{EI}, \quad \varphi_2 = -3{,}48\cdot\frac{1}{EI}$

$M_{1A} = 4{,}87$ kNm

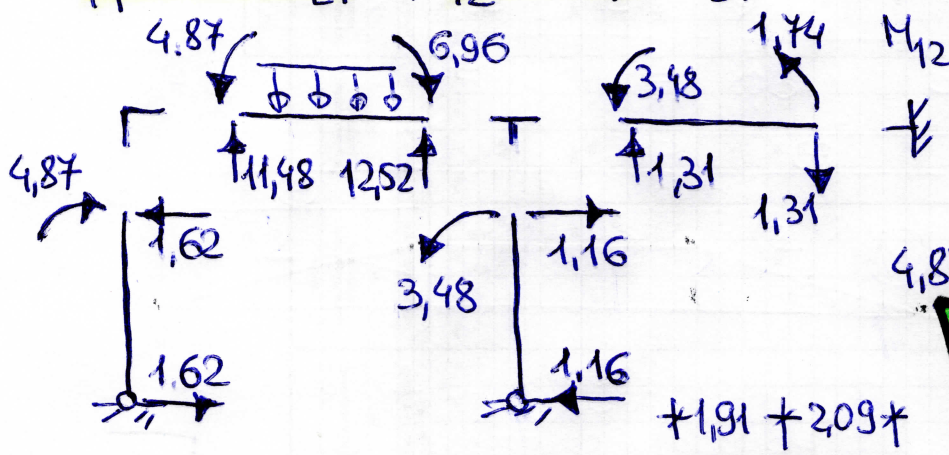
$M_{12} = -4{,}87$ kNm

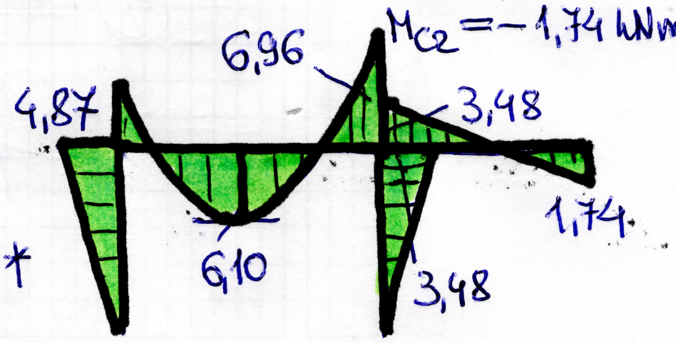
$M_{21} = 6{,}96$ kNm

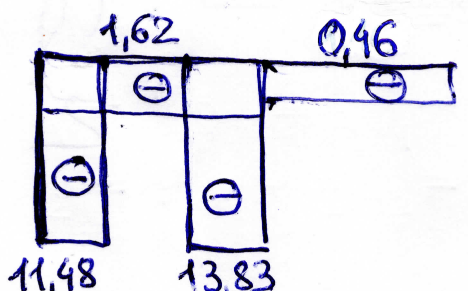
$M_{2B} = -3{,}48$ kNm

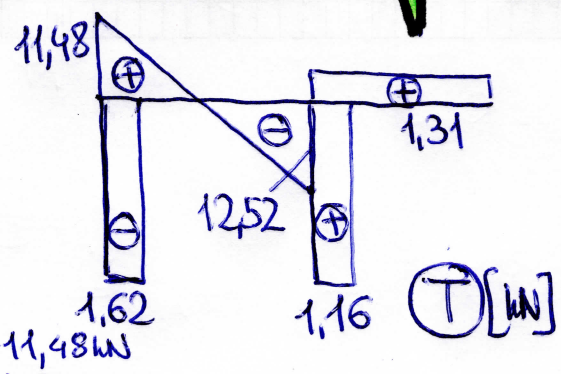
$M_{2C} = -3{,}48$ kNm

$M_{C2} = -1{,}74$ kNm







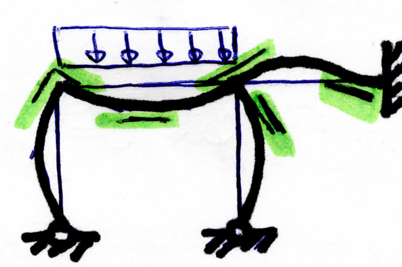


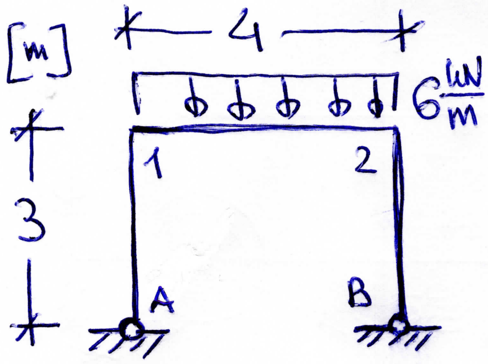
$N_{1A} = -11{,}48$ kN

$N_{12} = -1{,}62$ kN

$N_{2C} = -1{,}62 + 1{,}16 = -0{,}46$ kN

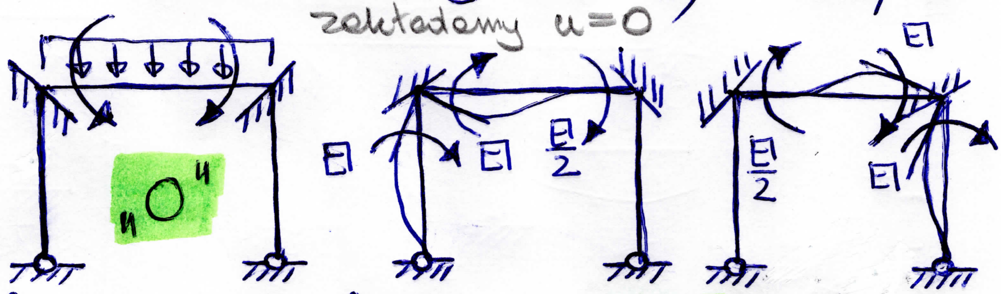
$N_{2B} = -12{,}52 - 1{,}31 = -13{,}83$ kN





bezpośrednio: $n_g = 2(\varphi_1, \varphi_2)$ ćw. 8/7

zakładamy $u=0$



$M_{12}^0 = -8\,kNm$ $M_{21}^0 = 8\,kNm$ $\varphi_1 = 1$ $\varphi_2 = 1$

sumaryczne momenty przywęzłowe:

$M_{1A} = EI\varphi_1$

$M_{12} = -8 + EI\varphi_1 + \frac{EI}{2}\varphi_2$

$M_{21} = 8 + \frac{EI}{2}\varphi_1 + EI\varphi_2$

$M_{2B} = EI\varphi_2$

równania równowagi:

$\Sigma M_1 = M_{1A} + M_{12} = -8 + 2EI\varphi_1 + \frac{EI}{2}\varphi_2 = 0 \Rightarrow 4\varphi_1 + \varphi_2 = \frac{16}{EI}$

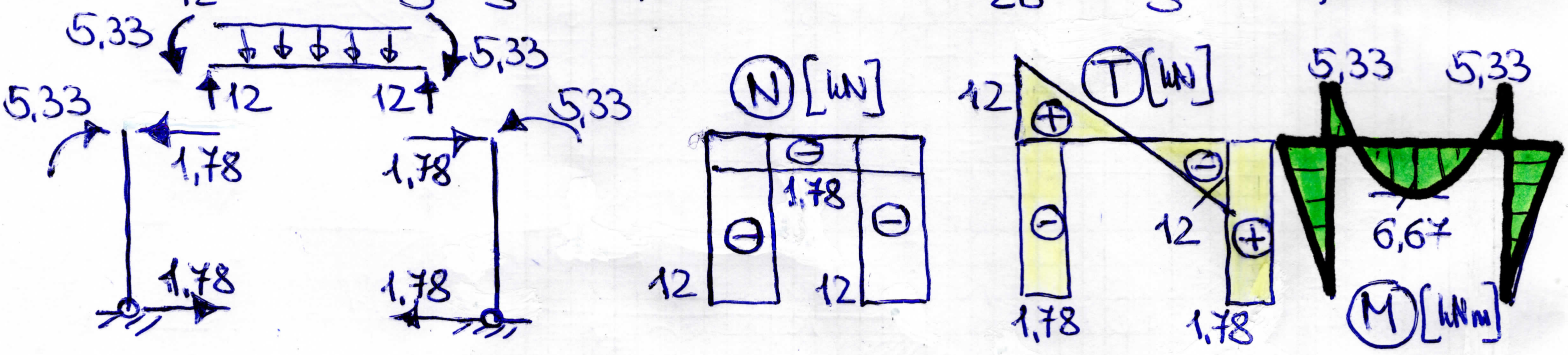
$\Sigma M_2 = M_{21} + M_{2B} = 8 + \frac{EI}{2}\varphi_1 + EI\varphi_2 = 0 \Rightarrow \varphi_1 + 4\varphi_2 = \frac{-16}{EI}$

$W = 15$ $W_1 = \frac{16}{EI}\begin{vmatrix} 1 & 1 \\ -1 & 4 \end{vmatrix} = \frac{80}{EI}$ $W_2 = \frac{16}{EI}\begin{vmatrix} 4 & 1 \\ 1 & -1 \end{vmatrix} = \frac{-80}{EI}$

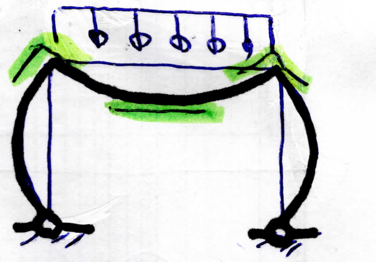
stąd $\varphi_1 = \frac{16}{3EI}$, $\varphi_2 = -\frac{16}{3EI}$. Wartości momentów przywęzłowych:

$M_{1A} = \frac{16}{3} = 5{,}333\,kNm$ $M_{21} = 8 + \frac{8}{3} - \frac{16}{3} = 5{,}333\,kNm$

$M_{12} = -8 + \frac{16}{3} - \frac{8}{3} = -5{,}333\,kNm$ $M_{2B} = -\frac{16}{3} = -5{,}333\,kNm$



Można wykorzystać symetrię układu i obciążenia



$u=0$, $\varphi_2 = -\varphi_1 \Rightarrow$ 1 $M_{1A} = EI\varphi_1$ $M_{12} = -8 + \frac{EI}{2}\varphi_1$

$\Sigma M_1 = M_{1A} + M_{12} = -8 + \frac{3EI}{2}\varphi_1 = 0 \Rightarrow \varphi_1 = \frac{16}{3EI}$

stąd $M_{1A} = 5{,}333\,kNm$, $M_{12} = -5{,}333\,kNm$, reszta symetrycznie