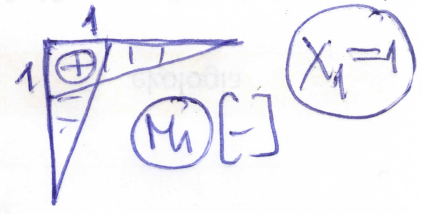
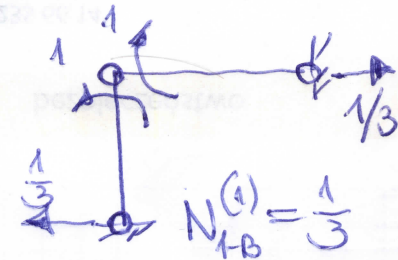
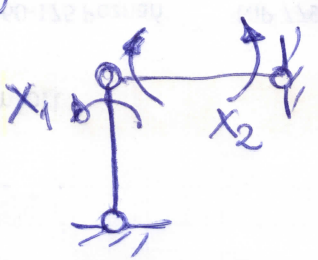
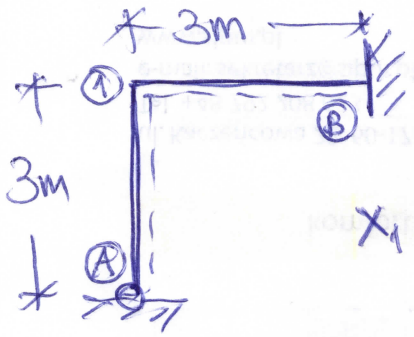


W montażowej pręcie 1-B dłuższego o  $\Delta l = 7 \text{ cm}$   
 $EI = 3000 \text{ kNm}^2 = \text{const}$



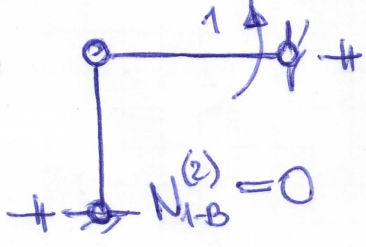
$$\delta_{10} = 0,07 \cdot \frac{1}{3} = \frac{7}{300}$$

$$\delta_{20} = 0$$

$$\delta_{11} = \frac{2}{3EI} \cdot 3 \cdot 1 \cdot 1 = \frac{2}{EI}$$

$$\delta_{22} = \frac{1}{3EI} \cdot 3 \cdot 1 \cdot 1 = \frac{1}{EI}$$

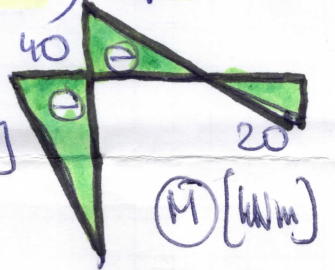
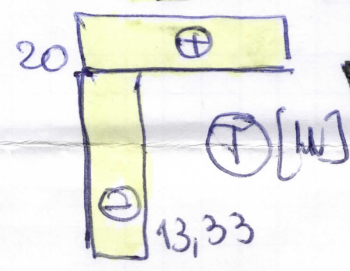
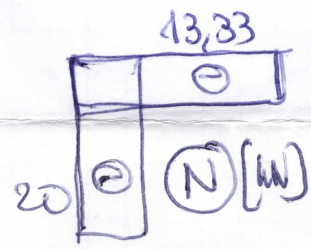
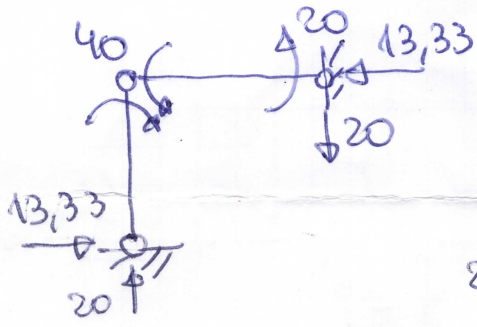
$$\delta_{12} = \delta_{21} = \frac{1}{6EI} \cdot 3 \cdot 1 \cdot 1 = \frac{1}{2EI}$$



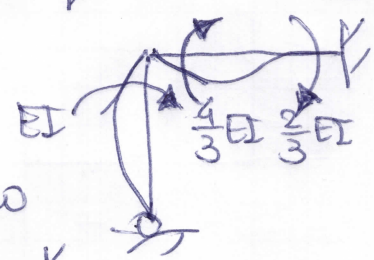
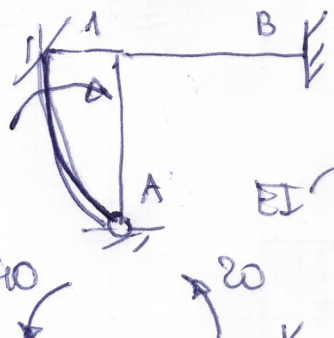
$$\delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \Rightarrow \frac{2}{EI}X_1 + \frac{1}{2EI}X_2 = -\frac{7}{300} \Rightarrow 4X_1 + X_2 = \frac{-14}{300} \cdot 3000 = -140$$

$$\delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \Rightarrow \frac{1}{2EI}X_1 + \frac{1}{EI}X_2 = 0 \Rightarrow X_1 = -2X_2$$

$$-7X_2 = -140 \Rightarrow X_2 = 20 \text{ kNm}, X_1 = -40 \text{ kNm}$$



$$M_{1A}^0 = \frac{3 \cdot 3000}{9} \times 0,07 = 70 \text{ kNm}$$



$$M_{1A} = 70 + EI\varphi$$

$$M_{1B} = \frac{4}{3}EI\varphi$$

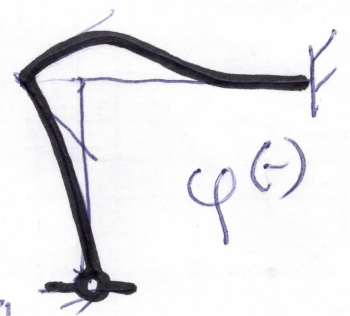
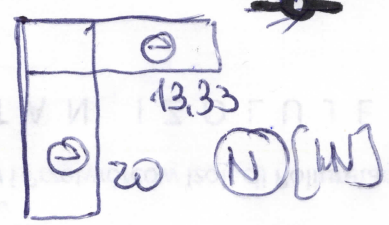
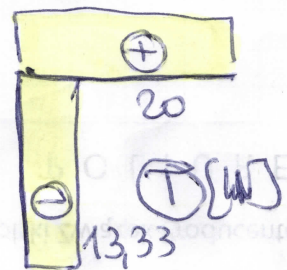
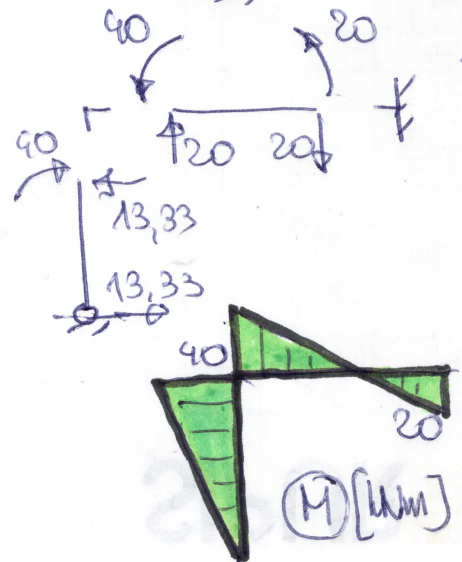
$$M_{B1} = \frac{2}{3}EI\varphi$$

$$\sum M_1 = 0 \Rightarrow \frac{7}{3}EI\varphi = -70 \Rightarrow \varphi = -\frac{30}{EI}$$

$$M_{1A} = 70 - 30 = 40 \text{ kNm}$$

$$M_{1B} = -\frac{4}{3} \cdot 30 = -40 \text{ kNm}$$

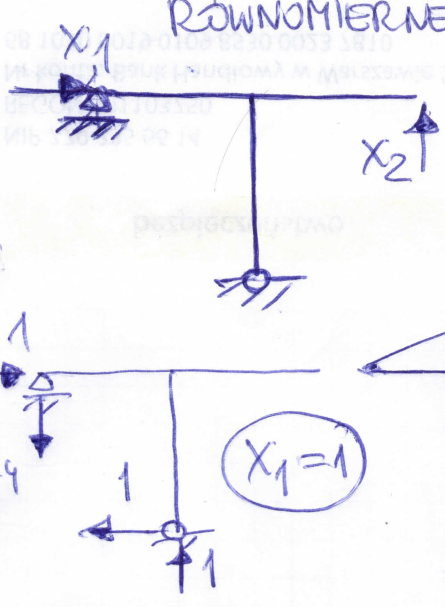
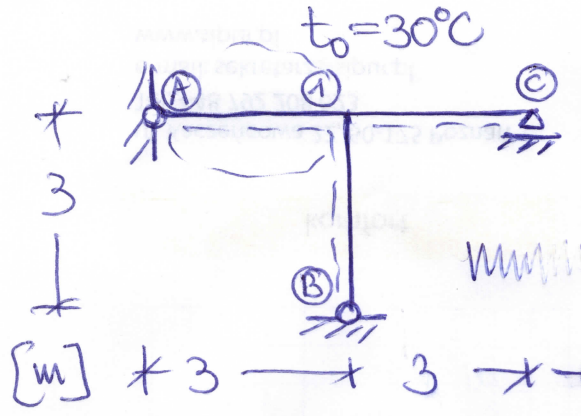
$$M_{B1} = -\frac{2}{3} \cdot 30 = -20 \text{ kNm}$$



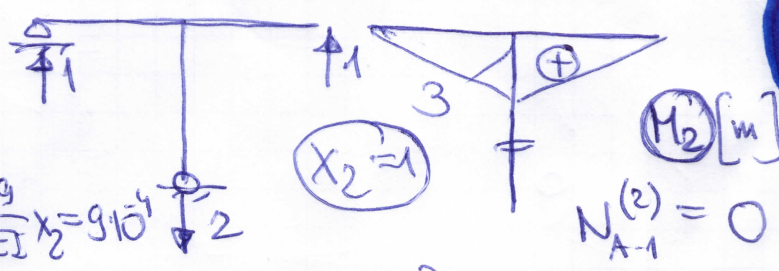


RÓWNOCIERNE OGRZANIE A-1

$\alpha_t = 10^{-5} \frac{1}{^\circ\text{C}}$   
 $EI = 30000 \text{ kNm}^2 = 3 \cdot 10^4 \text{ kNm}^2$

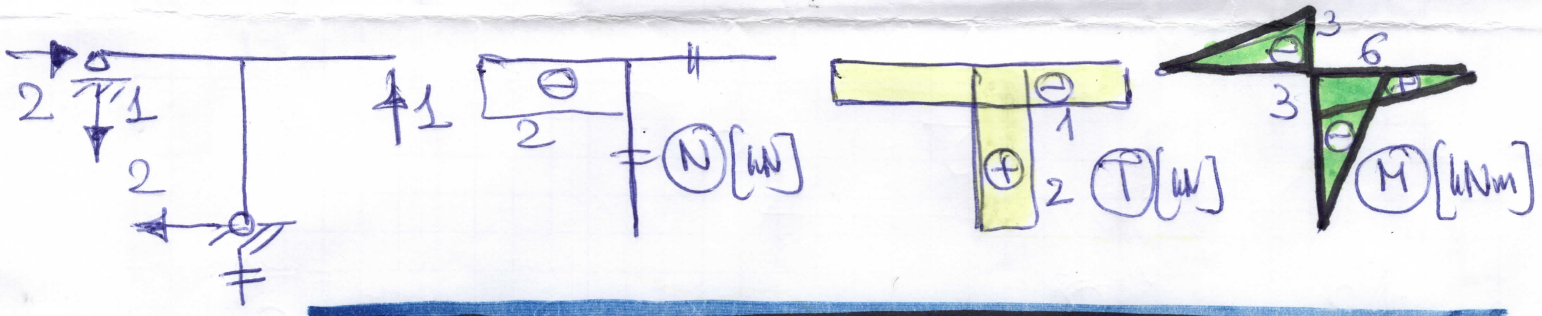


$\delta_{10} = 10^{-5} \cdot 30 \cdot (-1) \cdot 3 = -9 \cdot 10^{-4}$   
 $\delta_{20} = 0$   
 $\delta_{11} = \delta_{22} = \frac{2}{3EI} \cdot 3 \cdot 3 \cdot 3 = \frac{18}{EI}$   
 $\delta_{12} = \delta_{21} = -\frac{1}{3EI} \cdot 3 \cdot 3 \cdot 3 = -\frac{9}{EI}$



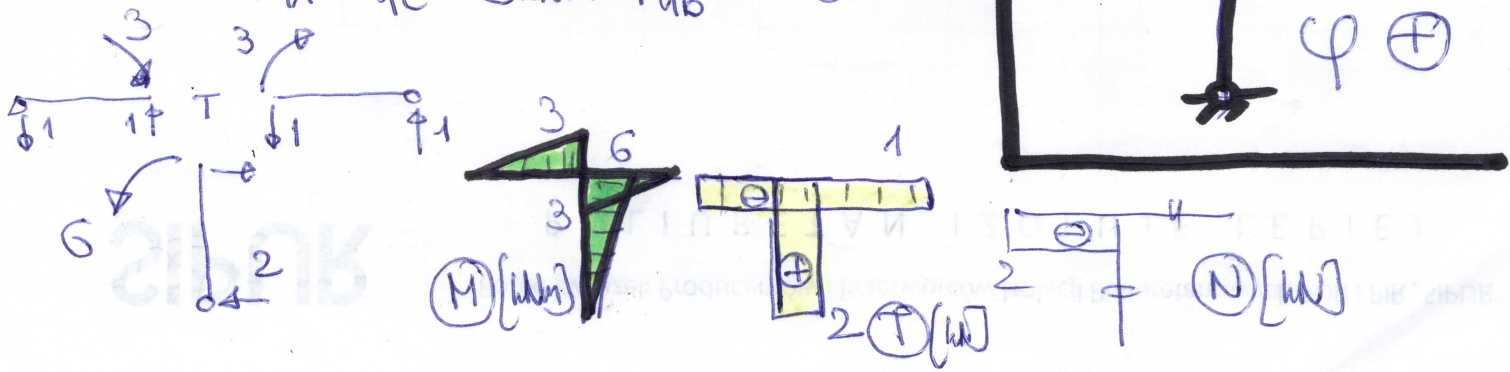
$\delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \Rightarrow \frac{18}{EI}X_1 - \frac{9}{EI}X_2 = 9 \cdot 10^{-4}$   
 $2X_1 - X_2 = 3$   
 $\delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \Rightarrow -\frac{9}{EI}X_1 + \frac{18}{EI}X_2 = 0 \Rightarrow X_1 = 2X_2$

$X_2 = 1 \text{ kN}$   
 $X_1 = 2 \text{ kN}$

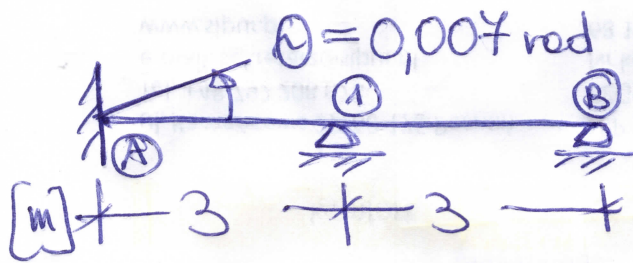


$M_{1B}^0 = -\frac{3 \cdot 3 \cdot 10^4}{9} \cdot 10^{-5} \cdot 30 \cdot 3 = -9 \text{ kNm}$   
 $M_{1A} = M_{1C} = EI\varphi$      $M_{1B} = -9 + EI\varphi$

$\sum M_1 = 0 \Rightarrow 3EI\varphi = 9 \Rightarrow \varphi = \frac{3}{EI}$   
 $M_{1A} = M_{1C} = 3 \text{ kNm}$      $M_{1B} = -6 \text{ kNm}$

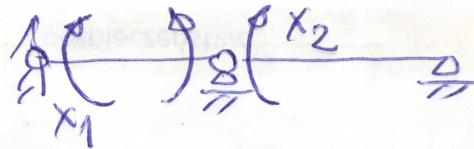






WYMUSZONY KĄT OBROTU

$EI = 3000 \text{ kNm}^2$



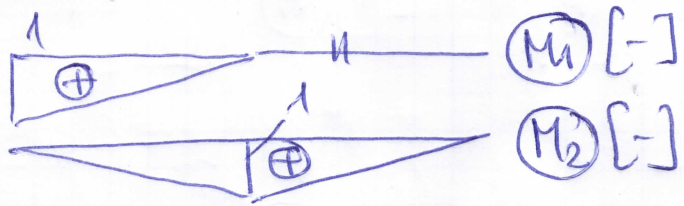
$\delta_{10} = 0,007 \cdot 1 = 0,007$

$\delta_{20} = 0$

$\delta_{11} = \frac{1}{3EI} \cdot 3 \cdot 1 \cdot 1 = \frac{1}{EI}$

$\delta_{22} = \frac{2}{3EI} \cdot 3 \cdot 1 \cdot 1 = \frac{2}{EI}$

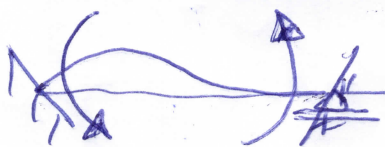
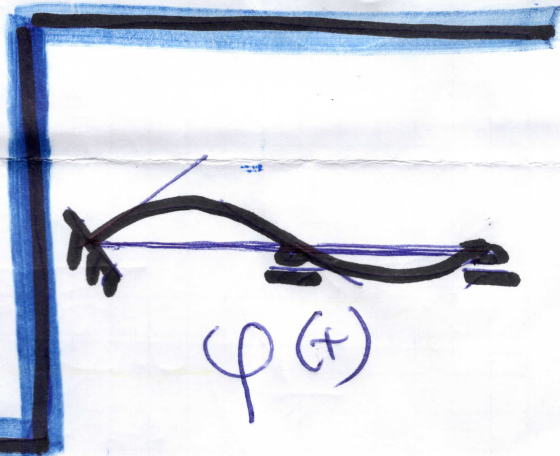
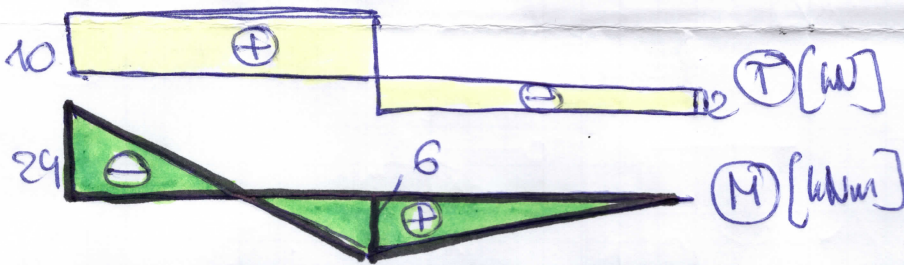
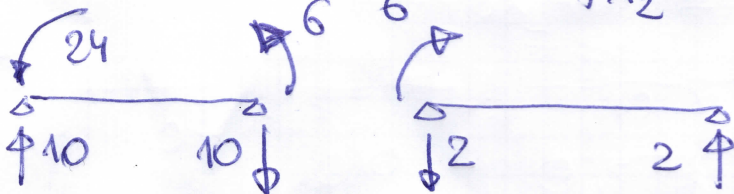
$\delta_{12} = \delta_{21} = \frac{1}{6EI} \cdot 3 \cdot 1 \cdot 1 = \frac{1}{2EI}$



$\delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \Rightarrow \frac{1}{EI}X_1 + \frac{1}{2EI}X_2 = -0,007 \Rightarrow 2X_1 + X_2 = -42$

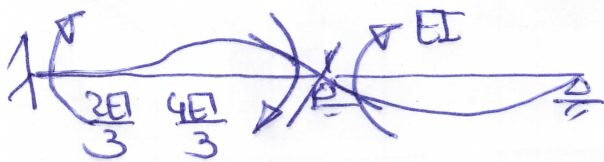
$\delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \Rightarrow \frac{1}{2EI}X_1 + \frac{2}{EI}X_2 = 0 \Rightarrow X_1 = -4X_2$

$-7X_2 = -42 \Rightarrow X_2 = 6 \text{ kNm}, X_1 = -24 \text{ kNm}$



$M_{A1}^0 = \frac{4 \cdot 3000}{3} \cdot 0,007 = -28 \text{ kNm}$

$M_{A0}^0 = -\frac{2 \cdot 3000}{3} \cdot 0,007 = -14 \text{ kNm}$



$M_{A1} = -14 + \frac{4EI}{3}\varphi$        $M_{1B} = EI\varphi$

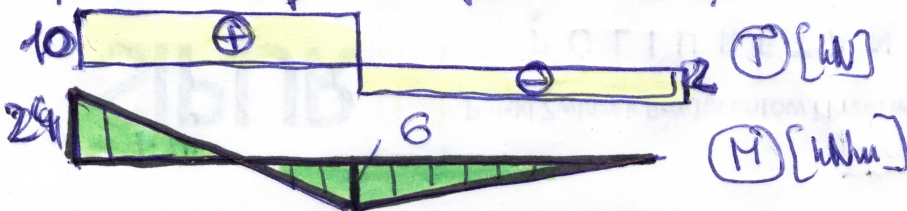
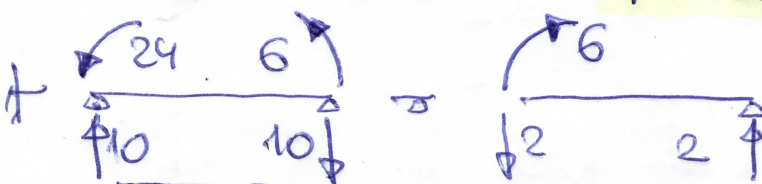
$M_{A1} = -28 + \frac{2EI}{3}\varphi$

$\sum M_1 = 0 \Rightarrow \frac{7EI}{3}\varphi = 14 \Rightarrow \varphi = \frac{6}{EI}$

$M_{1A} = -14 + 8 = -6 \text{ kNm}$

$M_{1B} = 6 \text{ kNm}$

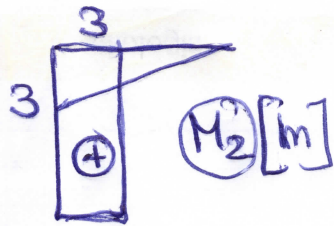
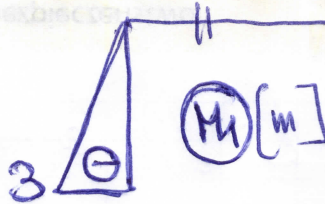
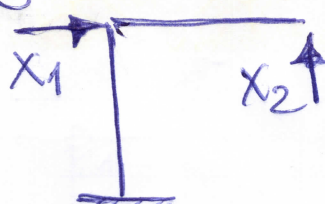
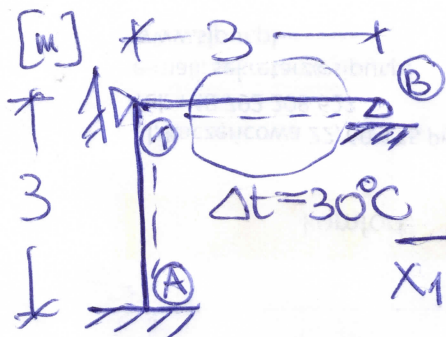
$M_{A1} = -28 + 4 = -24 \text{ kNm}$





# NIERÓWNOCIERNE OGRZANIE 1-B

$\alpha_t = 10^{-5} \frac{1}{^\circ\text{C}}$      $h = 0,3 \text{ m}$      $EI = 35000 \text{ kNm}^2$



$$\delta_{10} = 0 \quad \delta_{20} = \frac{30 \cdot 10^{-5}}{0,3} \cdot \frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2} \cdot 10^{-3}$$

$$\delta_{11} = \frac{1}{3EI} \cdot 3 \cdot 3 \cdot 3 = \frac{9}{EI}, \quad \delta_{22} = \frac{1}{EI} \left( \frac{1}{3} \cdot 3 \cdot 3 \cdot 3 + 3 \cdot 3 \cdot 3 \right) = \frac{36}{EI}$$

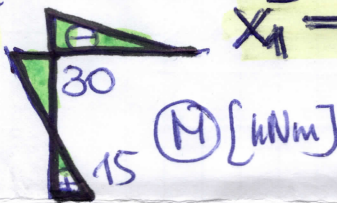
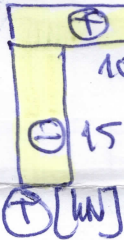
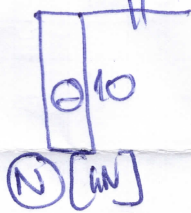
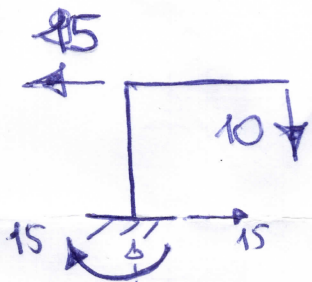
$$\delta_{12} = \delta_{21} = -\frac{1}{2EI} \cdot 3 \cdot 3 \cdot 3 = -\frac{13,5}{EI}$$

$$\delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \Rightarrow \frac{9}{EI}X_1 - \frac{13,5}{EI}X_2 = 0 \Rightarrow X_1 = \frac{3}{2}X_2$$

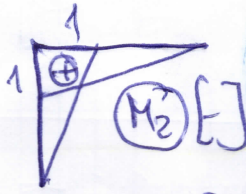
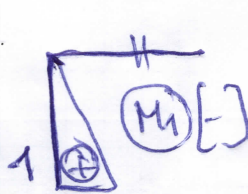
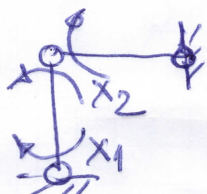
$$\delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \Rightarrow -\frac{13,5}{EI}X_1 + \frac{36}{EI}X_2 = \frac{-4,5}{10^3} \Rightarrow -3X_1 + 8X_2 = -35$$

$$-4,5X_2 + 8X_2 = -35 \Rightarrow X_2 = -10 \text{ kNm}$$

$$X_1 = -15 \text{ kNm}$$



innym układ podskony mebody sit:



$$\delta_{10} = 0$$

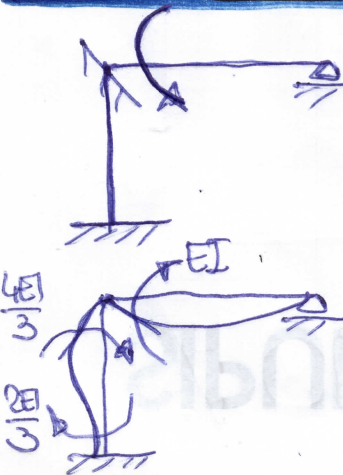
$$\delta_{20} = \frac{10^{-5} \cdot 30}{0,3} \cdot \frac{1}{2} \cdot 1,3 = 1,5 \cdot 10^{-3}$$

$$\delta_{11} = \frac{1}{3EI} \cdot 3 \cdot 1 \cdot 1 = \frac{1}{EI}, \quad \delta_{22} = \frac{1}{EI}; \quad \delta_{12} = \delta_{21} = \frac{1}{2EI}$$

$$\delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \Rightarrow \frac{1}{EI}X_1 + \frac{1}{2EI}X_2 = 0 \Rightarrow X_2 = -2X_1$$

$$\delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \Rightarrow \frac{1,5}{10^3}X_1 + \frac{1}{EI}X_2 = \frac{-1,5}{10^3} \Rightarrow X_1 + 4X_2 = -105$$

$$X_1 = 15 \text{ kNm} \quad X_2 = -30 \text{ kNm}$$



$$M_{1B}^0 = -\frac{3}{2} \cdot 35000 \cdot \frac{10^{-5} \cdot 30}{0,3} = -52,5 \text{ kNm}$$

$$M_{1A} = \frac{4EI}{3} \varphi$$

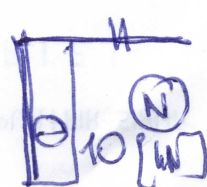
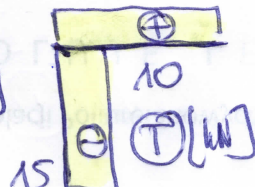
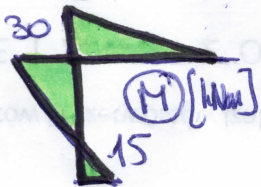
$$M_{1B} = -52,5 + EI \varphi$$

$$M_{A1} = \frac{2EI}{3} \varphi$$

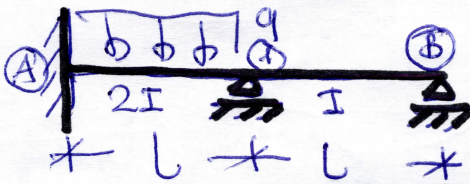
$$\sum M_1 = 0 \Rightarrow \frac{7EI}{3} \varphi = 52,5 \Rightarrow \varphi = \frac{22,5}{EI}$$

$$M_{1A} = 30 \text{ kNm} \quad M_{A1} = 15 \text{ kNm}$$

$$M_{1B} = -52,5 + 22,5 = -30 \text{ kNm}$$



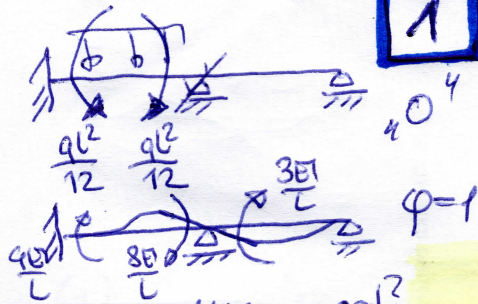




$$M_{1A} = + \frac{qL^2}{12} + \frac{8EI}{L} \varphi$$

$$M_{1B} = \frac{3EI}{L} \varphi$$

$$M_{A1} = - \frac{qL^2}{12} + \frac{4EI}{L} \varphi$$

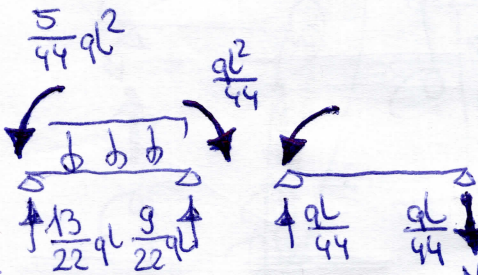


$$\sum M_1 = 0 \Rightarrow \frac{11EI}{L} \varphi = - \frac{2qL^2}{12}, \varphi = - \frac{qL^3}{132EI}$$

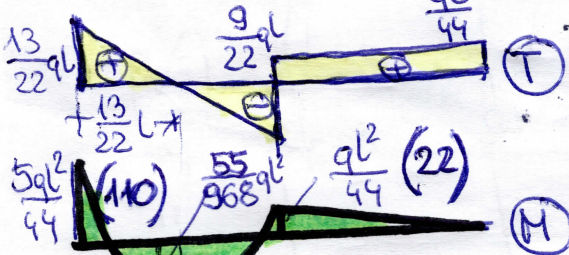
$$M_{1A} = \frac{qL^2}{12} - \frac{2}{33} qL^2 = \frac{qL^2}{44}$$

$$M_{1B} = - \frac{qL^2}{44}$$

$$M_{A1} = - \frac{qL^2}{12} - \frac{qL^2}{33} = - \frac{5}{44} qL^2$$



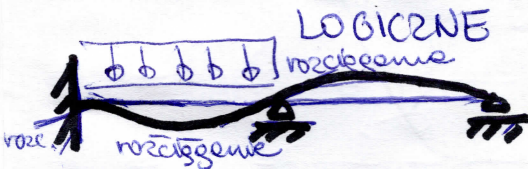
$$M_{max} = \left( - \frac{5qL^2}{44} + \frac{1}{2} \cdot \frac{13}{22} qL \cdot \frac{13}{22} L \right) = \frac{59}{968} qL^2$$



$$M_{avg} = - \frac{1}{88} qL^2 (5+1) - \frac{qL^2}{8} = - \frac{5}{88} qL^2 = - \frac{55}{968} qL^2$$

$$\frac{59}{968} qL^2 (59)$$

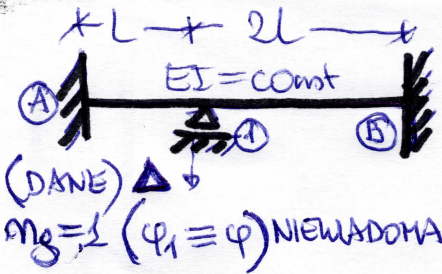
(\*)



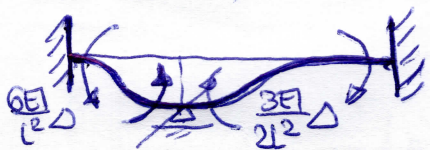
kgf  $\varphi < 0$

OD RAZU PO OBLICZENIU NIEMIŁYCH GEOMETRYCZNYCH (TU: KĄTA  $\varphi$ ), NARYSOWAĆ STAN PRZEMIESZCZEN, WERDYFIKOWAĆ WIZUALNIE, PROSTA LOGIKA

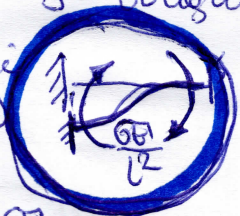




blokada przesunięć poziomych  
wspólna w całym układzie -  
podpora (B) może być utwardzona  
przesunięć, bez dodatkowej reakcji  
(przy zadanym obciążeniu  $H_A = 0$ )



przyjmując  
wzrost ogólny



**2**

$$M_{A1} = -\frac{6EI}{L^2} \Delta + \frac{4EI}{L} \varphi$$

$$M_{B1} = \frac{3EI}{2L^2} \Delta + \frac{2EI}{L} \varphi$$

$$\sum H = 0 \Rightarrow -\frac{9EI}{2L^2} \Delta + \frac{6EI}{L} \varphi = 0$$

$$\varphi = \frac{3}{4} \frac{\Delta}{L}$$

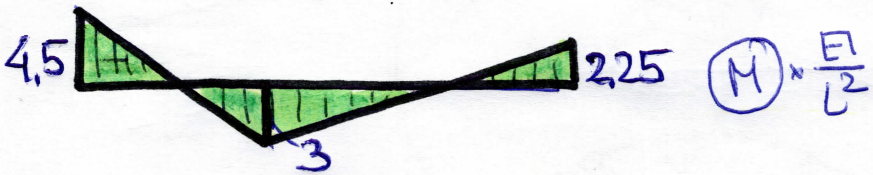
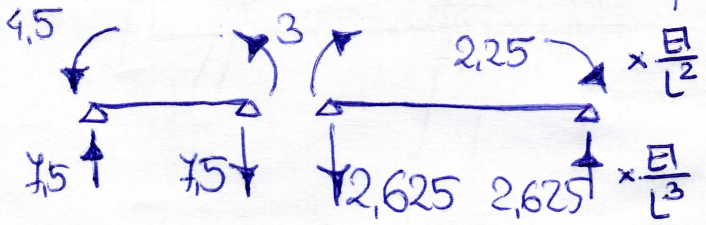
$$M_{A1} = -\frac{6EI}{L^2} \Delta + \frac{2EI}{L} \varphi$$

$$M_{B2} = \frac{3EI}{2L^2} \Delta + \frac{EI}{L} \varphi$$

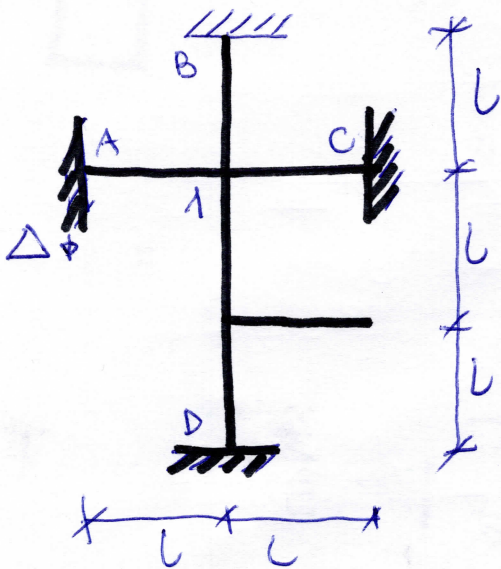


dodatkowi kąt φ - logarytmie,  
przebieg A-1 symetryczny od 1-B

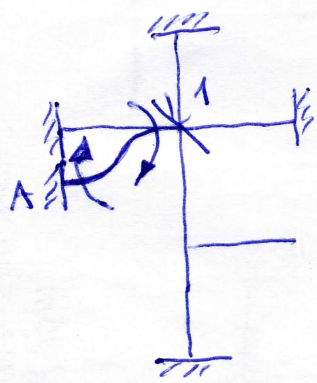
$$M_{A1} = -3 \frac{EI}{L^2} \Delta \quad M_{B1} = 3 \frac{EI}{L^2} \Delta \quad \left\{ \begin{array}{l} M_{A1} = -\frac{9EI}{2L^2} \Delta \\ M_{B2} = \frac{9EI}{4L^2} \Delta \end{array} \right.$$







$m_0 = 1 (\varphi_1 \equiv \varphi)$   
 TILKO WIKRES (M) 3



$$M_{MA}^0 = M_{A1}^0 = \frac{6EI}{L^2} \Delta$$

$$\sum M_1 = 0 \Rightarrow \frac{6EI}{L^2} \Delta + \frac{14EI}{L} \varphi = 0$$

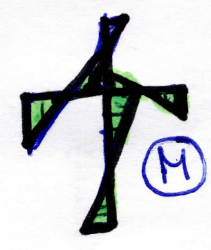
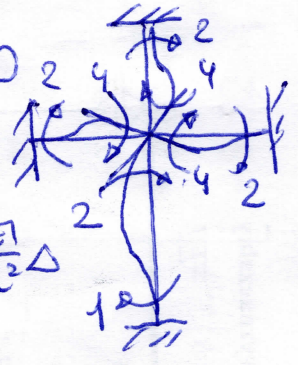
$$\varphi = -\frac{3}{7} \frac{\Delta}{L}$$

$$M_{MA} = \frac{6EI}{L^2} \Delta - \frac{12EI}{7L^2} \Delta = \frac{30EI}{7L^2} \Delta$$

$$M_{MB} = M_{MC} = -\frac{12EI}{7L^2} \Delta$$

$$M_{MD} = -\frac{6EI}{7L^2} \Delta$$

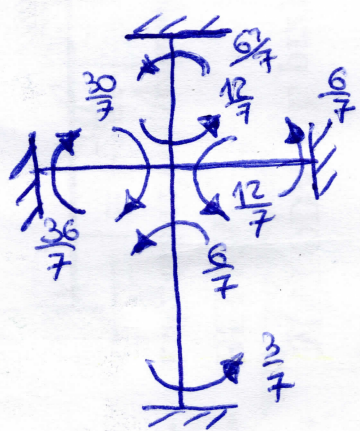
$\varphi = 1$



$$M_{A1} = \frac{6EI}{L^2} \Delta - \frac{6EI}{7L^2} \Delta = \frac{36EI}{7L^2} \Delta$$

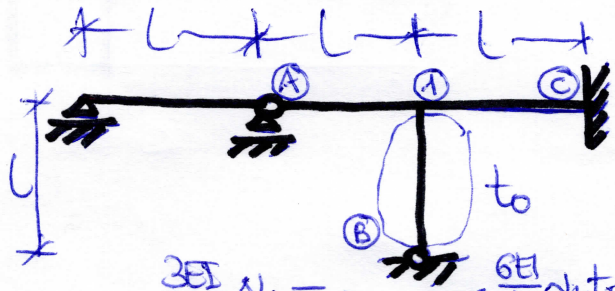
$$M_{B1} = M_{C1} = -\frac{6EI}{7L^2} \Delta$$

$$M_{D1} = -\frac{3EI}{7L^2} \Delta$$





4



Dane:  $\alpha_t, t_0, l, EI$

$\Delta_t = \alpha_t t_0 l$   
SWobodnie wywołanie term.

$$\frac{3EI}{L^2} \Delta_t = \frac{6EI}{L} \alpha_t t_0$$

$$= \frac{3EI}{L} \alpha_t t_0$$

$\sum M_1 = 0 \Rightarrow$

$$-\frac{EI}{L} \alpha_t t_0 + \frac{10EI}{L} \varphi = 0$$

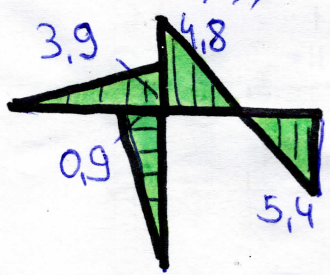
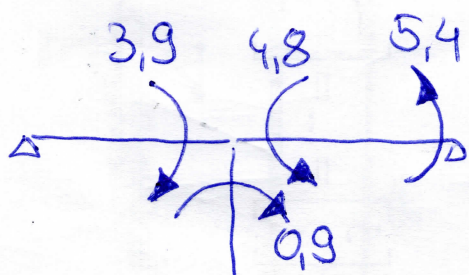
$\varphi = 0,3 \alpha_t t_0$

$$M_{1A} = \frac{3EI}{L} \alpha_t t_0 + 0,9 \frac{EI}{L} \alpha_t t_0 = \frac{3,9EI}{L} \alpha_t t_0$$

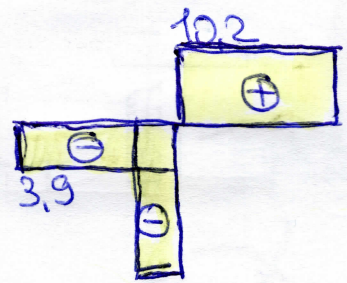
$$M_{1B} = 0,9 \frac{EI}{L} \alpha_t t_0$$

$$M_{1C} = -\frac{6EI}{L} \alpha_t t_0 + \frac{1,2EI}{L} \alpha_t t_0 = \frac{-4,8EI}{L} \alpha_t t_0$$

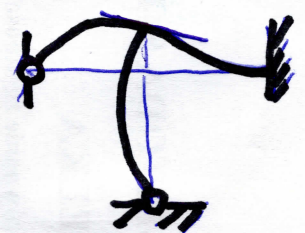
$$M_{21} = -\frac{6EI}{L} \alpha_t t_0 + 0,6 \frac{EI}{L} \alpha_t t_0 = \frac{-5,4EI}{L} \alpha_t t_0$$



$M \times \frac{EI}{L} \alpha_t t_0$



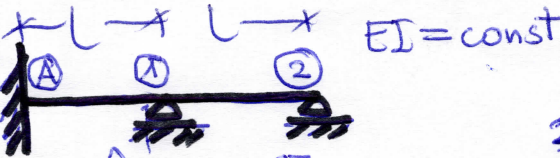
$T \times \frac{EI}{L^2} \alpha_t t_0$



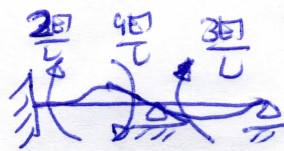
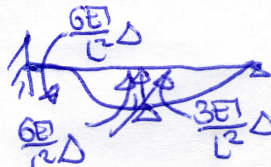
PRZĘSTO A-1  
MNIĘ SZTYWNE  
OD 1-C, STĄD  
OBRÓT

ZADANIE DOMOWE: PODP. C





I  $m_0 = 1$  ( $\varphi_1$ )

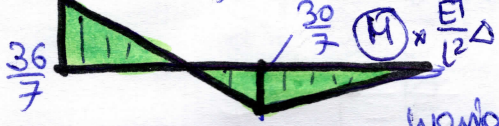
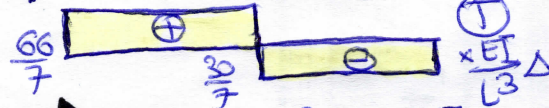
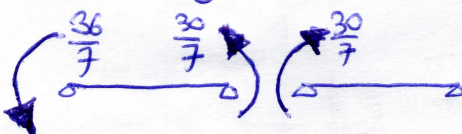


$$M_1 = -\frac{3EI}{L^2} \Delta + \frac{EI}{L} \varphi = 0 \Rightarrow \varphi = \frac{3}{7} \frac{\Delta}{L}$$

$$M_{A1} = -\frac{36}{7} \frac{EI}{L^2} \Delta$$

$$M_{1A} = -\frac{30}{7} \frac{EI}{L^2} \Delta$$

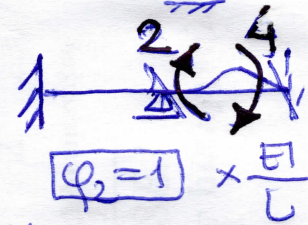
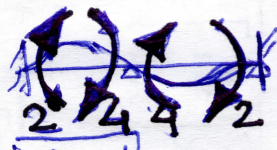
$$M_{12} = \frac{30}{7} \frac{EI}{L^2} \Delta$$



KĄT DODATNI - PRZĘSŁO A-1 SZTYWIEJSZE, NIŻ 1-2



II  $m_0 = 2$  ( $\varphi_1, \varphi_2$ )



wszystkie równie  $\frac{6EI}{L^2} \Delta$

$$\varphi_1 = 1$$

$$\varphi_2 = 1 \times \frac{EI}{L}$$

$$\sum M_1 = \frac{8EI}{L} \varphi_1 + \frac{2EI}{L} \varphi_2 = 0 \Rightarrow \varphi_2 = -4\varphi_1$$

$$\sum M_2 = \frac{6EI}{L^2} \Delta + \frac{2EI}{L} \varphi_1 + \frac{4EI}{L} \varphi_2 = 0 \Rightarrow \varphi_1 + 2\varphi_2 = -\frac{3\Delta}{L}$$

$$-7\varphi_1 = -\frac{3\Delta}{L} \Rightarrow \varphi_1 = \frac{3}{7} \frac{\Delta}{L}, \varphi_2 = -\frac{12}{7} \frac{\Delta}{L}$$

$$M_{A1} = \frac{EI}{L^2} \Delta \left( 6 + \frac{6}{7} \right) = -\frac{36}{7} \frac{EI}{L^2} \Delta$$

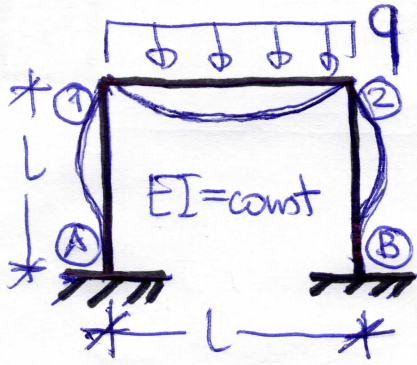
$$M_{12} = \frac{EI}{L^2} \Delta \left( 6 + \frac{12}{7} - \frac{24}{7} \right) =$$

$$M_{1A} = \frac{EI}{L^2} \Delta \left( -6 + \frac{12}{7} \right) = -\frac{30}{7} \frac{EI}{L^2} \Delta$$

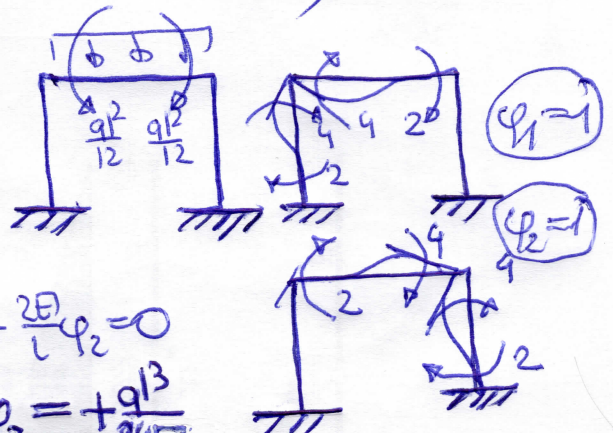
$$= \frac{EI}{L^2} \cdot \frac{30}{7}$$

$$M_{21} = \frac{EI}{L^2} \Delta \left( 6 + \frac{6}{7} - \frac{48}{7} \right) = 0$$





$$M_0 = 2(\varphi_1, \varphi_2)$$



$$\sum M_1 = -\frac{qL^2}{12} + \frac{8EI}{L}\varphi_1 + \frac{2EI}{L}\varphi_2 = 0$$

$$4\varphi_1 + \varphi_2 = +\frac{qL^3}{24EI}$$

$$\sum M_2 = \frac{qL^2}{12} + \frac{2EI}{L}\varphi_1 + \frac{8EI}{L}\varphi_2 = 0$$

$$\varphi_1 + 4\varphi_2 = -\frac{qL^3}{24EI}$$

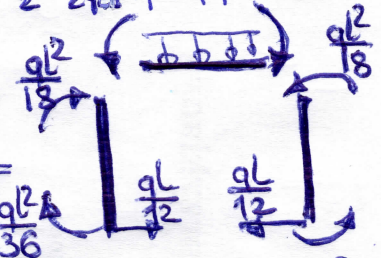
$$\varphi_1 = \frac{qL^3}{72EI}$$

$$\varphi_2 = -\frac{qL^3}{72EI}$$

$$W = 15$$

$$W_1 = \frac{qL^3}{24EI} \begin{vmatrix} 1 & 1 \\ -1 & 4 \end{vmatrix} = \frac{5qL^3}{24EI}$$

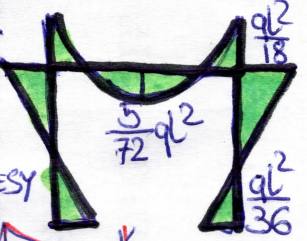
$$W_2 = \frac{qL^3}{24EI} \begin{vmatrix} 4 & 1 \\ 1 & -1 \end{vmatrix} = -\frac{5qL^3}{24EI}$$



$$M_{1A} = \frac{qL^2}{18}$$

$$M_{12} = qL^2 \left( -\frac{1}{12} + \frac{1}{18} - \frac{1}{36} \right) = \frac{qL^2}{36} \frac{-3+2-1}{1} = -\frac{qL^2}{18}$$

$$M_{21} = qL^2 \left( \frac{1}{12} + \frac{1}{36} - \frac{1}{18} \right) = \frac{qL^2}{36} (3+1-2) = \frac{qL^2}{18}$$

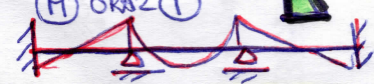


$$M_{2B} = -\frac{qL^2}{18}$$

$$M_{A1} = \frac{qL^2}{36}$$

$$M_{B2} = -\frac{qL^2}{36}$$

TE SAMEWYKRESY (M) ORAZ (T)



wykorzystanie symetrii:  $\varphi_2 = -\varphi_1$

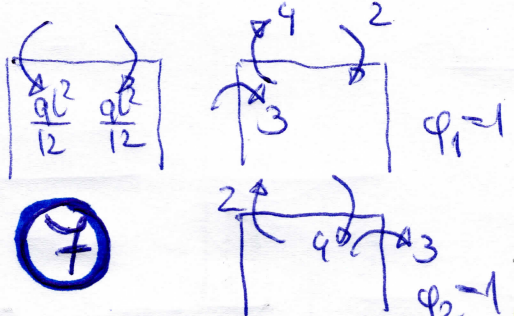
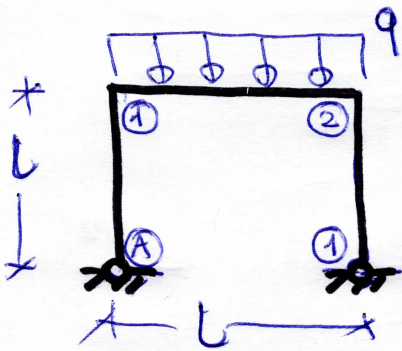
$$M_{12} = -\frac{qL^2}{12} + \frac{4EI}{L}\varphi_1 + \frac{2EI}{L}\varphi_2 = -\frac{qL^2}{12} + \frac{2EI}{L}\varphi_1$$

$$M_{1A} = \frac{4EI}{L}\varphi_1 \quad M_{A1} = \frac{2EI}{L}\varphi_1$$

momenty symetryczne

$$\left. \begin{aligned} \sum M_1 &= 0 \Rightarrow \varphi_1 = \frac{qL^3}{72EI} \\ M_{12} &= -\frac{qL^2}{18} \\ M_{1A} &= \frac{qL^2}{18} \quad M_{A1} = \frac{qL^2}{36} \end{aligned} \right\}$$





To rozszerzono bez wychylenia  
warunku symetrii  
 $\varphi_2 = -\varphi_1$

$W = 45$

$$W_1 = \frac{qL^3}{12EI} \begin{vmatrix} 1 & 2 \\ 1 & 7 \end{vmatrix} = \frac{3qL^3}{4EI}$$

$$W_2 = \frac{qL^3}{12EI} \begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix} = -\frac{3qL^3}{4EI}$$

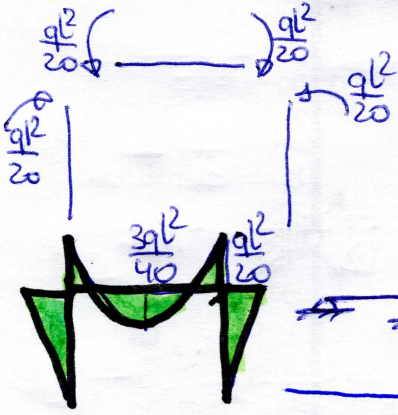
$$\varphi_1 = \frac{qL^3}{60EI} \quad \varphi_2 = -\frac{qL^3}{60EI}$$

$$\sum M_1 = -\frac{qL^2}{12} + \frac{7EI}{L}\varphi_1 + \frac{2EI}{L}\varphi_2 = 0$$

$$7\varphi_1 + 2\varphi_2 = \frac{qL^3}{12EI}$$

$$\sum M_2 = \frac{qL^2}{12} + \frac{2EI}{L}\varphi_1 + \frac{7EI}{L}\varphi_2 = 0$$

$$2\varphi_1 + 7\varphi_2 = -\frac{qL^3}{12EI}$$

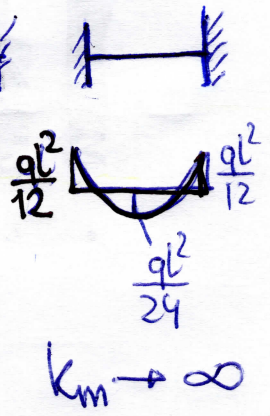
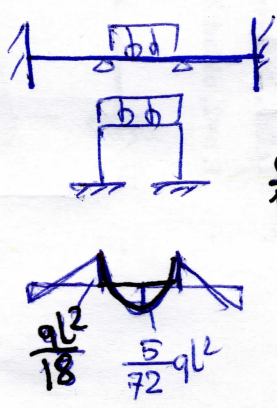
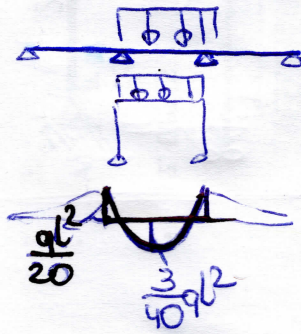
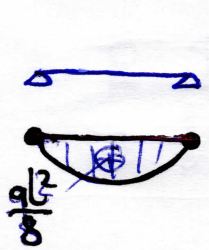
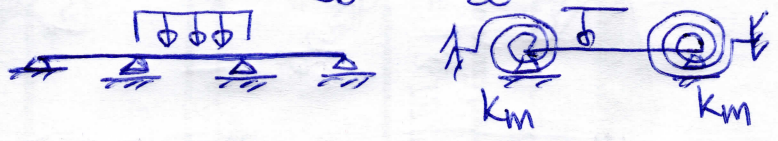


$$M_{1A} = \frac{qL^2}{20}$$

$$M_{12} = qL^2 \left( -\frac{1}{12} + \frac{4}{60} - \frac{2}{60} \right) = -\frac{qL^2}{20}$$

$$M_{21} = qL^2 \left( \frac{1}{12} + \frac{2}{60} - \frac{4}{60} \right) = \frac{qL^2}{20}$$

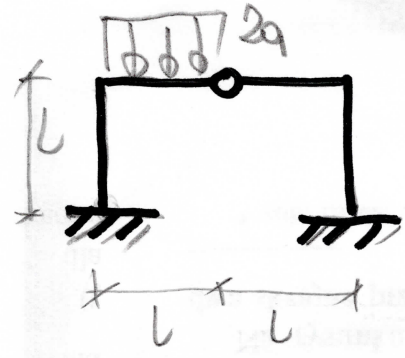
$$M_{2B} = -\frac{qL^2}{20}$$



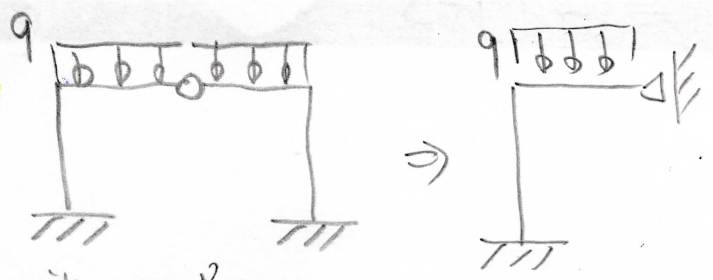
$k_m = 0$

$k_m \rightarrow \infty$



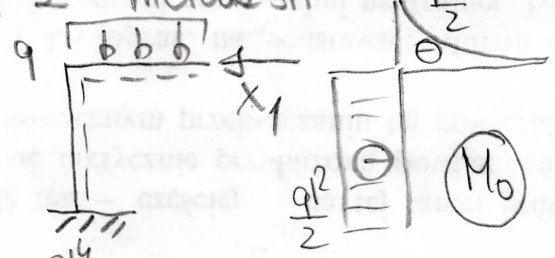


**symetrisne:**



$EI = \text{const}$

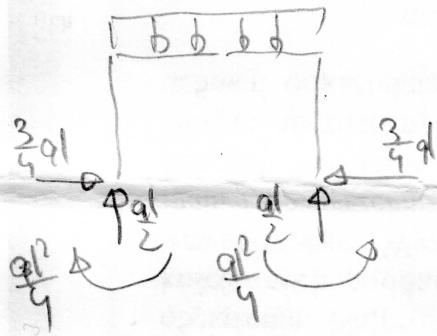
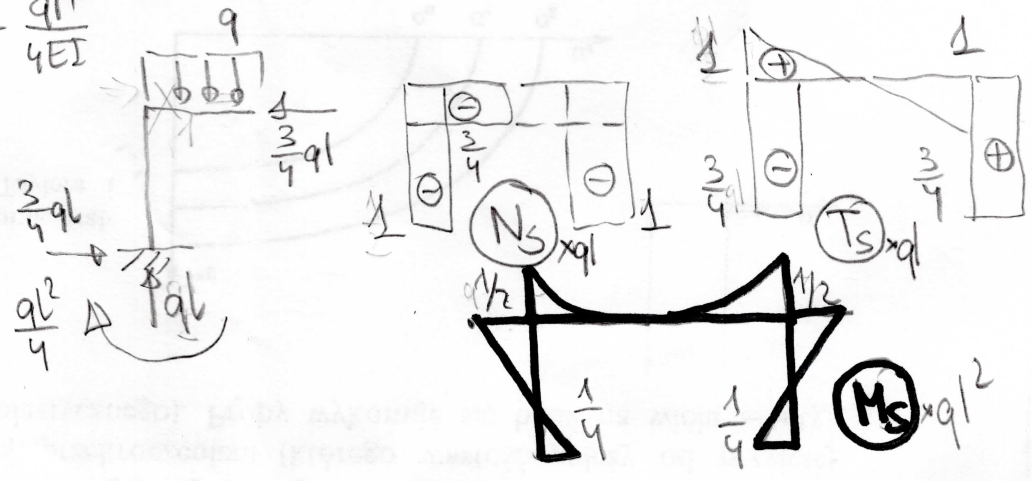
$m_s = 1$  metoda sit



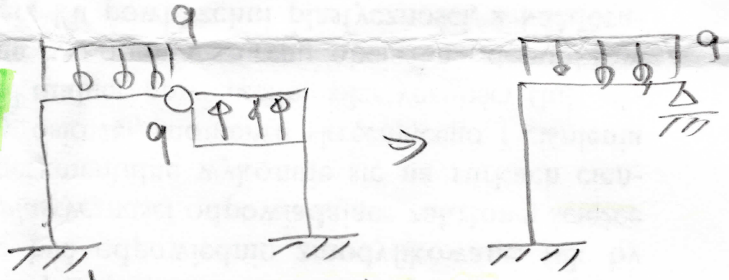
$$\delta_{10} = -\frac{1}{EI} \cdot \frac{1}{2} \cdot l \cdot l \cdot \frac{ql^2}{2} = -\frac{ql^4}{4EI}$$

$$\delta_{11} = \frac{1}{EI} \cdot \frac{1}{3} \cdot l \cdot l \cdot l = \frac{l^3}{3EI}$$

$\Rightarrow X_1 = \frac{3}{4} ql$



**antisym...**

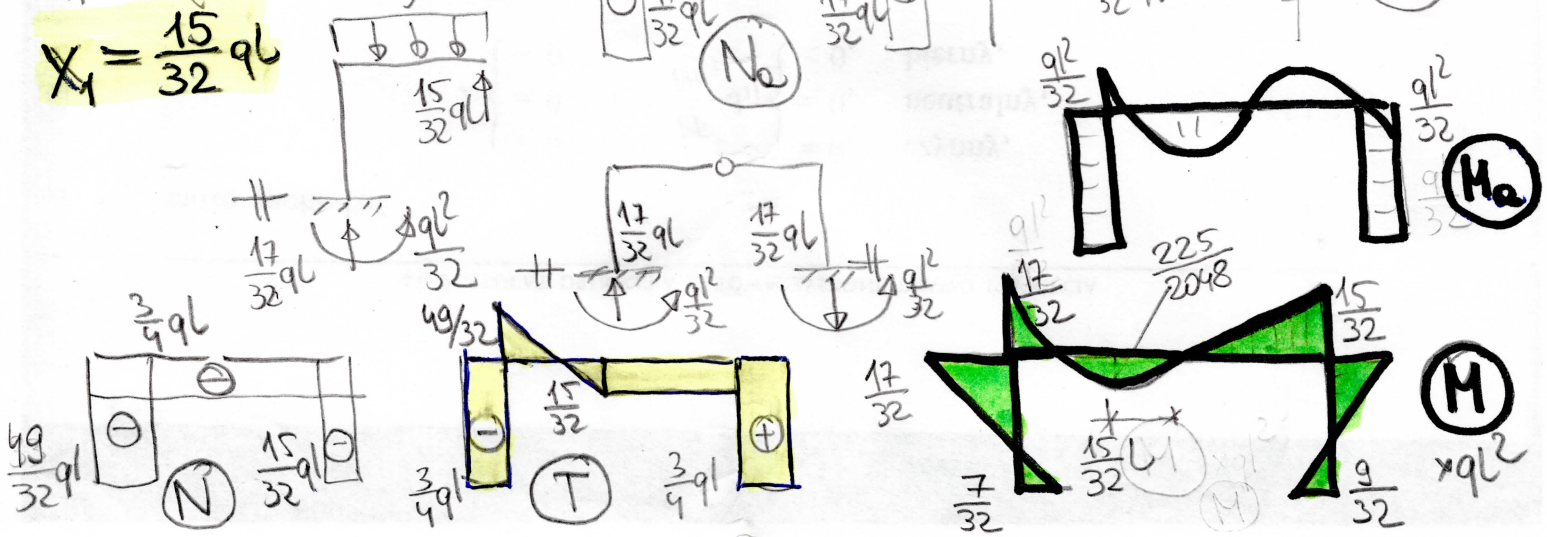
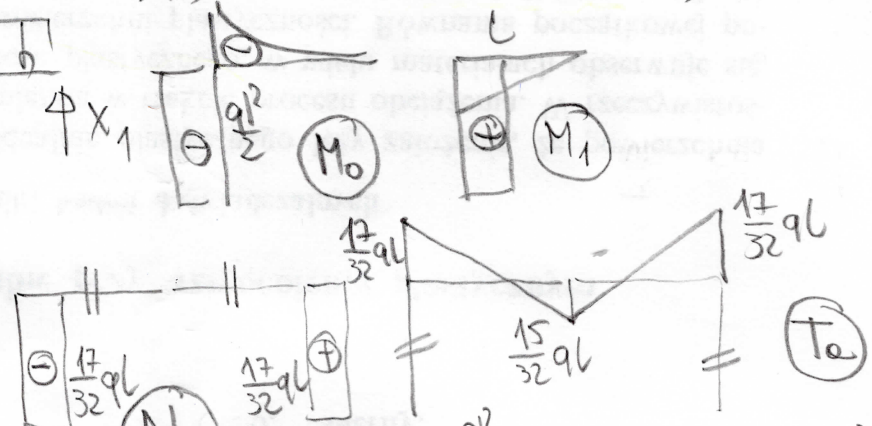


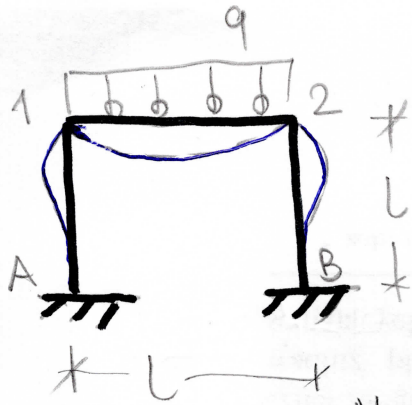
$m_s = 1$  metoda sit

$$\delta_{10} = -\frac{1}{EI} \left( \frac{1}{4} \cdot l \cdot l \cdot \frac{ql^2}{2} + l \cdot l \cdot \frac{ql^2}{2} \right) = -\frac{5ql^4}{8EI}$$

$$\delta_{11} = \frac{1}{EI} \left( \frac{1}{3} \cdot l \cdot l \cdot l + l \cdot l \cdot l \right) = \frac{4l^3}{3EI}$$

$X_1 = \frac{15}{32} ql$





Metoda premuljenja:  $u=0, \varphi_2 = -\varphi_1$  (symetrično)

$$M_{1A} = \frac{4EI}{L} \varphi_1$$

$$M_{A1} = \frac{2EI}{L} \varphi_1$$

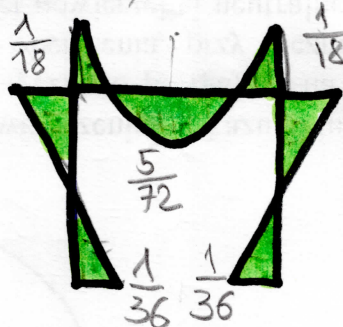
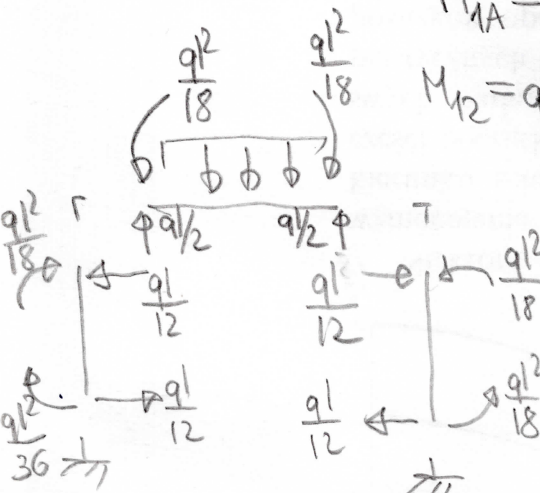
$$M_{12} = -\frac{qL^2}{12} + \frac{4EI}{L} \varphi_1 + \frac{2EI}{L} \varphi_2 = -\frac{qL^2}{12} + \frac{2EI}{L} \varphi_1$$

romanele romanezi:  $\sum M_1 = -\frac{qL^2}{12} + \frac{6EI}{L} \varphi_1 = 0$

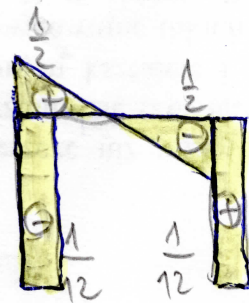
$$\varphi_1 = \frac{qL^3}{72EI} \text{ dodatni}$$

$$M_{1A} = \frac{qL^2}{18} \quad M_{A1} = \frac{qL^2}{36}$$

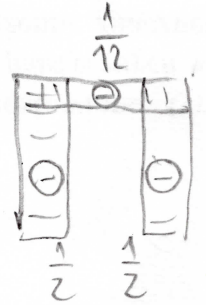
$$M_{12} = qL^2 \left( -\frac{1}{12} + \frac{1}{36} \right) = -\frac{qL^2}{18} \text{ OK.}$$



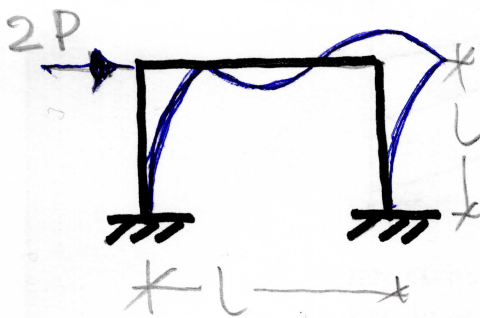
$(M) \times qL^2$



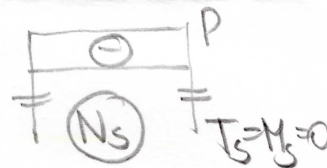
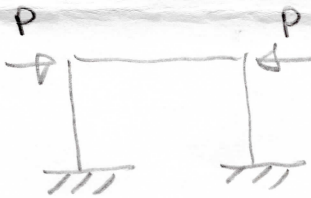
$(T) \times qL$



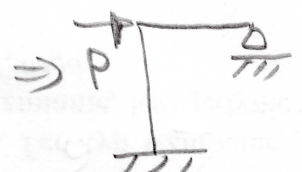
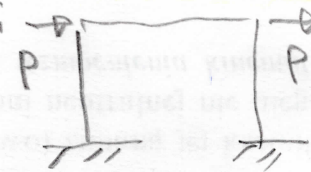
$(N) \times qL$



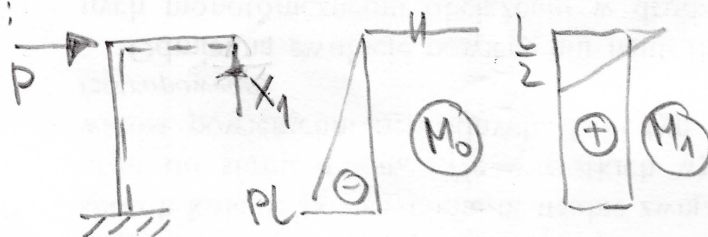
stan simetrični:



stan asimetrični:



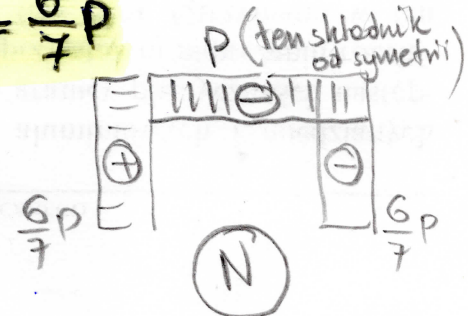
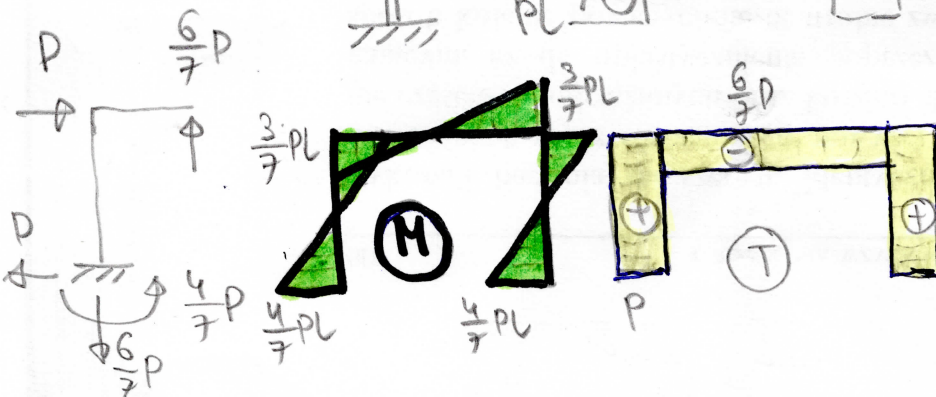
razmigranje potonj ukladu  
metoda sit:



$$\delta_{10} = \frac{-1}{EI} \cdot \frac{1}{2} L \cdot \frac{L}{2} \cdot PL = -\frac{PL^3}{4EI}$$

$$\delta_{11} = \frac{1}{EI} \left( \frac{1}{3} \cdot \frac{L}{2} \cdot \frac{L}{2} \cdot \frac{L}{2} + 1 \cdot L \cdot \frac{L}{2} \cdot \frac{L}{2} \right) = \frac{7L^3}{24EI}$$

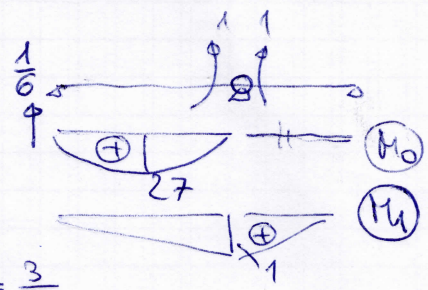
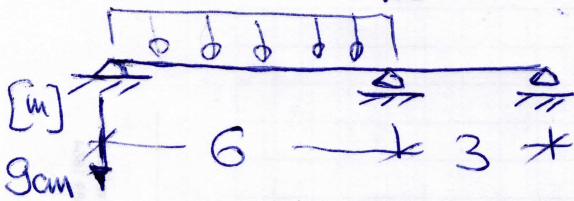
$$X_1 = \frac{6}{7} P$$





$$6 \frac{kN}{m}$$

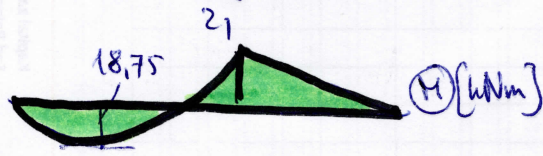
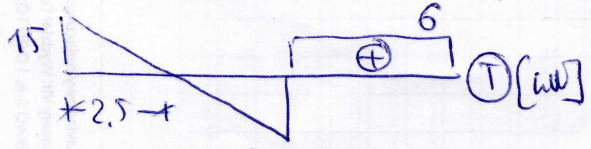
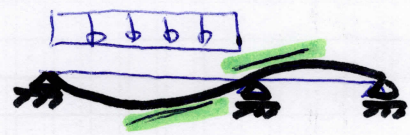
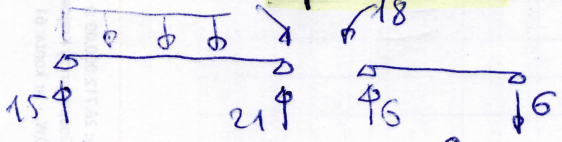
$$EI = 10^3 kNm^2$$



$$\delta_{10} = \frac{1}{EI} \cdot \frac{1}{3} \cdot 6 \cdot 27 \cdot 1 = \frac{54}{EI}$$

$$\delta_{11} = \frac{1}{EI} \left( \frac{1}{3} \cdot 6 \cdot 1 \cdot 1 + \frac{1}{3} \cdot 3 \cdot 1 \cdot 1 \right) = \frac{3}{EI}$$

$$X_1 = -18 kNm$$



osledenie:  $\delta_{10} = \frac{0,09}{6} = 0,015 \Rightarrow X_1 = -\frac{10^3}{3} \cdot 0,015 = -5 kNm$

