
RELIABILITY – BASED OPTIMIZATION

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Introduction

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RELIABILITY OF ENGINEERING STRUCTURES

Three levels (methods) of design incorporating input data variability:

Level 1 - design standards employing partial safety factors

Level 2 - numerical parameters reflect load and resistance uncertainty

Level 3 - direct application of load and resistance random variables

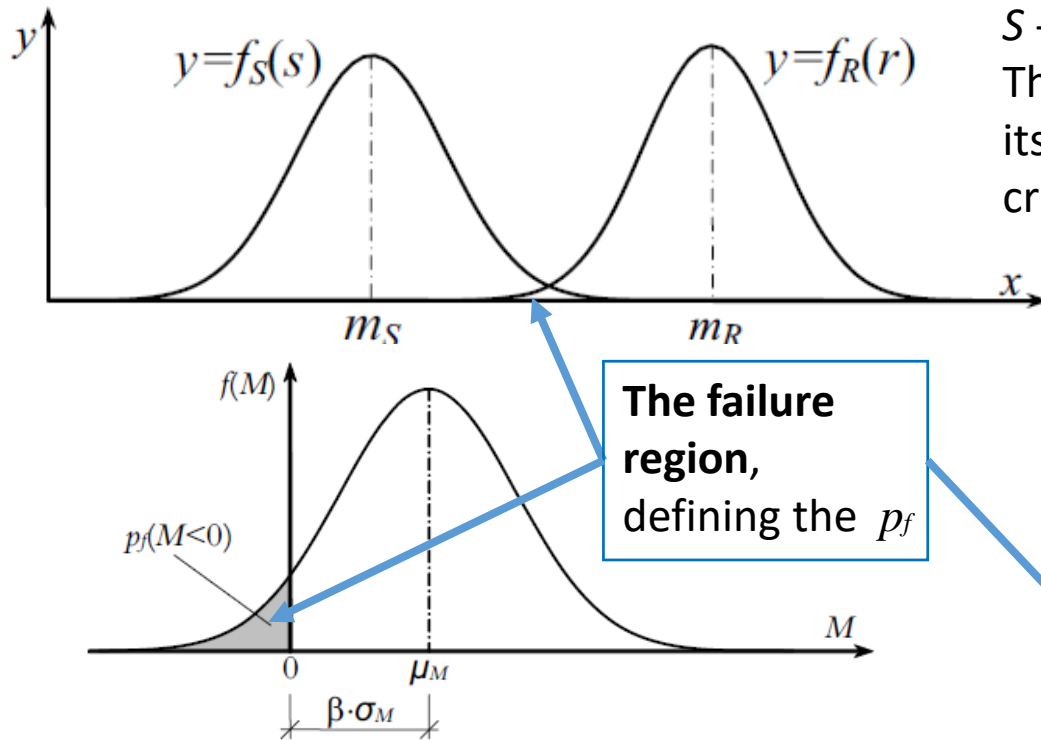
Design controlled by Level 1 methods forms the domain of engineering design subjects (steel, concrete, timber structures, etc.)

Structural design according to Levels 2 and 3 is regarded in the Reliability course only.

Note, that the Eurocodes include reliability assessment criteria based on levels higher than the first.

Most of the computations employ the Monte Carlo method worked by the Polish scientist **Stanisław Ulam** in the course of Project Manhattan, at Los Alamos, 1947

The Level 2 methods



The Level 2 methods

R - structural resistance

S - structural load (load effect)

The so called limit state function, its simplest form, involved in the failure criterion :

$$M(\cdot) = R - S < 0$$

Reliability index

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

Failure probability

$$p_f = \Phi(-\beta)$$

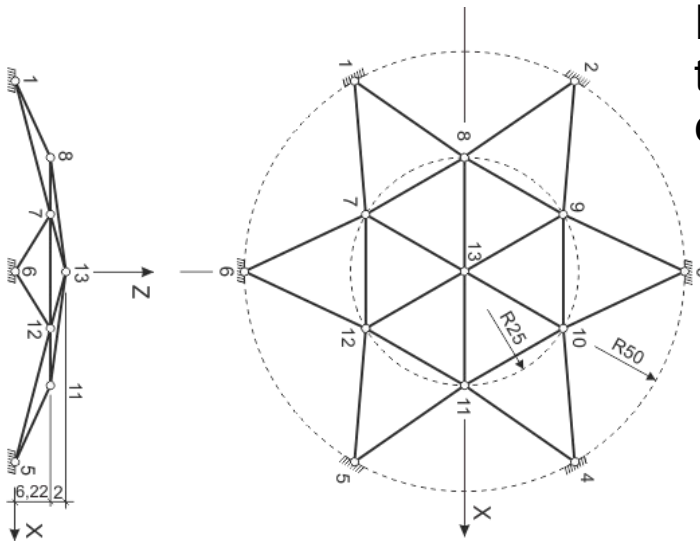
Φ - standard Gaussian CDF

Tablica 2.1. Orientacyjne wartości docelowego wskaźnika niezawodności β według [91]

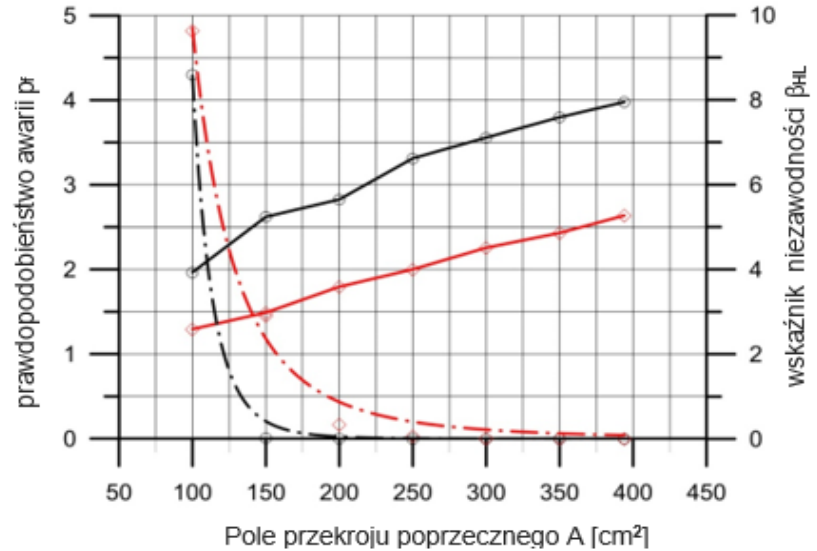
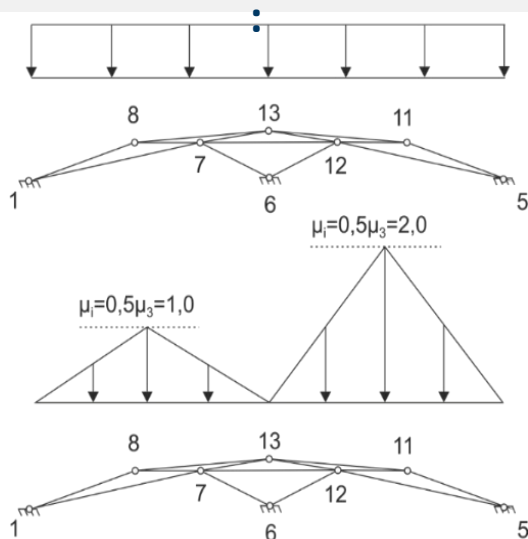
Stan graniczny	Docelowy indeks niezawodnościowy (obliczeniowy okres użytkowania)	Docelowy indeks niezawodnościowy (jeden rok)
nośności	3,8	4,7
zmęczenia	1,5–3,8	–
użytkowalności (nieodwracalny)	1,5	3,0

Example - reliability assessment of a 3D bar-element dome

Estimation of reliability time change, determining the minimum required cross-sectional area A of elements



Snow load, symmetric and anti-symmetric cases



According to PN-EN-190 standard:

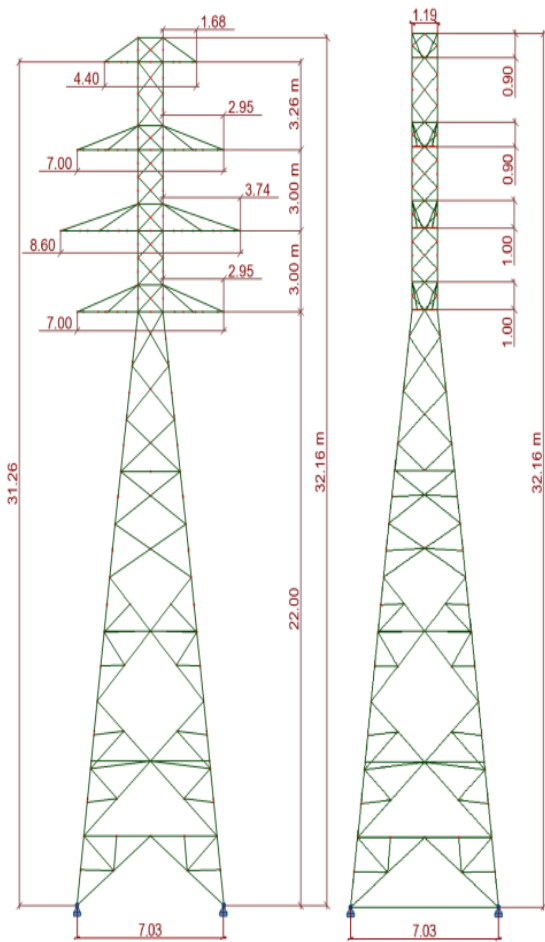
reliability class **RC3**

$\beta_{HL} = 4,3$

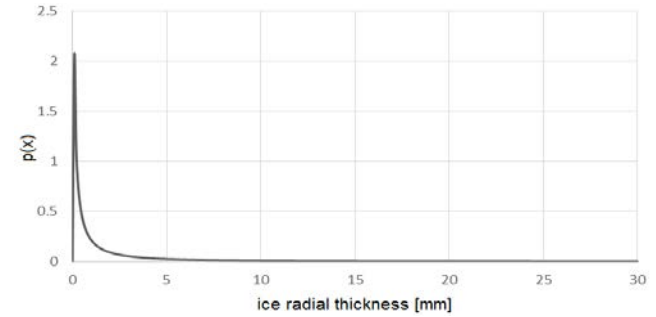
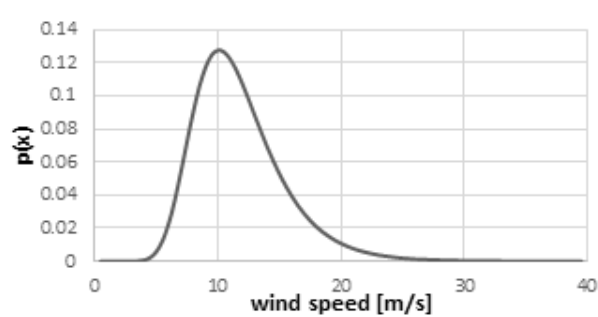
$A^{sym} = 100 \text{ cm}^2$

$A^{asym} = 300 \text{ cm}^2$

Example - reliability assessment of a power transmission tower

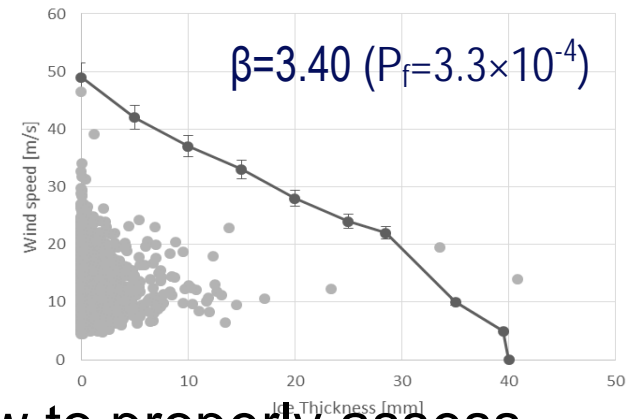
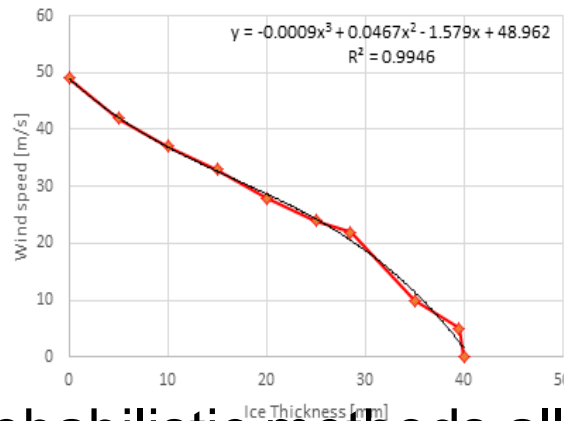


Two random variables assumed - wind and ice accretion



Their combination is proved to possibly cause structural failure or catastrophe

The tower reliability is assessed by Monte Carlo simulation method

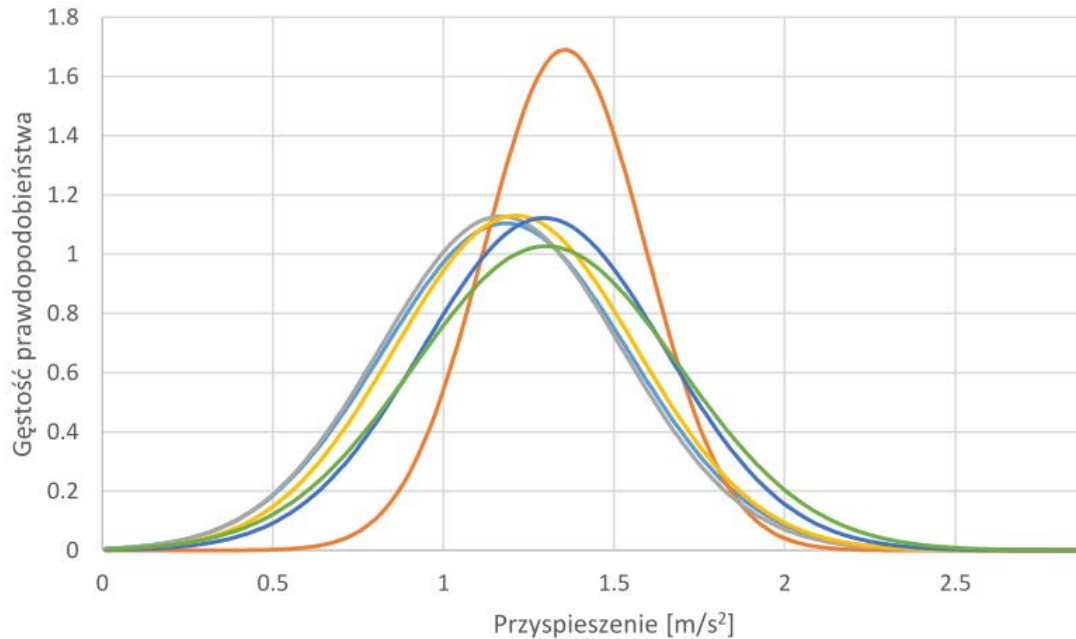


Conclusion: simple probabilistic methods allow to properly assess the reliability of engineering structures incorporating Level 1 and 2 methods and to draw guidelines for reliability improvement

Example: probabilistic sensitivity analysis of a footbridge

Five random variables have been assumed to represent the cross-sections of:

1. main load-carrying cables
2. hangers
3. the columns and spandrel beams of pylons
4. the main girders of the deck
5. longitudinal and transversal beams



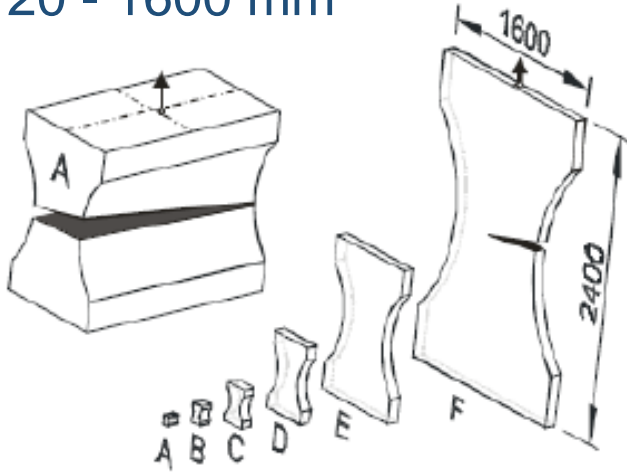
— Średni — Id. Lina główna — Id. Wieszaki
 — Id. Pylon — Id. Belki główne — Id. Podłużnice

Conclusion: the main cables show the highest impact on footbridge serviceability

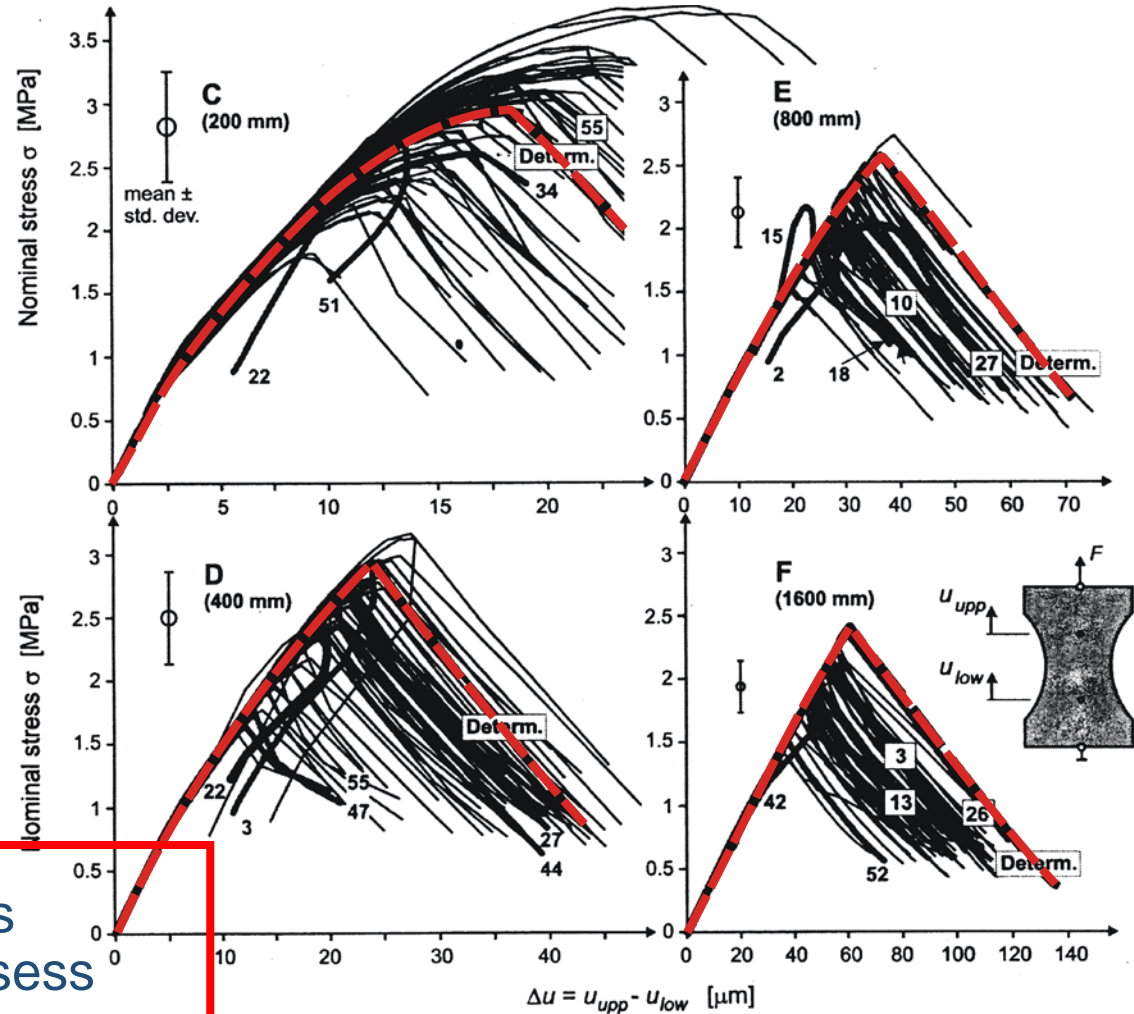
Their cross-sections may be adjusted to provide optimal footbridge design

Example: statistical scale effect of concrete specimens at tension

Specimen dimensions:
20 - 1600 mm



Deterministic results: -----



Conclusions:

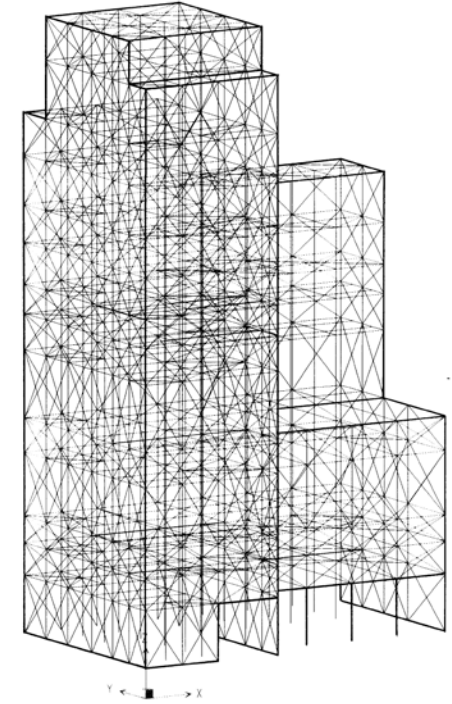
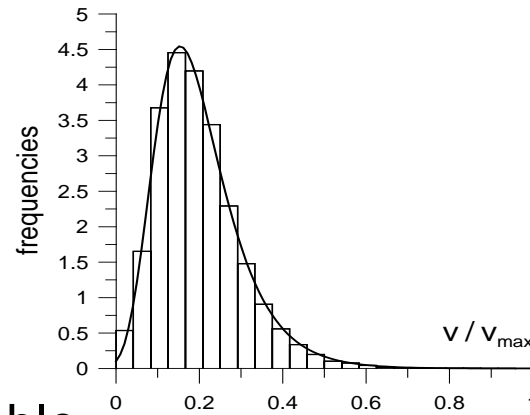
- deterministic results of large specimens reach their maximum values

- the probabilistic approach is the only one to relevantly assess materials and structure in this case

The designed multistorey structure
at the vicinity of the Northern Port, Gdańsk

The serviceability limit state (SGU)
- the horizontal deflection u
of the upper structural deck is investigated
 u_0 - allowable deflection

The wind speed histogram
in the Northern Port, Gdańsk
courtesy of the Institute of meteorology
and Water Management, 1990-99
probabilistic model:
the extreme type I (Gumbel) variable



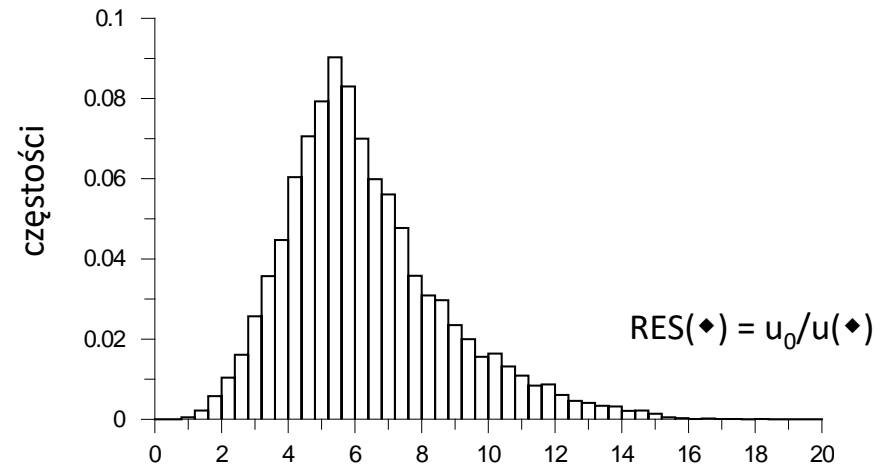
The histogram of the variable

$$RES(\omega) = \frac{u_0}{u(\omega)}$$

is the structural limit state histogram

The probability of allowable
deflection exceedance:

$$p_f = P(RES(\omega) < 1.0) = 0.0002$$

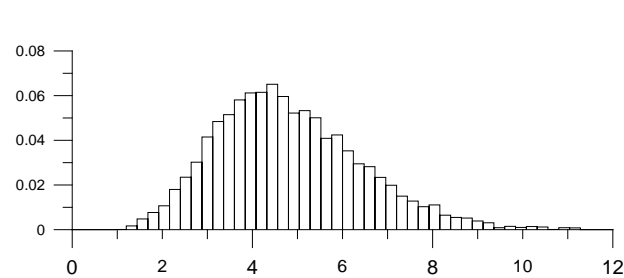
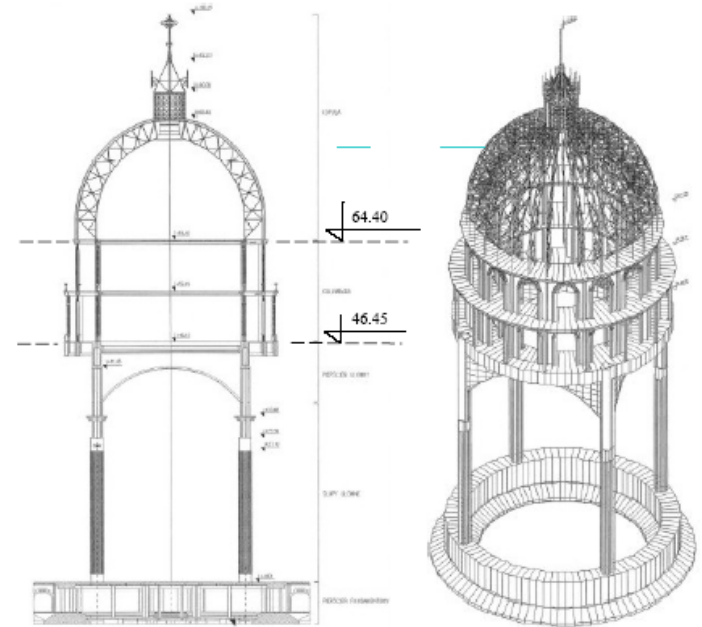


SGU - relative deflections of upper and lower floors of the Licheń Basilica colonnade

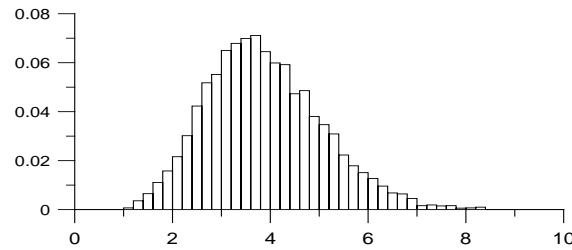
The wind load on both colonnade storeys - transformed wind speed variables (Gumbel-distributed)

Different correlation variants of both wind load variables

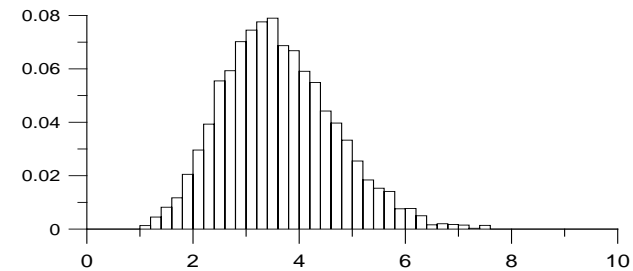
The limit state histograms - the variable λ
 $\lambda > 1$ – allowable deflection not exceeded
 $\lambda < 1$ – allowable deflection exceeded



no correlation ($\rho = 0$)



partial correlation ($\rho = 0.64$)



full correlation ($\rho = 1$)

Each case produces the probability of allowable deflection exceedance

$$p_f = P(\lambda < 1.0) < 10^{-4}$$