EXAMPLE – MC, PEM RSM and other methods

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ORIGINAL PAPER

Reliability-Based Stability Analysis of a Baltic Cliff by the Combined Response Surface Method

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Abstract The study presents a probabilistic stability analysis of a Baltic cliff in Jastrzębia Góra, Poland. Progressive slope erosion is a threat to adjacent buildings, so safety assessment of the slope is essential. The cliff shows a compound, multi-layered geological structure, which makes the analysis of its reliability a complex multivariate problem. A simple, successfully applied in geotechnical computations characterized by dispersion and uncertainty of soil data as well as a relatively high damage probability.

Keywords Probabilistic sensitivity · Reliability · Cliff stability · Monte Carlo method · Response Surface Method · Point Estimate Method

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Jastrzębia Góra cliff landslide 2002

The average rate of the cliff regression landwards:

- in the 70s of 20th century – 0.3 m/year
- in the 80s – 1.6 m/year
- 2002: a series of several landslides developed – 8-10 m landwards

Historical data and results of long-term measurements

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- Sikora Z., Subotowicz W., Wyroślak M., Ossowski R. (2015), Emergency condition of the cliff shore in Jastrzębia Góra, XXVII Scientific and Technical Conference "Construction Failures 2015", Miedzyzdroje, 449-460 (in Polish).
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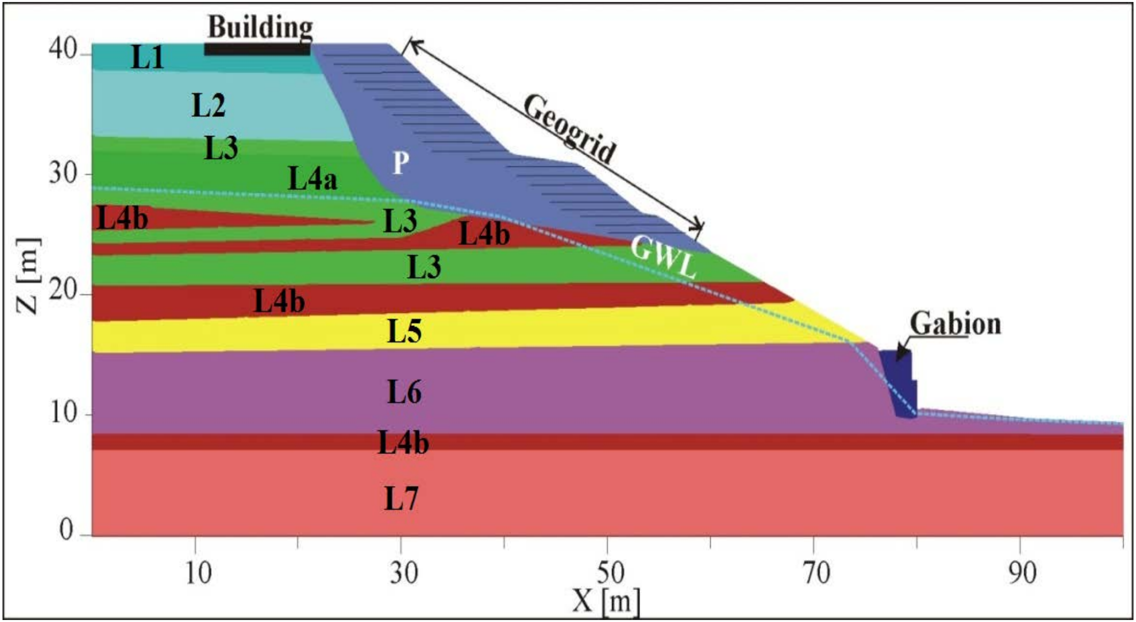
Geological cross-sections



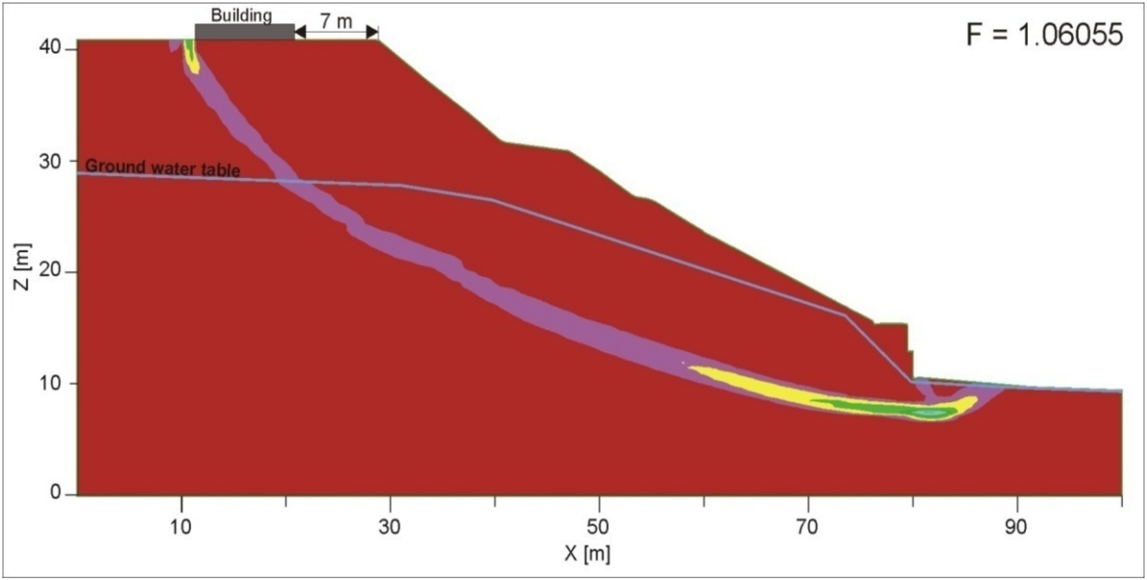
Data

Layer	Soil	Bulk density ρ_i [g/cm^3]	Cohesion c_i [kPa]			Internal friction angle ϕ_i [°]		
			designation	mean value	stan. dev.	designation	mean value	stan. dev.
L1	silty loam	2.05	x_1 [c L1]	$\mu_1 = 40.0$	$\sigma_1 = 12.0$	x_2 [ϕ L1]	$\mu_2 = 15.0$	$\sigma_2 = 1.5$
L2	sandy loam	1.80	x_3 [c L2]	$\mu_3 = 15.0$	$\sigma_3 = 4.5$	x_4 [ϕ L2]	$\mu_4 = 29.0$	$\sigma_4 = 2.9$
L3	clay	2.05	x_5 [c L3]	$\mu_5 = 45.0$	$\sigma_5 = 13.5$	x_6 [ϕ L3]	$\mu_6 = 10.0$	$\sigma_6 = 1.0$
L4a	fine sand	1.80	–	0	0	x_7 [ϕ L4a]	$\mu_7 = 33.0$	$\sigma_7 = 3.3$
L4b	fine sand	1.80	–	0	0	x_8 [ϕ L4b]	$\mu_8 = 35.0$	$\sigma_8 = 3.5$
L5	silty loam	2.05	x_9 [c L5]	$\mu_9 = 58.0$	$\sigma_9 = 17.4$	x_{10} [ϕ L5]	$\mu_{10} = 19.0$	$\sigma_{10} = 1.9$
L6	loamy sand	2.18	x_{11} [c L6]	$\mu_{11} = 21.8$	$\sigma_{11} = 6.54$	x_{12} [ϕ L6]	$\mu_{12} = 27.0$	$\sigma_{12} = 2.7$
L7	clay	2.15	x_{13} [c L7]	$\mu_{13} = 50.0$	$\sigma_{13} = 15.0$	x_{14} [ϕ L7]	$\mu_{14} = 17.0$	$\sigma_{14} = 1.7$

Deterministic calculations – undrained and drained state

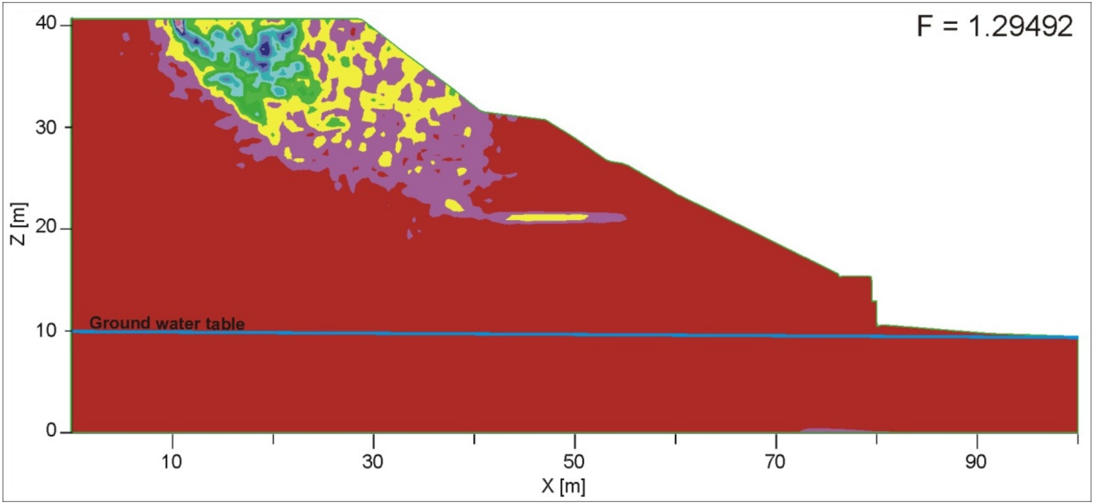
Itasca FLAC (FLAC 4.0, 2000)

- Finite Difference (FD) Method
- the Mohr-Coulomb failure criterion
- trial factor *F* (to reduce the cohesion *c*, friction angle *f*, and tensile strength s_t



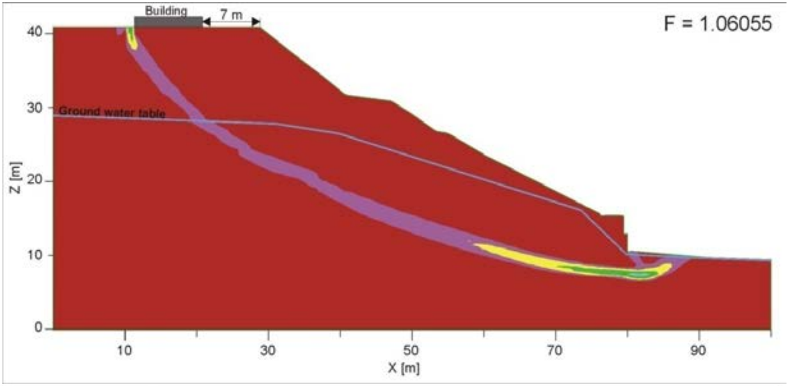
Location of the potential slip zone natural (undrained) state $F_{undr} = 1.06055$

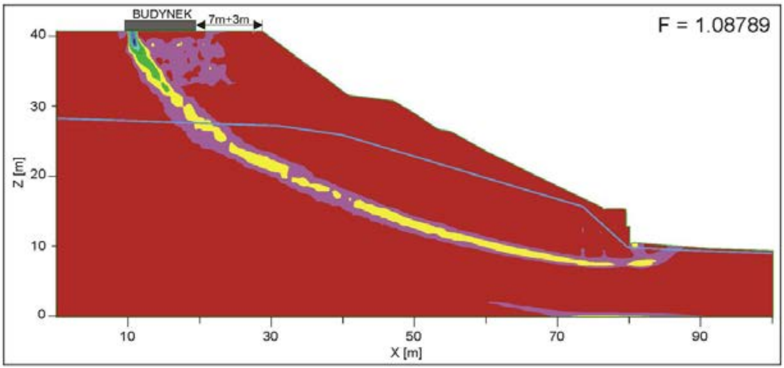
Hypothetical drained state

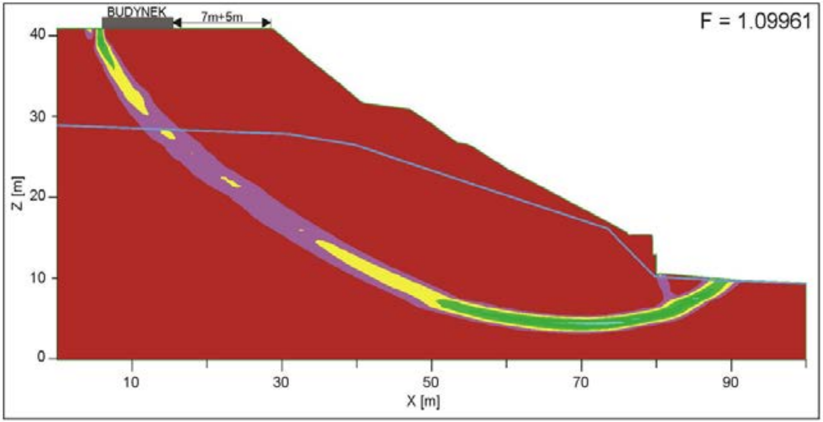


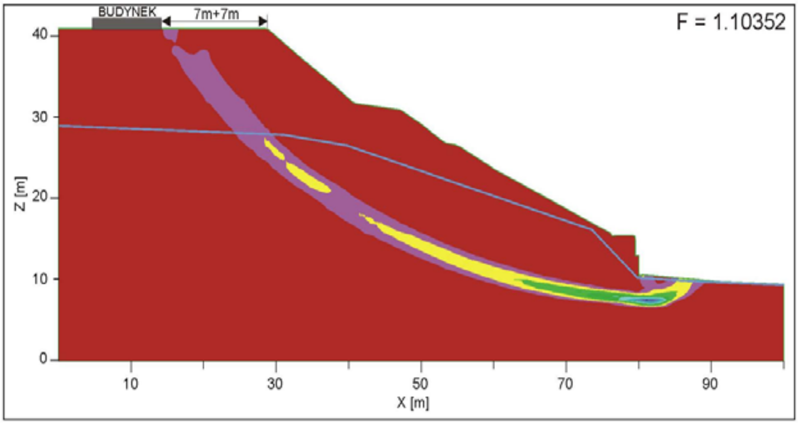
$F_{dr} = 1.29492$

Deterministic calculations – Location of the potential slip zone
Building locations: 7, 10, 12, 14 m






BUDYNEK
7m+5m
F = 1.09961
Z [m]
X [m]


BUDYNEK
7m+7m
F = 1.10352
Z [m]
X [m]

Stability assessment of a cliff in Jastrzębia Góra

The aim of the work:

The standard cliff analysis should be broadened by a random approach which will take into consideration the strength parameters variation assessed on the basis of the laboratory tests
to check the safety margin and the reliability of the slope

Sensitivity and probabilistic analysis

Combined Response Surface Method (CRSM):

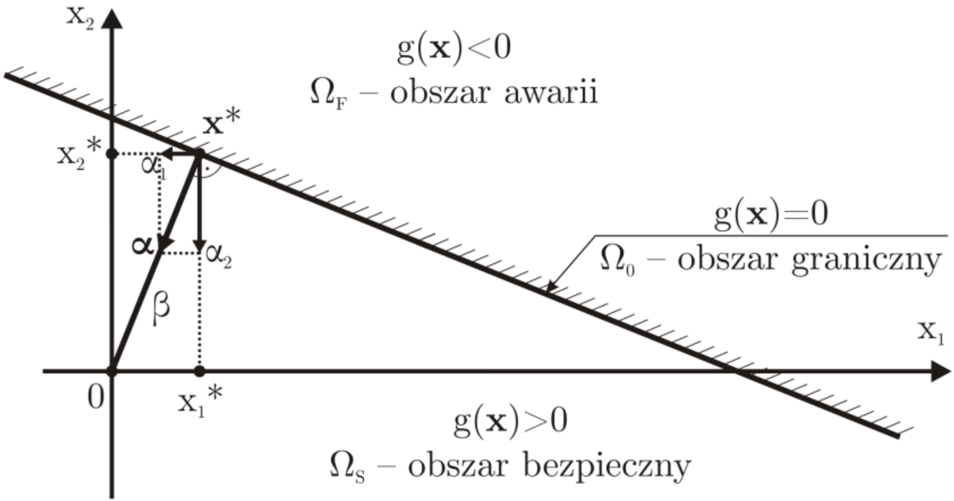
- Monte Carlo Method (MC)
- Point Estimate Method (PEM)
- Response Surface Method (RSM)

Response Surface Method (RSM)
approximation function of the actual structural response

$$\bar{y} \equiv \bar{y}(\mathbf{x}) = f(x_1; x_2; x_3; ...; x_n) + \varepsilon$$

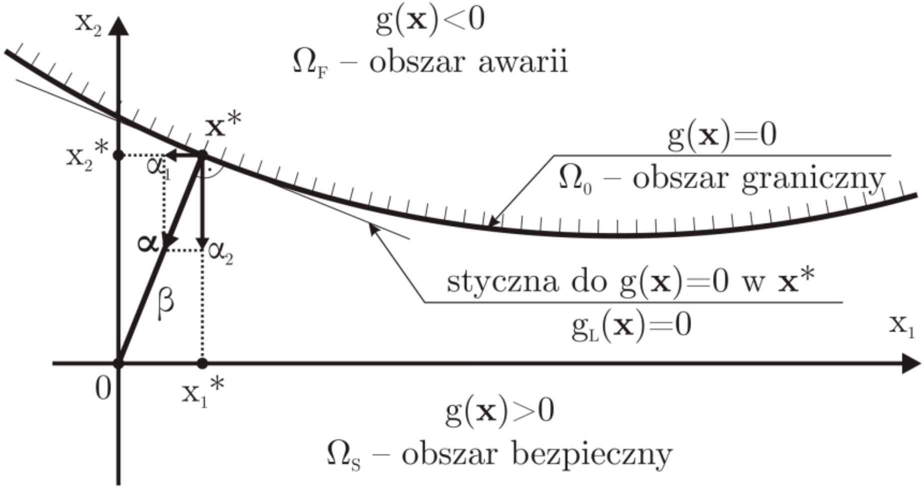
First order model FORM

$$\bar{y}(\mathbf{x}) = b_0 + \sum_{i=1}^{n} b_i x_i + \varepsilon$$



Second-order model SORM

$$\bar{y}(\mathbf{x}) = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_i^2 + \sum_{i<j}^{n} \sum_{j=2}^{n} b_{ij} x_i x_j + \varepsilon$$



The response surface should minimize the least-squares lack-of-fit ratio

$$\varepsilon_{RSM} = \sum_{i=1}^{NS} \left[y_i(\mathbf{x}) - \overline{y}_i(\mathbf{x}) \right]^2$$

The design point $\mathbf{x}^*$ is found by iteratively solving the set of equations:

$$\overline{y}(\mathbf{x}^*) = \overline{y}\left(x_1^*; x_2^*; x_3^*; \ldots; x_n^* \right) = 0$$

$$x_1^* = -\beta \left(\frac{\partial \overline{y}(\mathbf{x})}{\partial x_i} \right)_{x_i^*} \left(\sum_{i=1}^{n} \left(\frac{\partial \overline{y}(\mathbf{x})}{\partial x_i} \right)_{x_i^*}^2 \right)^{-1/2}$$

All parameters are described by a normal distribution
mean values – measured in situ
standard deviations – according to literature
coefficients of variation
- 0.3 for cohesion
- 0.1 for the internal friction angle

Layer	Soil	Bulk density ρ_i [g/cm^3]	Cohesion c_i [kPa]			Internal friction angle ϕ_i [°]		
			designation	mean value	stan. dev.	designation	mean value	stan. dev.
L1	silty loam	2.05	x_1 [c L1]	$\mu_1 = 40.0$	$\sigma_1 = 12.0$	x_2 [ϕ L1]	$\mu_2 = 15.0$	$\sigma_2 = 1.5$
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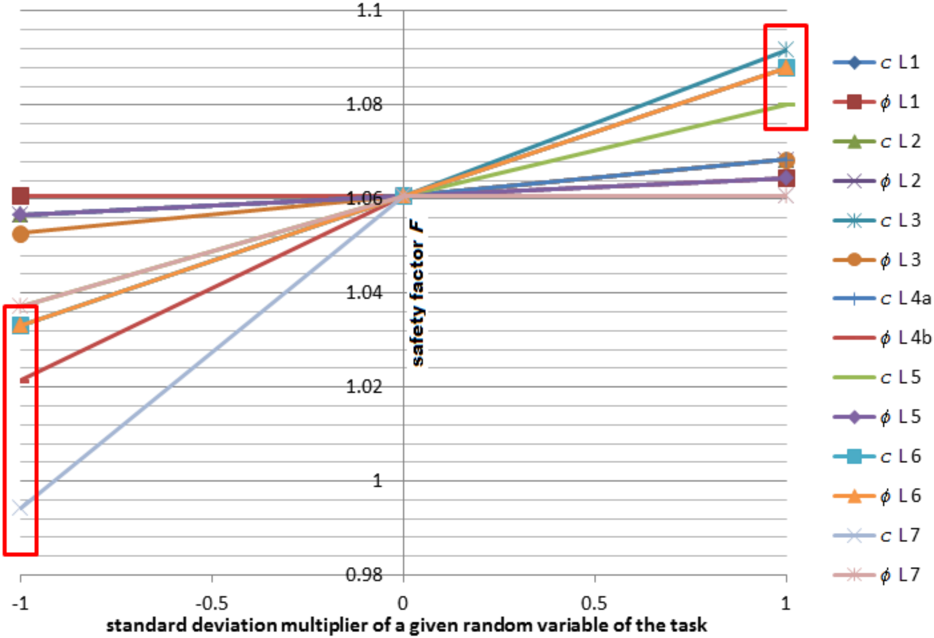
The high diversity of soil cohesion and friction angles is intended to produce a high dispersion of the cliff's response results.

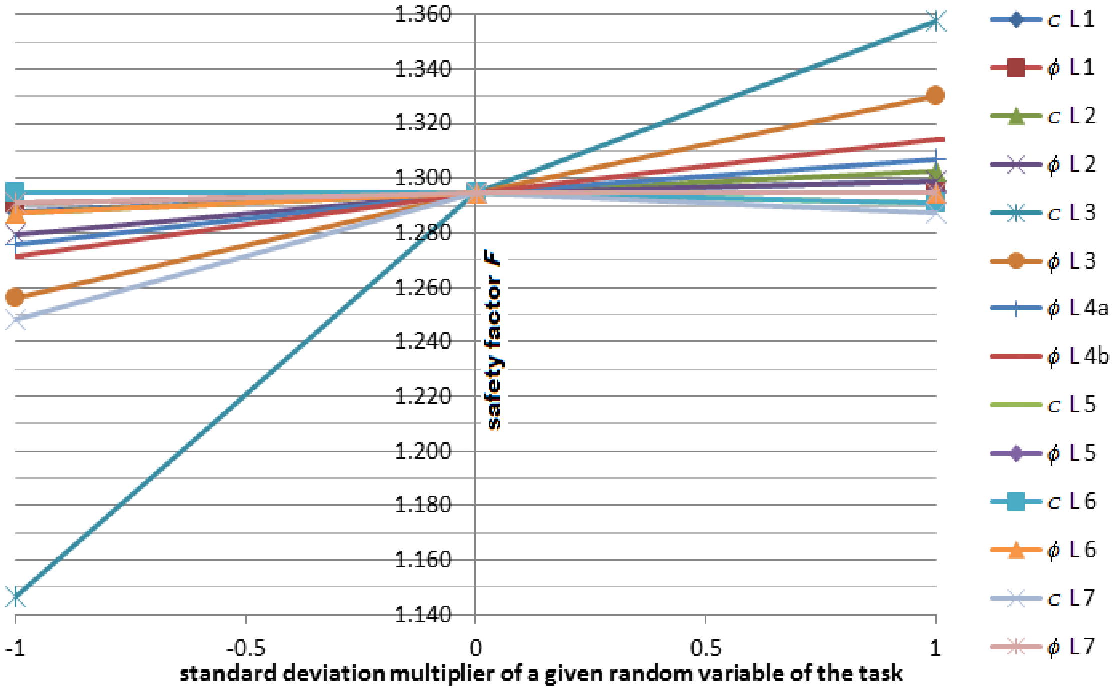
Parameter sensitivity analysis - PEM samples undrained state

$$x_i = \mu_i \quad x_i = \mu_i \pm \sigma_i$$



$F_{\max} = 1.09180[-]$ $F_{\min} = 0.99414[-]$

Drained state



On the basis of the **sensitivity analysis**:

- a basic prediction of the structural response surface curvature (linear or non-linear) may be estimated,
- a proper order of an RS approximation model may be made,
- variables with small impact on the structural response may be eliminated.

The impact of each random parameter on the value of the safety factor

$$\delta_i = \frac{1}{2} \cdot \left(\left| \frac{F_{\max,i}}{\overline{F}} - 1 \right| + \left| \frac{F_{\min,i}}{\overline{F}} - 1 \right| \right) \cdot 100\%$$

Undrained state – 7 random variables
Drained state – 5 random variables

PEM – 14 variables, 28 samples

$$\Delta F = F_{\max} - F_{\min} = 1.09180 - 0.99414 = 0.09766$$

$$P_{f\ PEM-14} = 0.402$$

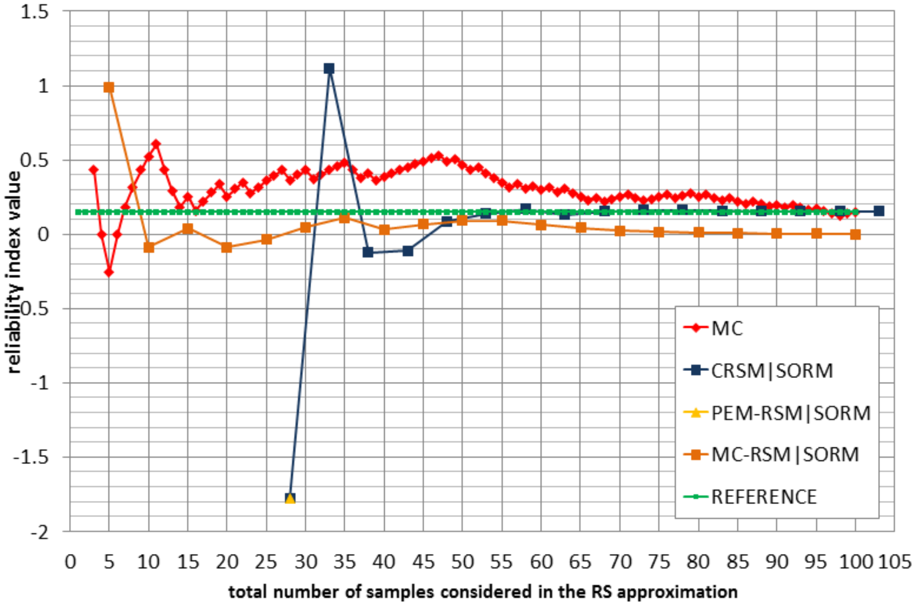
$$\beta_{PEM-14} = 0.25$$

Variable	Designation	Parameter	Present state		Drained state	
			Impact	Significant?	Impact	Significant?
x_1	c L1	cohesion of L1 silty loams	0.18 %	NO (13)	0.43 %	NO (8)
x_2	ϕ L1	angle of friction of L1 silty loams	0.18 %	NO (14)	0.15 %	NO (10)
x_3	c L2	cohesion of L2 sandy loams	0.55 %	NO (9)	0.60 %	NO (7)
x_4	ϕ L2	angle of friction of L2 sandy loams	0.55 %	NO (10)	0.75 %	NO (6)
x_5	c L3	cohesion of L3 clays	2.76 %	**YES (3)**	8.14 %	**YES (1)**
x_6	ϕ L3	angle of friction of L3 clays	0.74 %	NO (8)	2.87 %	**YES (2)**
x_7	ϕ L4a	angle of friction of L4a fine sands	0.55 %	NO (11)	1.21 %	**YES (5)**
x_8	ϕ L4b	angle of friction of L4b fine sands	3.13 %	**YES (2)**	1.66 %	**YES (4)**
x_9	c L5	cohesion of L5 silty loams	2.03 %	**YES (6)**	0.15 %	NO (11)
x_{10}	ϕ L5	angle of friction of L5 silty loams	0.37 %	NO (12)	0.00 %	NO (14)
x_{11}	c L6	cohesion of L6 loamy sands	2.58 %	**YES (4)**	0.15 %	NO (12)
x_{12}	ϕ L6	angle of friction of L6 loamy sands	2.58 %	**YES (5)**	0.30 %	NO (9)
x_{13}	c L7	cohesion of L7 clays	3.13 %	**YES (1)**	2.11 %	**YES (3)**
x_{14}	ϕ L7	angle of friction of L7 clays	1.11 %	**YES (7)**	0.15 %	NO (13)

Probabilistic calculations - natural (undrained) state – 14 variables

Monte Carlo approach, NS = 100 samples were calculated - reference value

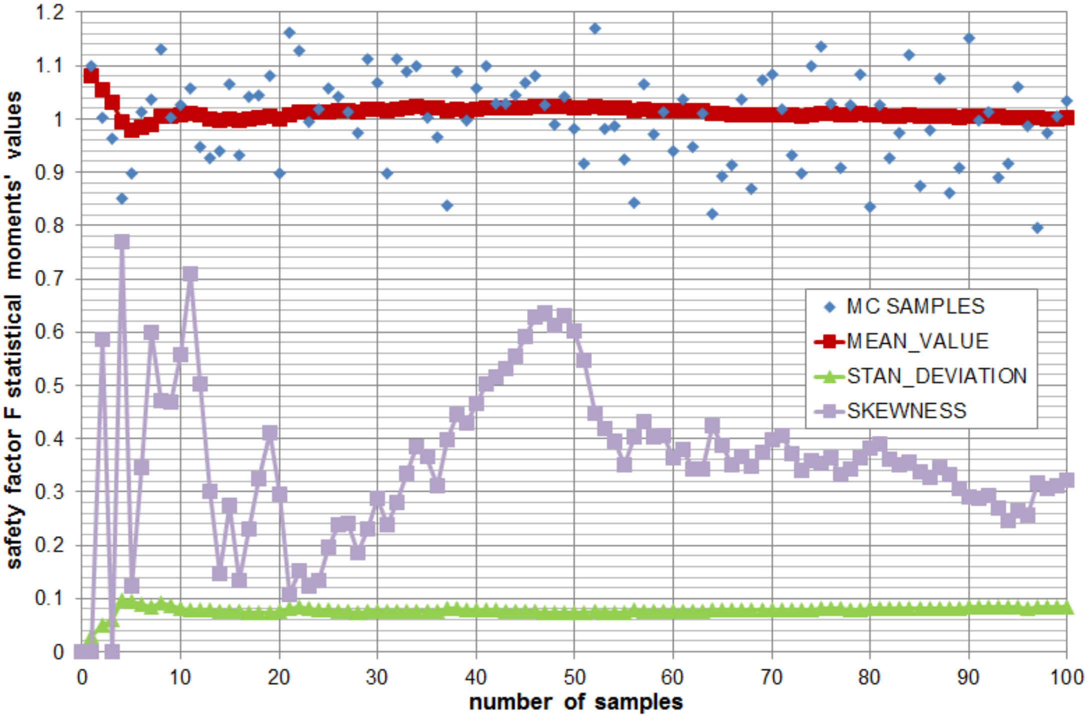
$P_{f\,MC} = 0.44 \qquad \beta_{MC} = 0.16$



F_{mean} = 1.00131 (F_{undr} = 1.06055) Standard deviation 0.083377

Probabilistic calculations - natural (undrained) state – 14 variables

Monte Carlo approach, *NS* = 100 samples were calculated



F_{mean} = 1.00131 Standard deviation 0.083377

Probabilistic calculations - natural (undrained) state – 14 variables

Sprawdzenie poprawności rozwiązania – Student's *t*-distribution

Monte Carlo approach, NS = 100 samples were calculated

$\bar{F} = 1.00131$

$\sigma_F = 0.08338$

$\gamma_F = 0.32272$

$$\frac{(\bar{F} - F)\sqrt{NS}}{\sigma_F}$$

$t_{\alpha/2, NS-1}$

Student's *t*-distribution

$$\bar{F} - \frac{\sigma_F}{\sqrt{NS}} t_{\alpha/2, NS-1} < F < \bar{F} + \frac{\sigma_F}{\sqrt{NS}} t_{\alpha/2, NS-1}$$

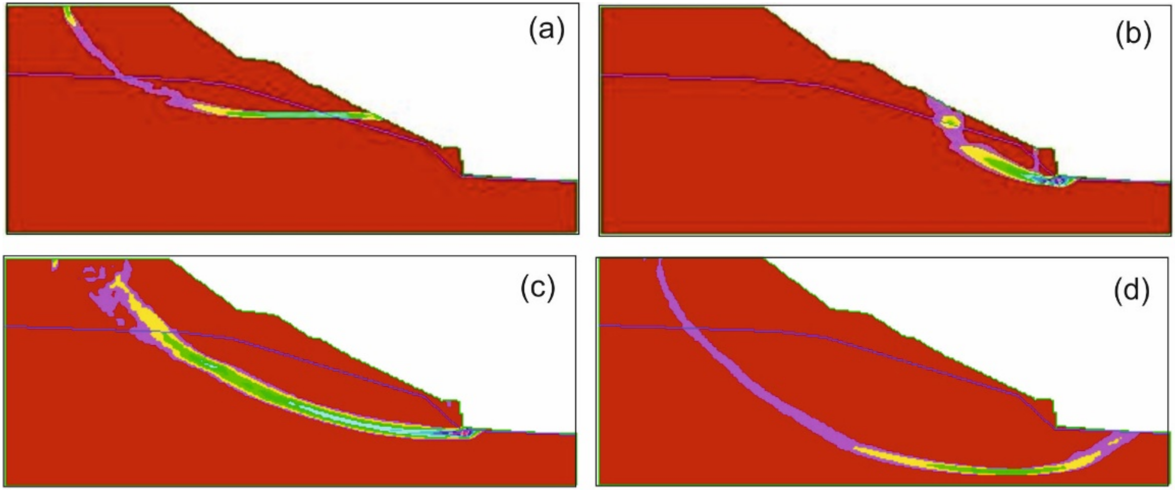
Assuming a 99% confidence interval

$\alpha = 0.01$

$t_{0.005,\,99} = 2.5674$

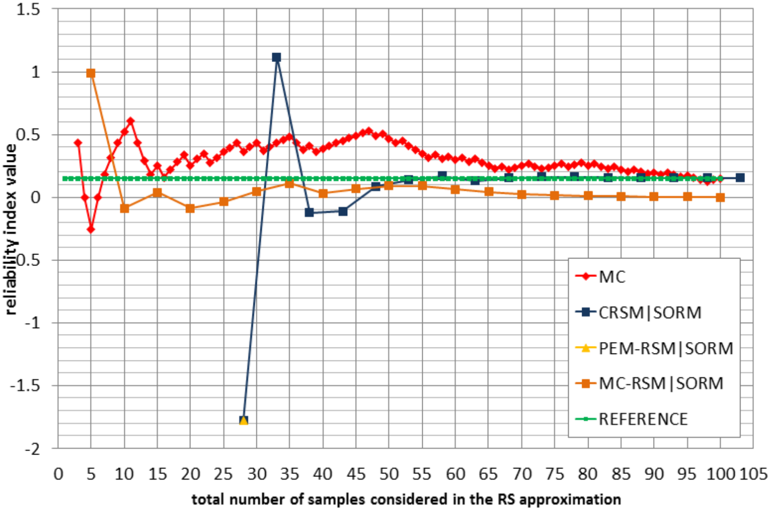
$$0.9798 < F < 1.0228$$

Probabilistic calculations - natural (undrained) state
Monte Carlo approach, NS = 100 samples were calculated
Selected variants of the slip surface and the shape of the slipping mass



Combined Response Surface Method (CRSM) - natural (undrained) state

129 samples used in direct calculations (one central sample as the starting point of the analysis, 2×14 PEM samples and 100 MC samples)



54 samples in total (29 from PEM and 25 from MC)

$\beta_{CRSM} = 0.15$ $P_{f\ CRSM} = 0.44$

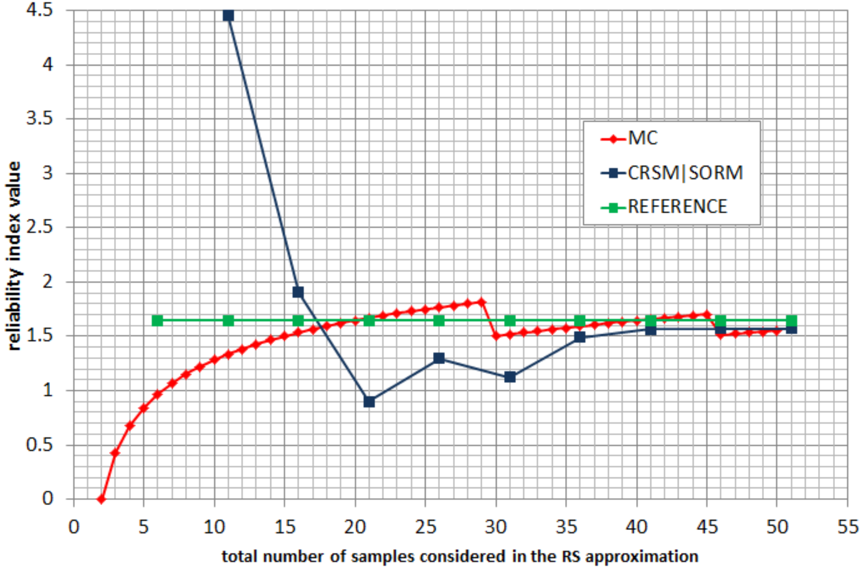
Reference value (100) –

$\beta_{MC} = 0.16$ $P_{f\ MC} = 0.44$

Combined Response Surface Method (CRSM) - drained state

– 5 variables, two parallel computational

- direct MC
- RSM based on 41 samples: 11 PEM and 30 MC
- 111 samples: one central sample, 2×5 PEM points and 100 MC points



$P_{f\,MC} = 0.44 \quad p_{f,CRSM,41} = 0.059$

$$\beta_{MC,41} = 1.657 \qquad (p_{f,MC,41} = 0.049)$$

$$\beta_{CRSM,41} = 1.567 \qquad p_{f,CRSM,41} = 0.059$$

$$\beta_{CRSM,111} = 1.569 \qquad p_{f\ CRSM,111} = 0.058.$$

error: 4.73%

This is not a simple or easy task!

The values of the slope's reliability index calculated using the RSM approach

undrained state

	RSM-C+PEM [15]/[29]			RSM-C+MC [101]			RSM-CS [115]/[129]			
	SIM	FORM	SORM	SIM	FORM	SORM	SIM	FORM	SORM	
VAR = 7	0.638	0.635	N/A	0.124	0.122	N/A	0.250	0.247	N/A	1 order
	0.112	0.243	0.078	0.340	0.877	0.550	-0.487	-0.401	-0.483	2 order
VAR = 14	0.639	0.638	N/A	0.397	0.396	N/A	0.397	0.396	N/A	1 order
	-1.743	-1.619	-1.780	-0.040	-0.279	-0.001	0.123	0.221	0.150	2 order

Drained state

	RSM-C+PEM [11]/[29]			RSM-C+MC [41]			RSM-CS [51]/[69]			
	SIM	FORM	SORM	SIM	FORM	SORM	SIM	FORM	SORM	
VAR = 5	4.753	5.115	N/A	1.703	1.705	N/A	1.750	1.753	N/A	1 order
	2.406	2.408	N/A	2.198	2.204	N/A	2.220	2.226	N/A	
	1.694	1.537	1.705	1.574	1.900	1.689	1.571	1.900	1.689	2 order
	4.465	4.424	4.461	1.538	1.730	1.587	1.521	1.725	1.569	
VAR = 14	3.019	3.030	N/A	1.027	1.025	N/A	1.458	1.458	N/A	1 order
	0.185	0.801	0.519	0.001	0.094	0.006	0.547	0.561	0.544	2 order

Conclusions and remarks - from the probabilistic standpoint

- PEM-analogous sampling makes it possible to perform a simple sensitivity analysis
- The result of the MC method is used as a reference value for all results obtained
- Without introducing new samples, a CRSM approximation may be performed on a combined set of PEM and MC samples
- The parallel MC and RSM calculations are stopped once the convergence has been achieved
- The analysis proves that PEM calculations alone fail to produce correct results
- The calculations have shown that the proposed method of improving the cliff's stability is correct because the probability of stability loss decreases from 45% to 5% as a result of changes in soil parameters induced by drainage.
- A preliminary sensitivity analysis can identify the strata that contribute the most to the stability of the slope.