Random Fields – example – composite shell of low elevation

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THE USE OF RESPONSE SURFACE METHODOLOGY FOR RELIABILITY ESTIMATION OF COMPOSITE ENGINEERING STRUCTURES

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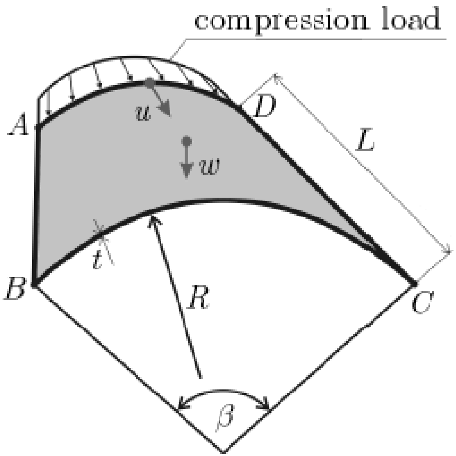
Stability loss is one of the key issues in the analysis of shells made of composite materials. This paper aims at finding the variation of the critical or limit value of the load resulting from geometric imperfections. An axially compressed 16-layer cylindrical panel model is considered. The imperfections of the panel are simulated as random fields. The parameters of these fields are used as the basic variables of the reliability problem of the model. The reliability analysis is based on the Response Surface Methodology (RSM). On the basis of the derived equation of the response surface, a reliability index of the model is determined using an author's program – both the Monte Carlo method (MC) and the Hasofer-Lind--Rackwitz-Fiessler (HLRF) reliability index formulation analyses.

Keywords: reliability, response surface methodology, composite panel

The example is aimed at assessing the impact of geometric imperfection random fields on structural reliability.

The model

Example: low elevation 16-layer composite shell, reinforcement – carbon fibres, matrix – epoxy resin, subjected to compression along the generator



Straight sides AB and CD are simply supported.

Curved sides AD and BC are fully clamped

The loaded side AD, capable of deflecting.

Panel thickness $t = 16 \times 0{,}125\,\mathrm{mm} = 2{,}0\,\mathrm{mm}$.

Radius of curvature $R = 250,0\,\text{mm}$.

The length of straight sides $L = 540,0\,\text{mm}$, curved sides $S = 421,2\,\text{mm}$.

The inclination angle between straight sides $\beta = 1,6848\,\text{rad}$

Composite layer structure $\left[45 \middle| -45_2 \middle| 45 \middle| 0_4\right]_S$.

The shell is made of a XAS-914C composite of the following parameters:

$E_a = 130 \cdot 10^6\,\text{kPa}$, $E_b = 10 \cdot 10^6\,\text{kPa}$, $\upsilon_{ab} = 0,3$, $G_{ab} = G_{ac} = G_{bc} = 5 \cdot 10^6\,\text{kPa}$.

Making use of the data above **material parameters of panels** may be assessed (e.g. based on physical equations of composites in the „off-axis" system [Christensen, 1979; German, 2001; Woźniak, 2001]).

The software NX Nastran v.10.1.1. is applied here

Numerical model: 1600 four-node shell elements QUAD (40x40), a mesh of 1681 nodes (41x41), defined in a polar coordinate system.

The analysis assumes geometric and material non-linearity, based on large rotation theory.

The random field modelling geometric imperfections

Geometric imperfctions of the shell are considered only (random variation is neglected of loads, panel supporting, discrepancies in material layer location and material parameters of composite laminae)

The input data includes generated random fields representing geometric imperfections w.

The **truncated Gaussian variable** is applied here.

The dispersions are represented by a correlation function of a **homogeneous random field**.

$$K(x_1, x_2) = s_w^2 \times e^{-\lambda_{x_1}\Delta x_1}(1+\lambda_{x_1}\Delta x_1)e^{-\lambda_{x2}\Delta x_2}(1+\lambda_{x_2}\Delta x_2)$$

here Δx_1 and Δx_2 are distances between field points, coefficients λ_{x_1} and λ_{x_2} are parameters marking **correlation range** in both directions, the standard deviation represents field variability.

The distances between field points Δx_1 i Δx_2 are adjusted to coincide with the FE mesh.

Unit standard deviation s_w^2 is assumed here.

The coefficients λ_{x_1} and λ_{x_2} are represented by a random variable.

The conditional simulation algorithm is applied, making use of the **acceptance and rejection method**.

The FEM application requires **averaging of the values** of **generated imperfection field**.

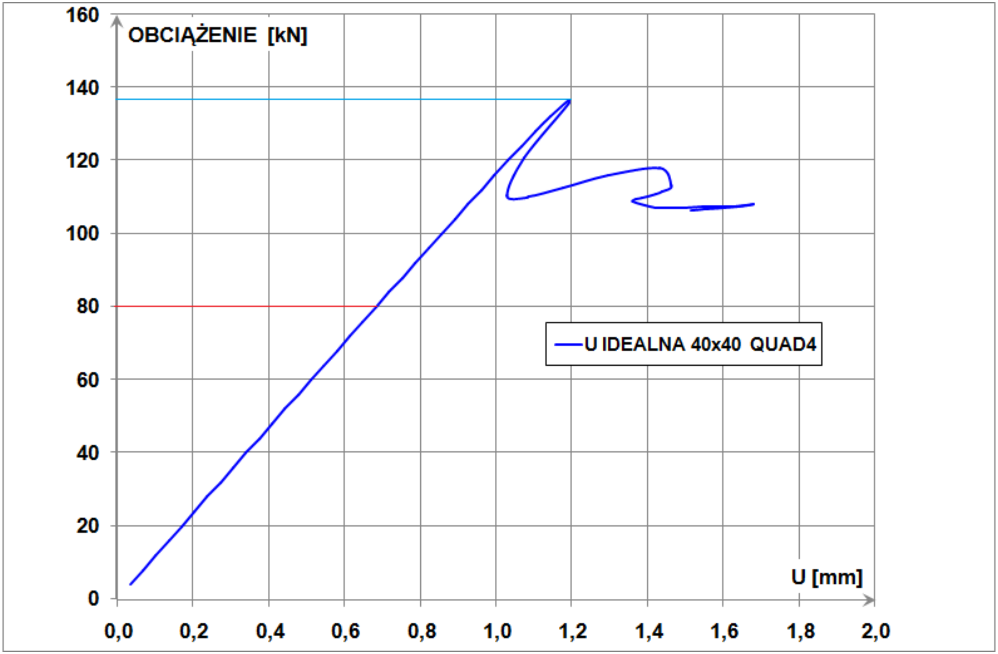
$$D_w(\Delta x, \Delta y) = \frac{2}{\lambda_x \Delta x}\left[2 + e^{-\lambda_x \Delta x} - \frac{3}{\lambda_x \Delta x}\left(1 - e^{-\lambda_x \Delta x}\right)\right]\frac{2}{\lambda_y \Delta y}\left[2 + e^{-\lambda_y \Delta y} - \frac{3}{\lambda_y \Delta y}\left(1 - e^{-\lambda_y \Delta y}\right)\right]$$

$$K_w(\Delta x, \Delta y) = \frac{e^{\lambda_x \Delta x}}{(\lambda_x \Delta x)^2}\left\{\left[\cos(\lambda_x \Delta x) - \sin(\lambda_x \Delta x)\right] + 2\lambda_x \Delta x - 1\right\}$$

$$\cdot \frac{e^{\lambda_y \Delta y}}{(\lambda_y \Delta y)^2}\left\{\left[\cos(\lambda_y \Delta y) - \sin(\lambda_y \Delta y)\right] + 2\lambda_y \Delta y - 1\right\}$$

Numerical computations

Results: equilibrium paths of axial deflection U .



The deflection equilibrium path U according to panels of **perfect geometry**

Two distinct points are considered (Figure):

– **the central node deflection of the curved side** U_{101}**, the resultant load level** $80\ \mathrm{kN}$ (assuring the avoidance of stability loss, regadless of the level of geometric deformation),

– **the point of stability loss, corresponding to appropriate critical or limit load**

Reliability assessment is performed by **the response surface method (RSM)**

There are two factors of the highest impact to the **panel stability loss**:

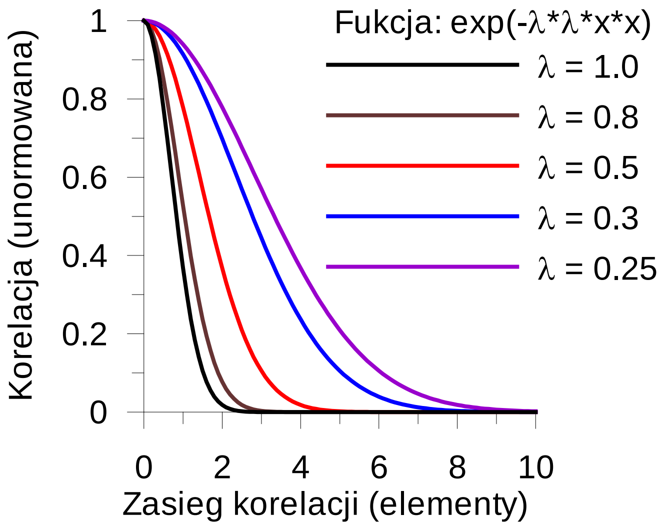
The first factor – variable x_1 – **imperfection depth**.

The magnitude of field amplitude is taken from the uniformly distributed random variable $x_1 \in \langle 0,0\,;1,0 \rangle$ – a multiplier imposed on the generated normalized field – the point of maximum imperfection field amplitude sampled from 0 cm (shell of ideal geometry) and 1 cm (shell of the highest imperfection field amplitude).

The second factor – correlation range in both directions

It is assumed that the second random variable x_2 (correlation variable $\lambda = \lambda_{x_1} = \lambda_{x_2}$) is taken directly on the basis of the uniformly distributed variable of the following domain $x_2 \in \langle 0,0\,;1,0 \rangle$.

The impact of correlation parameter range to the random field features



The figure presents selected generated random fields of diverse correlation range parameters λ.

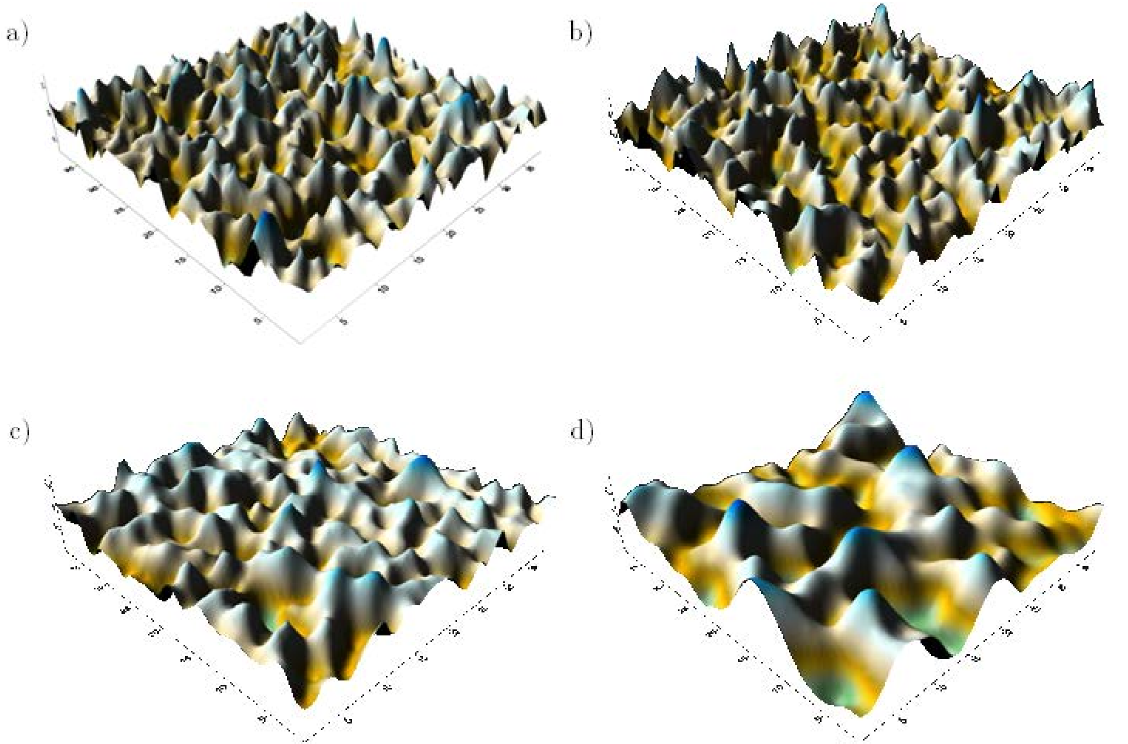
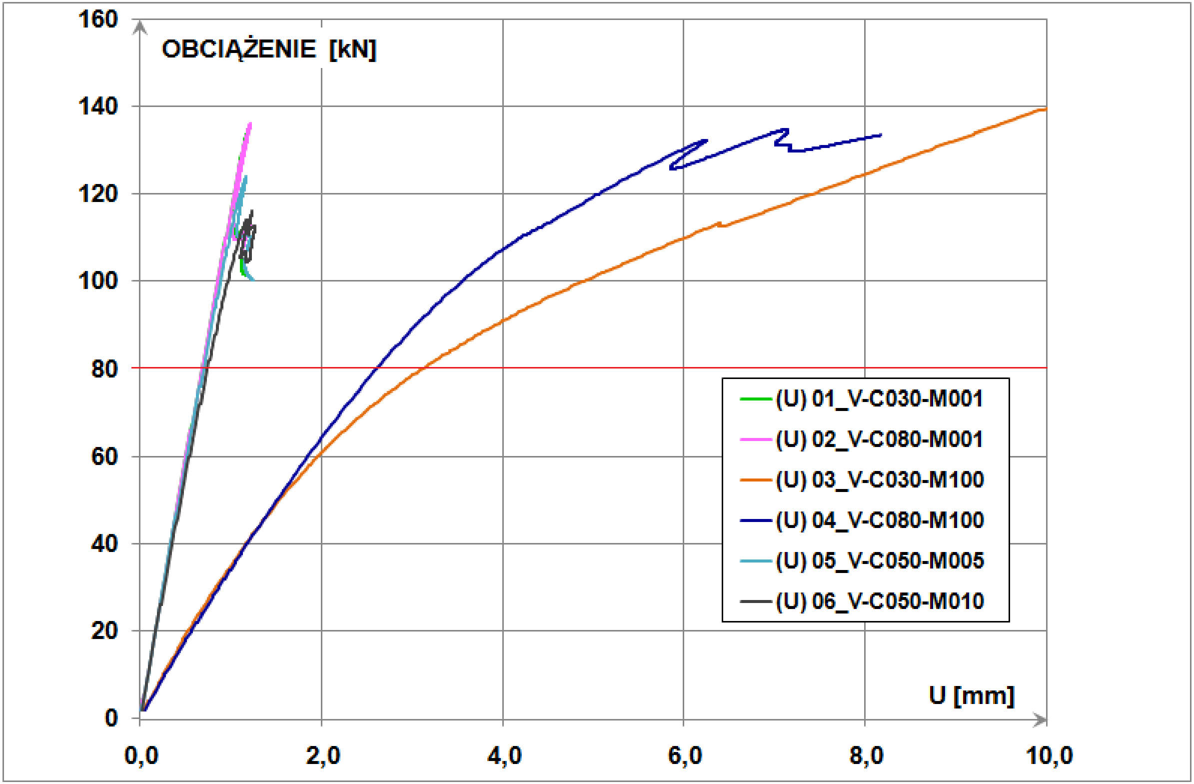


Figure: Field examples of various correlation range parameters λ:
a) very weakly correlated field, $\lambda = \lambda_{x_1} = \lambda_{x_2} = 1{,}00$
b) weakly correlated field, $\lambda = \lambda_{x_1} = \lambda_{x_2} = 0{,}80$
c) strongly correlated field, $\lambda = \lambda_{x_1} = \lambda_{x_2} = 0{,}50$
d) very strongly correlated field, $\lambda = \lambda_{x_1} = \lambda_{x_2} = 0{,}30$.

A series of 24 computations has been conducted, in three stages adjusted to the assumed analysis, employing the response surface method.

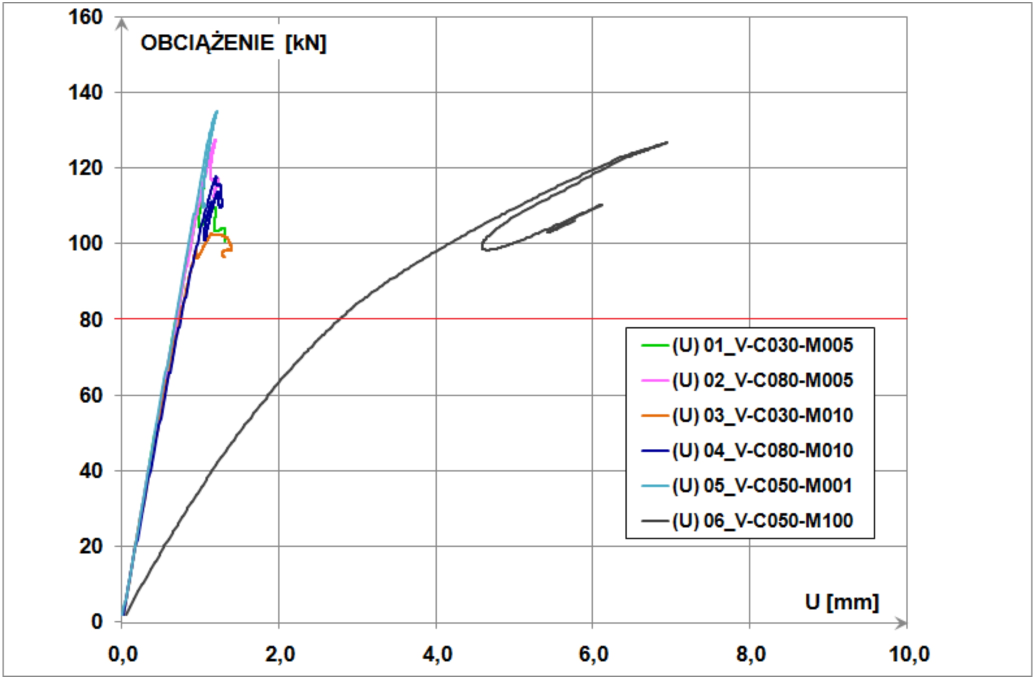
Each history is accompanied by displacement U_{101} results achieved at the $80kN$ and the load leading to stability loss.

The first stage – analysis of 6 samples – four points close to the surface vertices $(m\,;\lambda)$, their parameters are: $(0{,}01\,;0{,}30)$, $(0{,}01\,;0{,}80)$, $(1{,}00\,;0{,}30)$ i $(1{,}00\,;0{,}80)$ respectively and two central points $(0{,}05\,;0{,}50)$ and $(0{,}10\,;0{,}50)$.

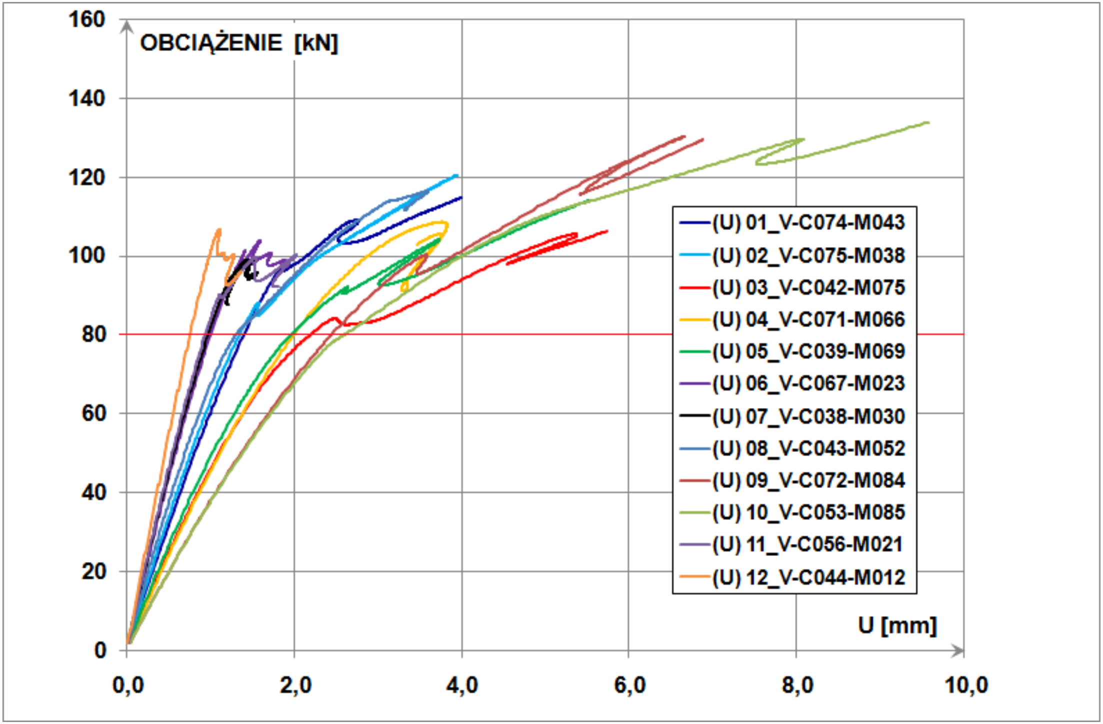


OBCIĄŻENIE [kN]
160
140
120
100
80
60
40
20
0
0,0
2,0
4,0
6,0
8,0
10,0
U [mm]
(U) 01_V-C030-M001
(U) 02_V-C080-M001
(U) 03_V-C030-M100
(U) 04_V-C080-M100
(U) 05_V-C050-M005
(U) 06_V-C050-M010

Stage two – analysis of the next 6 samples, selected from the realization surface to densify the information on the structural response surface close to its edge: $(0,05; 0,30)$, $(0,10; 0,30)$, $(0,05; 0,80)$, $(0,10; 0,80)$, $(0,01; 0,50)$ and $(1,00; 0,50)$.



The third stage – 12 samples totally randomly chosen from the realization surface (direct Monte Carlo sampling) to complement the points on the response surface in its central region.



The limit state definition.

The first analytical case: The **deflection of central node** of a curved edge U_{101} in the case of resultant load equal $80kN$ **cannot reach its limit value** $U_{101,\lim} = 2,0\,\mathrm{mm}$, i.e. the thickness of a composite element.

The second case – **stability loss** – the force corresponding to the critical (buckling) or limit load **cannot exceed the resultant load** equal 100 kN.

Wyniki przedstawiono w Tablicy; wprowadzono zero-jedynkowy indeks $I_{0/1}$ (o wartości 1 w przypadku spełnienia warunku granicznego).

The results are presented in the Table; a binary (0/1) index $I_{0/1}$ is introduces, 1 means the limit state condition satisfied

Table. The computational results

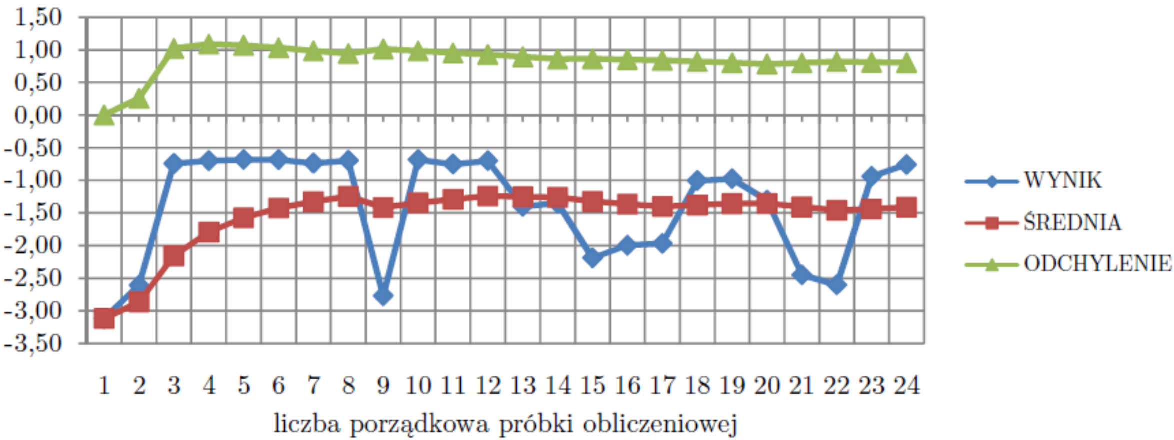
No.	RSM - PANELA		U101 [mm] (limit = -2,00)		Ncrit [kN] (limit = 100,00)	
	λ	m	80kN	I0/1	ucrit	I0/1
1.	0,30	1,00	-3,12008	0	113,120	1
2.	0,80	1,00	-2,60901	0	109,862	1
3.	0,50	0,10	-0,74544	1	114,164	1
4.	0,50	0,05	-0,69972	1	124,132	1

5.	0,30	0,01	-0,68375	1	134,790	1
6.	0,80	0,01	-0,68416	1	136,134	1
7.	0,30	0,10	-0,73906	1	98,824	0
8.	0,30	0,05	-0,69677	1	116,130	1
9.	0,50	1,00	-2,77036	0	82,708	0
10.	0,50	0,01	-0,68412	1	134,954	1
11.	0,80	0,10	-0,75164	1	117,732	1
12.	0,80	0,05	-0,70149	1	127,574	1
13.	0,74	0,43	-1,40191	1	96,490	0
14.	0,75	0,38	-1,35024	1	87,960	0
15.	0,42	0,75	-2,18863	0	84,114	0
16.	0,71	0,66	-1,99699	1	108,550	1
17.	0,39	0,69	-1,96836	1	92,106	0
18.	0,67	0,23	-1,00521	1	101,214	1
19.	0,38	0,30	-0,97521	1	90,574	0
20.	0,43	0,52	-1,30618	1	81,818	0
21.	0,72	0,84	-2,45007	0	100,184	1
22.	0,53	0,85	-2,60283	0	77,364	0
23.	0,56	0,21	-0,94040	1	90,188	0
24.	0,44	0,12	-0,76001	1	106,556	1

Convergnce analysis of results

Making use of all conducted computations, the solution correctness may be assessed, convergence analysis is investigated in the course of an increasing sample space.

This approach may be called **quasi-Monte Carlo**, while sampling is not random.

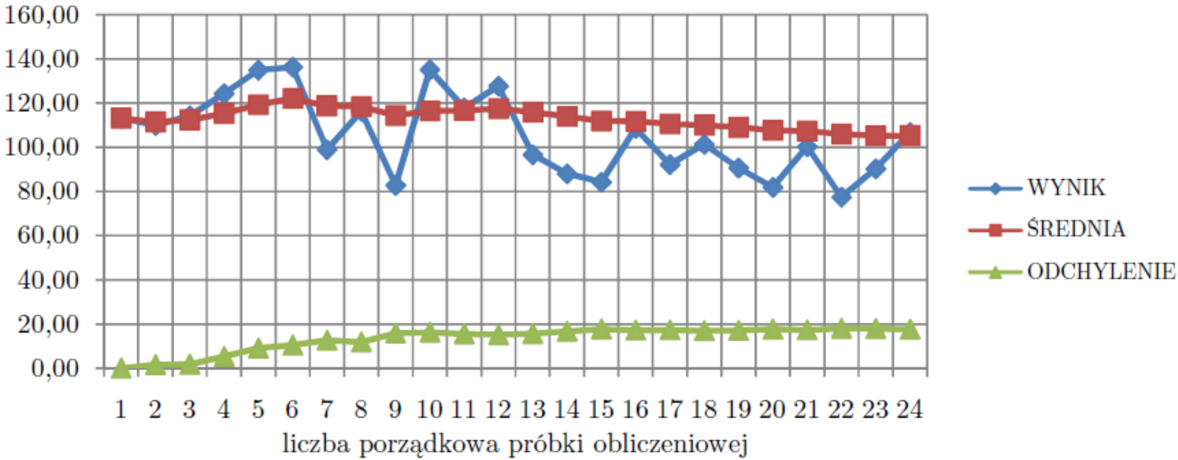


Rysunek. Figure. Computation convergence for the displacement U_{101} following variant, the total load equal 80 kN.

In the case of 24 samples convergence is sufficient: the relative difference between the second to last and the last mean displacement U_{101} is 2,01%, the relative difference of standard deviations is 0,68%.

The load level corresponding to stability loss

Convergence observation is conducted of probabilistic results of response moments due to increasing sample space, the variant following the overall force level N_{crit} leading to stability loss.



The analysis conducted on 24 samples shows convergence at the level of 10,4%.

The selection of response surface

Assuming high curvature of a real structural response surface the **2nd order model is applied**:

$$\hat{g}(\mathbf{x}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2$$

The response surface is estimated in three variants, incorporating 6, 12 and 24 design points.

The side deflection corresponding to load level of 80 kN.

Given 6 design points the surface is ę

$$\hat{g}(\mathbf{x}) = 1,268 + 0,267 x_1 - 1,197 x_2 - 0,253 x_1^2 - 1,558 x_2^2 + 1,033 x_1 x_2$$

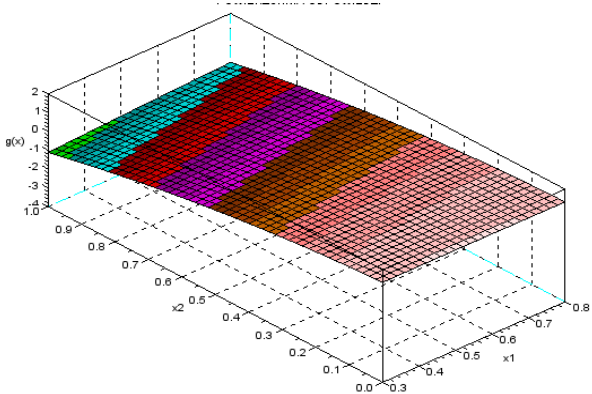
Given 12 design points

$$\hat{g}(\mathbf{x}) = 1,200 + 0,598 x_1 - 1,061 x_2 - 0,594 x_1^2 - 1,653 x_2^2 + 1,044 x_1 x_2$$

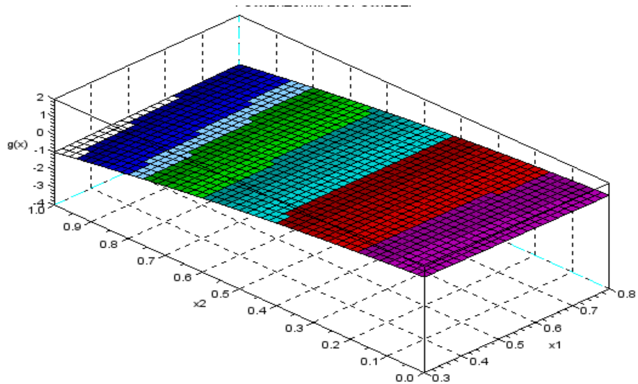
Given 24 design points

$$\hat{g}(\mathbf{x}) = 1,447 - 0,061 x_1 - 1,634 x_2 - 0,145 x_1^2 - 0,990 x_2^2 + 0,674 x_1 x_2$$

Outlook of a surface:



6 points



24 points

The overall loading level corresponding to stability loss.

Given 6 design points,

$$\hat{g}(\mathbf{x}) = 1,268 + 0,267x_1 - 1,197x_2 - 0,253x_1^2 - 1,558x_2^2 + 1,033x_1x_2$$

Given 12 design points

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Given 24 design points

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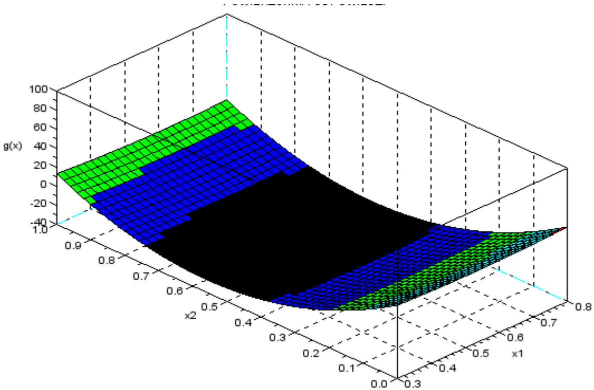
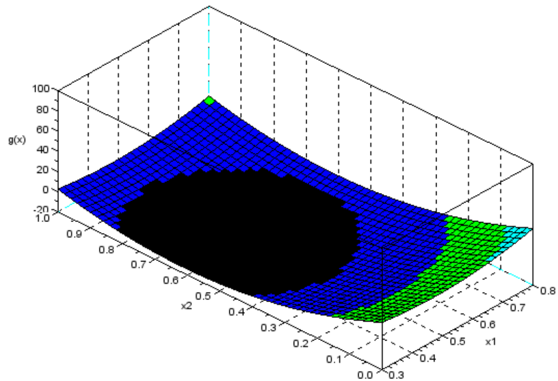


6 points



24 points

The probabilities of panel failure P_f and reliability indices Hasofera–Linda β_{HL} i Hasofera–Linda–Rackwitza–Fiesslera β_{HLRF}

Nine various parameters of probabilistic reliability assessment of the shell have been investigated:

- failire probability according to Monte Carlo approach $P_{f,MC}$ – the ratio of samples satisfying the limit criterion to the total sample domain,
- panel reliability according to the Monte Carlo approach $P_{s,MC} = 1 - P_{f,MC}$
- reliability index – the quasi Monte Carlo approach – $\beta_{C,MC} = -\Phi^{-1}(P_f)$
- the Hasofer–Lind reliability index β_{HL}, investigated on the approximated response surface,
- the probability of panel failure $P_{f,HL,RSM} = \Phi(-\beta_{HL,RSM})$,
- reliability of panels $P_{s,HL,RSM} = 1 - P_{f,HL,RSM}$,
- the Hasofer–Lind–Rackwitz–Fiessler reliability index β_{HLRF}, investigated on the response surface
- failure probability $P_{f,HLRF,RSM} = \Phi(-\beta_{HLRF,RSM})$,
- panel reliability as: $P_{s,HLRF,RSM}$ jako: $(P_{s,HLRF,RSM} = 1 - P_{f,HLRF,RSM})$.

Every parameter of the shell reliability assessment is estim ated in three variants, making use of 6, 12 and 24 computational points.

The table presents probabilistic parameters of safety assessment in the course of sample space increment, the variant following the deflection of a curved edge and the load level of 80 kN.

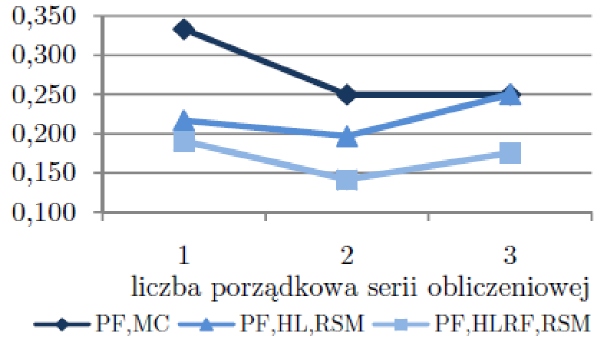
U_{101} 80kN	SERIES 1	SERIES 2	SERIES 3
	6 samples	12 samples	24 samples
$P_{f,MC}$	0,333	0,250	0,250
$P_{s,MC}$	0,667	0,750	0,750
$\beta_{C,MC}$	0,430	0,675	0,675
$P_{f,HL,RSM}$	0,217	0,197	0,250
$P_{s,HL,RSM}$	0,783	0,803	0,750
$\beta_{HL,RSM}$	0,783	0,853	0,673
$P_{f,HLRF,RSM}$	0,191	0,142	0,176
$P_{s,HLRF,RSM}$	0,809	0,858	0,824
$\beta_{HLRF,RSM}$	0,875	1,076	0,932

Collection of the results of conducted computational stages of a composite panel, the variant following the force level leading to stability loss

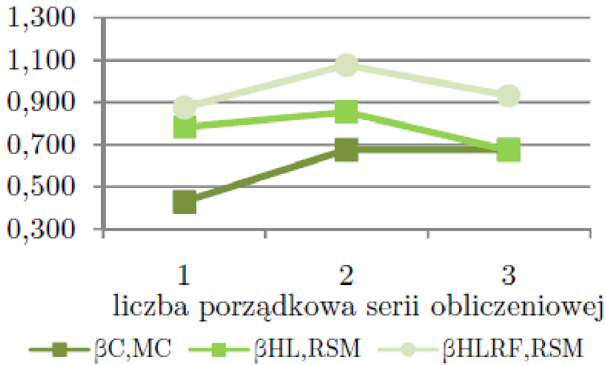
N_{CRIT}	SERIES 1	SERIES 2	SERIES 3
	6 samples	12 samples	24 samples
$P_{F,MC}$	0,000	0,167	0,417
$P_{S,MC}$	1,000	0,833	0,583
$\beta_{C,MC}$	3,999	0,970	0,210
$P_{F,HL,RSM}$	0,265	0,327	0,381
$P_{S,HL,RSM}$	0,735	0,673	0,619
$\beta_{HL,RSM}$	0,627	0,447	0,304
$P_{F,HLRF,RSM}$	0,374	0,394	0,406
$P_{S,HLRF,RSM}$	0,626	0,606	0,594
$\beta_{HLRF,RSM}$	0,320	0,270	0,139

Based on the data collected in tables **convergence analysis** has been conducted (two task variants taken separately).

The convergence after subsequent analytical stages of a displacement U_{101} following problem variant, the load equal 80 kN:

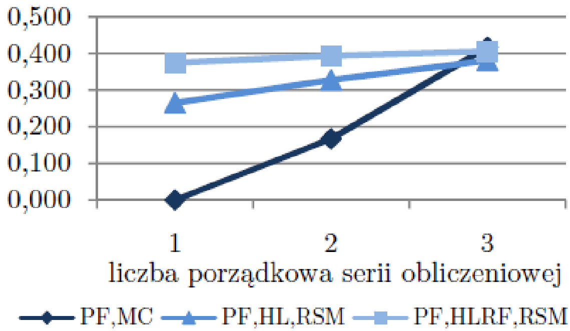


probability of failure P_f

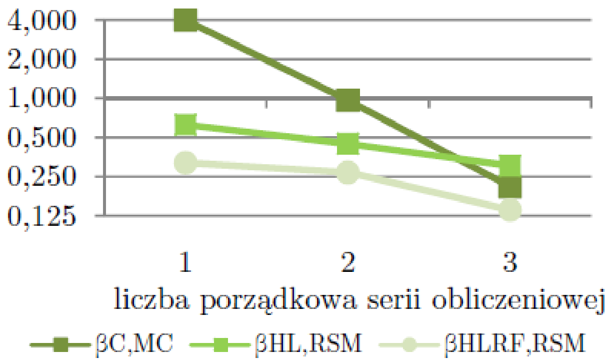


reliability indices β

Convergence analysis of displacement U_{101} under a load producing stability loss N_{crit}.



failure probabilities P_f



reliability indices β .