INFLUENCE OF GEOMETRIC RANDOM IMPERFECTIONS ON THE CAPACITIES OF ALUMINUM SILO STRUCTURE

J. Górski, T. Mikulski, M. Oziębło, K. Winkelmann

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Fachthemen

Jarosław Górski
Tomasz Mikulski
Magdalena Oziębło
Karol Winkelmann

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Effect of geometric imperfections on aluminium silo capacities

A significant stage in the design of slender shell structures is to establish **some assumptions concerning the dimensions, the shape, and the location of initial geometric imperfections**.

These initial defects are responsible for the occurrence of additional displacements and a change of stress distributions.

SILO MODEL

This paper includes a numerical limit state analysis of **a cylindrical aluminum silo** of $V = 324\ m^3$ capacity installed on a steel space frame.

The height of the silo shell is 25 m.

The shell is made up of 10 rings (sheets), each one is 2500 mm high.

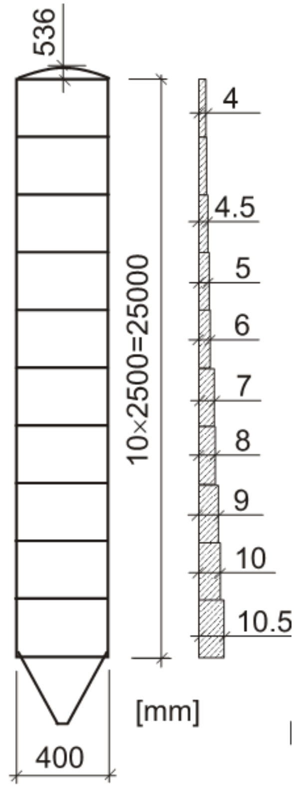
The internal diameter of the shell is constant and equal to $D_w = 4000$ mm.

The roof – a dome of a 536 mm maximum height and 5.5 mm thickness.

The bottom of the silo forms a hopper that is 3340 mm high and 10.5 mm thick.

The silo shell and roof are composed of $AlMg_3$ aluminum alloy of plasticity limit $f_{e,\min} = 70$ MPa.

The silo hopper is made of $AlMgSi_1$ aluminum alloy, of plasticity limit $f_{e,\min} = 120$ MPa.



The programs FEMAP with NX Nastran (v.7) and SOFiSTiK FEA (v.27) have been used to carry out the analysis.

The finite elements constituting the grid of the silo have the dimensions of 250×175 mm, where the first value corresponds to the vertical direction.

In the areas of the silo supporting construction, on the roof and the hopper, the mesh grid is denser, which is directly extorted by the silo geometry.

In total, there were used 11232 shell elements and 11304 nodes.

The elasticity modulus and the Poisson ratio were: E = 69 GPa, ν = 0.3 (as for aluminum).

LOCAL IMPERFECTIONS OF THE SHELL

It is practically impossible to produce a silo shell that would be ideally cylindrical.

Eurocodes provides a set of regulations making it possible to approve the silo construction for exploitation.

The limit values of the geometric imperfections of a cylindrical silo shell are calculated according to the formula

$$t_{v0} = 0.01\ell_g \tag{1}$$

The limit depth of the indentation value t_{v0} is measured on length l_g corresponding to the maximum spread of the local dent, given in millimetres.

Value l_g is calculated in compliance with the following formulas:

$$\ell_{gX} = 4\left(Rt\right)^{0.5} \text{ – along the vertical direction} \tag{2}$$

$$\ell_{g\theta} = 2.3\cdot(\ell^2 Rt)^{0.25} \text{ – along the horizontal direction} \tag{3}$$

$$\ell_g \le 2000\,\mathrm{mm} \text{ – for both cases} \tag{4}$$

where R is the silo radius, and t denotes the silo sheet thickness.

Silo loaded with negative pressure

The first stage of the analysis involves calculations made for a silo of a **perfect geometry**.

The value of the obtained limit pressure equals $p_{u,ideal} = 2.595$ kPa, and becomes a **reference point for further calculations**.

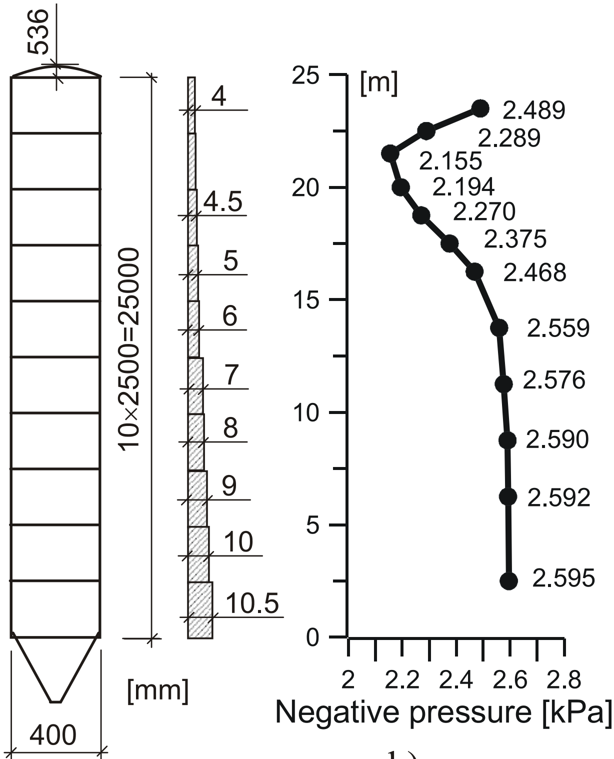
The next stage is aimed at analyzing the influence of the deviations of the lateral surface of the silo in the form of **local dents** on its load capacity.

It has been assumed that the remaining parts of the silo (roof, hopper, supporting structure) are not subject to any deformations.

The parameters of the dent are estimated according to Eurocode formulas (1)÷(4) where $\ell_g = 2000\,\text{mm}$ and the depth of the dent is $t_{vo} = 20$ mm.

An assumption has been made that the shape of the deformation reflects **an ellipsoid of equal axes**, with lengths expressed by ℓ_g .

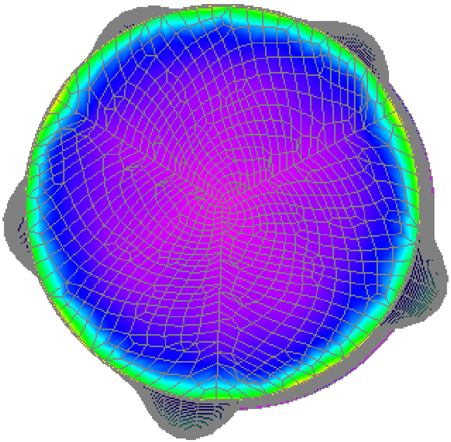
The dents are located at the following heights: 2.5, 6.25, 8.75, 11.25, 13.75, 16.25, 17.5, 18.75, 20.0, 21.5, 22.5, and 23.5 m.

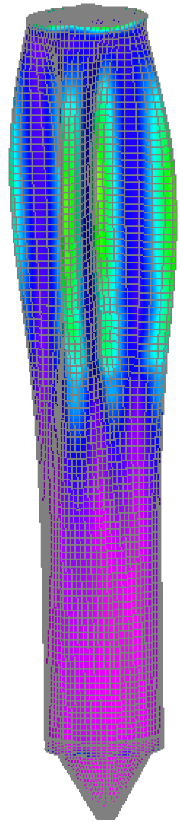


a) b)

Silo geometry and its sheet thicknesses (a), graph of the limit negative pressure values in reference to the placement of the elliptical imperfection (b).

The lowest load value obtained $p_{dent1} = 2.155$ kPa refers to the dent placed at an elevation of 21.5 m (ideal silo $p_{u,ideal} = 2.595$ kPa).

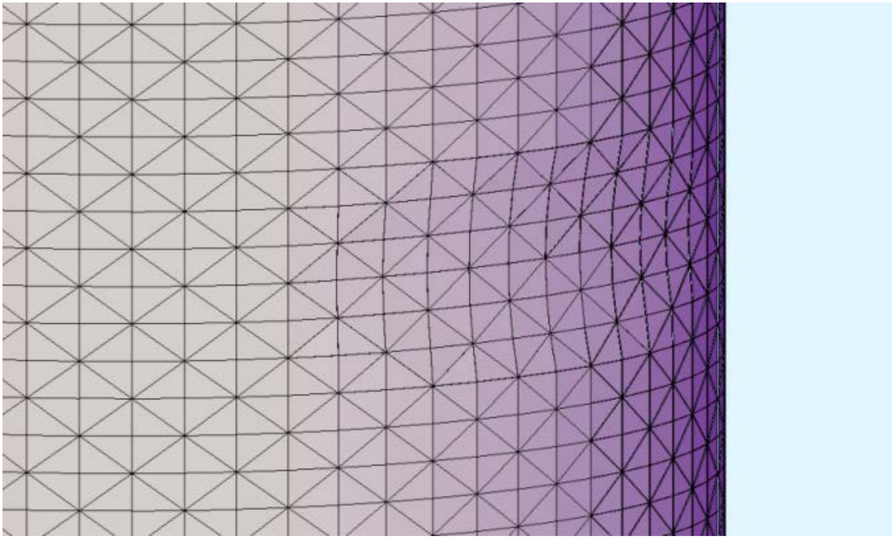




Rysunki: silos zniekształcony na skutek podciśnienia p=2,559 kPa (wstępne wgniecenie znajduję się na wysokości H=13,75m.

Additional calculations were made for this series.

The dent ellipsoid was changed, but the length of the dent along the perimeter ℓ_g and its depth t_{vo} remained the same, whereas **the length of the dent along the generatrix was shortened** to 600 mm.



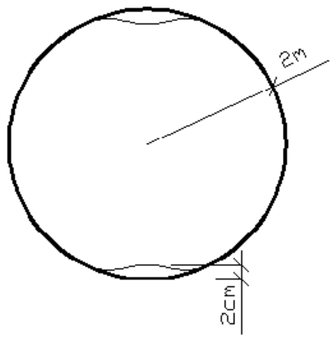
Finite element discretization of an ellipsoid local dent

The calculations were made for **three locations of the dents**:
20.0, 21.5, and 22.5 m above the bottom of the shell.

The maximum values of the calculated negative pressure reached $p_{wg2a} = 2.375\,\text{kPa}$, $p_{wg2b} = 2.355\,\text{kPa}$ and $p_{wg2c} = 2.388\,\text{kPa}$ respectively; they were higher than the values relating to the case involving a symmetric deformation and the ideal shell ($p_{u,ideal} = 2.595\ \text{kPa}$).

The next calculation series introduced an initial deformation in the form of **two dents situated on the opposite sides of the cross section of the silo** at a height of h = 21.5 m, where the area of the deformation is most dangerous.

The limit value of the negative pressure reached $p_{wg3} = 2.173\,\text{kPa}$.



Silo loaded with wind

36 measurements of wind direction and wind speed were made in the area of Gdynia-Oksywie airport between 2006 and 2008 by the Polish Institute of Meteorology and Water Management.

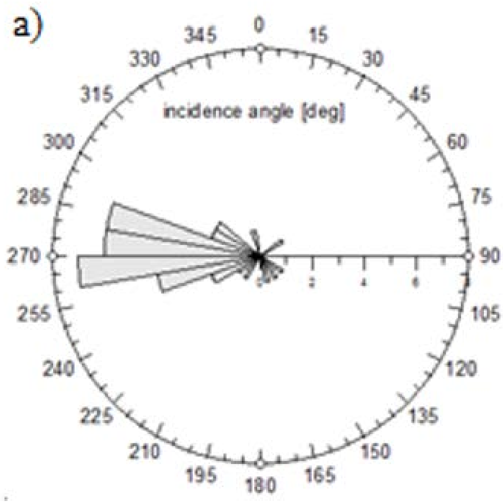


Figure. Wind load with the histogram of the incidence angle of the wind (a) and the adopted scheme of the wind loading – **a rose diagram** (b).

The measured wind speed is characterized by the following quantities:
minimal value $v_{\min} = 0$ m/s, maximal value $v_{\max} = 24$ m/s,
mean value $\mu_v = 10.77$ m/s, mode $m_v = 10.77$ m/s, median $v_v = 11.5$ m/s and standard deviation $\sigma_v = 3.7$ m/s.

All the above values are smaller than $v_{crit} = 26$ m/s, which is a **maximum suggested by the design standards**.

Consequently the latter value is used in further analysis.

On the basis of these observations, an assumption of arbitrariness of **the angle of wind approach in relation to the adopted initial imperfection** seems reasonable when analyzing an axisymmetric construction.

The characteristic value of the **wind load** used in the numerical model was calculated by applying the following formula

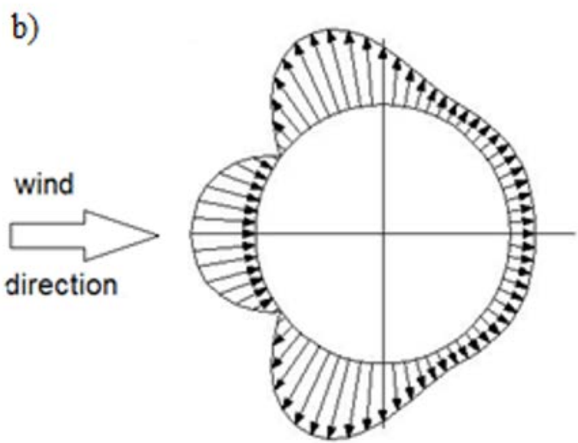
$$w_e = q_p(z_e)C_{pe} \tag{5}$$

The constant component of the wind pressure, is expressed by the formula

$$q_p(z_e) = 0.996 \text{ kPa} \approx 1 \text{ kPa} \tag{6}$$

The volatile component of the wind pressure value is a product of C_{pe} coefficient:

$$C_e(z_e) = -0.356 + 0,322\cos\alpha + 0,636\cos 2\alpha + 0,501\cos 3\alpha + ... + \\ + 0,058\cos 4\alpha - 0,128\cos 5\alpha - 0,034\cos 6\alpha$$



First, **the silo** of **a perfect geometry is analyzed**.

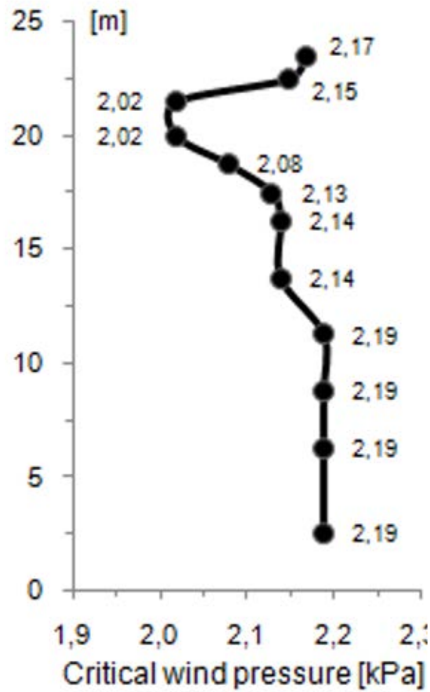
The ultimate load value of the wind pressure, resulting in the silo stability loss, is equal to $p_{w,ideal} = 2.595\,\text{kPa}$.

The next step is to specify **the limit value of the wind load applied directly to the geometric centre of the imperfections**.

A series of calculations was carried out, in which the location of the dent was changed.

Considering the limit pressure values of this test, it is possible to see that a fragment of the structure located in the area between 20 and 22.5 m above the bottom of the lateral shell is most vulnerable to initial deformations, just as in the negative pressure tests.

The minimum wind load value, resulting in the stability, is equal to $p = 2.02$ kPa.

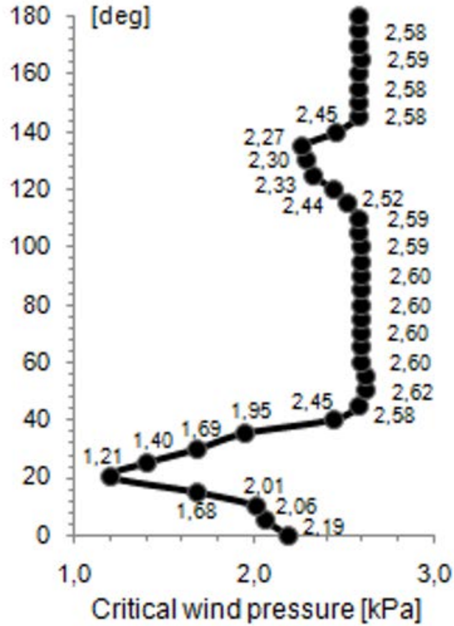


The next series of calculations include the limit wind pressure variation connected with the wind approaching the imperfection located on the elevation of 21.5 m when **the position of that imperfection is taken as variable**.

Its position is measured by an interior angle between the direction of the dent depth and the angle of the wind approach.

The relationship between the position of the dent and the approach angle of the wind load is presented in Figure.

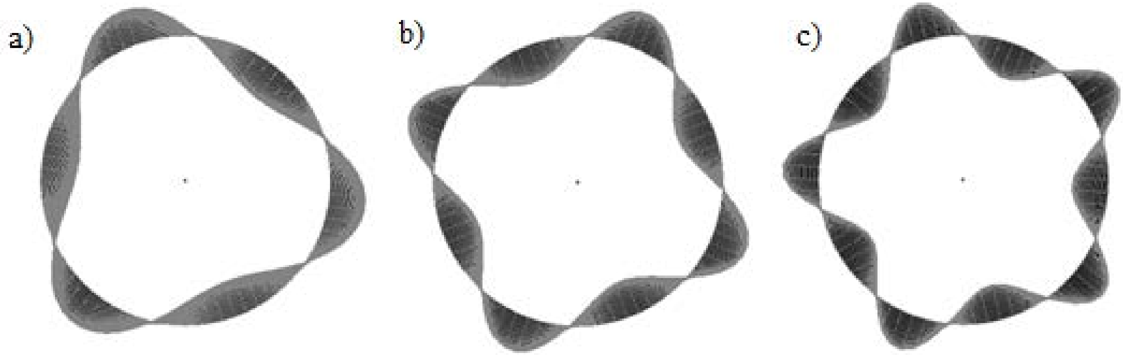
The lowest wind load limit value, is equal to $p = 1.21\ \text{kPa}$, which is much less than for a silo with perfect geometry.



IMPERFECTIONS IN THE SHAPE OF EIGENFORMS

Silo loaded with negative pressure

The initial imperfections are introduced in the form of the free vibration deformation of the shell, adapted on the basis of the 1st, 5th and 9th eigenform.



The individual eigenforms were obtained for eigenfrequencies of 4.50 Hz, 5.86 Hz, 7.76 Hz respectively.

The maximum value of node displacement amplitude t_{v0} = 20 mm.

The ultimate values of the negative pressure were equal to: 2.162 kPa (mode 1), 1.684 kPa (mode 5) and 1.49 kPa (mode 9) respectively.

Silo loaded with wind

The initial deflection of the shell loaded with wind is a deformation of the shell adopted by the use of the 1st, 5th and 9th eigenform.

The analysis was carried out for three values of the indentation amplitude namely 5, 10 and 20 mm.

The wind load description was the same as given by formulas (5) and (6).

The maximum wind pressure $p_{\max}$ is applied to the node of maximum displacement directed inwards.

The obtained ultimate load values are presented in Table.

EIGENFORM	DENT AMPLITUDE		
	5 *mm*	10 *mm*	20 *mm*
1st form	2.54 kPa	2.59 kPa	2.53 kPa
5th form	2.48 kPa	2.35 kPa	2.23 kPa
9th form	2.40 kPa	2.23 kPa	1.52 kPa

The least value was obtained for an imperfection situation corresponding to the 9th eigenform and an amplitude deviation of 20 mm.

IMPERFECTIONS IN THE SHAPE OF RANDOM FIELDS

Silo loaded with negative pressure

Geometric initial imperfections of any shell can be described by means of two-dimensional random fields.

Three types of random fields modelling the initial imperfections are analysed:
– non-correlated field,
– homogenous correlated field,
– and non-homogenous correlated field.

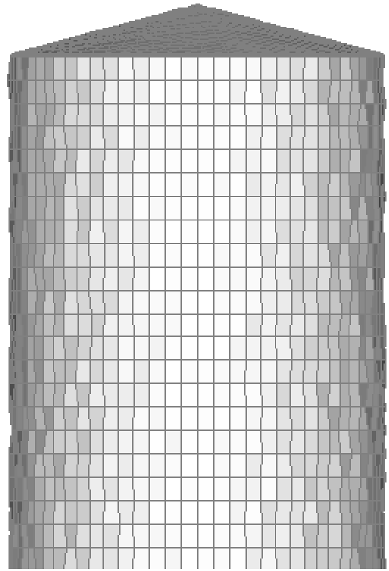
By lack of data describing real spatial dissipation of geometric imperfections, **the parameters of the correlation functions are assumed a priori**.

However, they reflect the maximal initial imperfections used earlier.

Non-correlated fields (white noise field)

The geometric deviations are defined using the uniform distribution.

The maximal imperfection $t_{v0} = 20\,\text{mm}$.



The standard deviation of the field described by the uniform distribution can be calculated according to the formula

$$\sigma = \frac{2t_{v0}}{\sqrt{12}} = \frac{2 \cdot 0.02}{\sqrt{12}} = 0.0115\,\text{m} \qquad (7)$$

The model including such initial imperfections is purely theoretical.

The obtained value of **the critical negative pressure** $p_{wn} = 4.204\,\text{kPa}$.

The chaotically distributed dents strengthen the shell.

Correlated fields

Two correlated random fields are generated.

Through lack of appropriate data the correlation function is chosen arbitrarily – it should confirm that the correlation between the random variables vanishes when the random point distance increases.

The initial geometric imperfections are described by means of the following **homogenous correlation function**

$$K\left(\Delta x_1, \Delta x_2\right) = \sigma^2 \exp\left(-\left(\beta \Delta x_1\right)^2 - \left(\gamma \Delta x_2\right)^2\right) \tag{8}$$

where Δx_1 and Δx_2 are distances between the points of the field along the horizontal and vertical axis, σ is a standard deviation describing the variability of the field, and β and γ are the decay coefficients.

In the example the following parameters have been assumed:
$\sigma = 0.01443$ m, $\beta = \gamma = 2,2$ m^{-1}.

Use is made of the same standard deviations as in the non-correlated field (7). The following range of truncated Gaussian distribution is applied

$$\pm 3\sigma = \pm 3 \cdot 0.0115 = 0.0345 \text{ m} \tag{9}$$

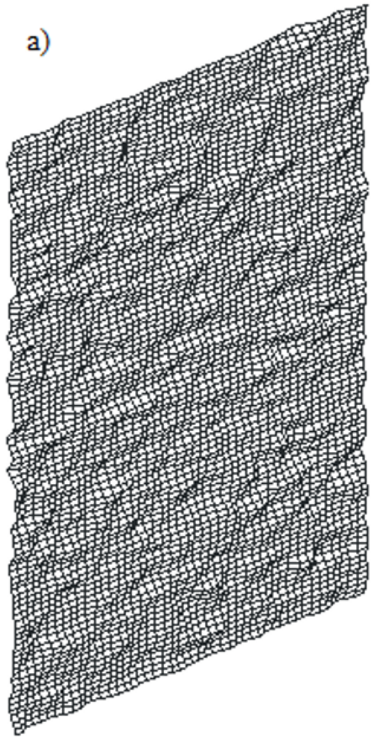
A set of samples for initial geometric imperfections of the silo are generated.

The field of imperfections, for which the calculations have been made, is illustrated by Figure.

The introduced deformations model the dented aluminum sheeting of the silo.

It should be pointed out that the imperfections, as shown in the illustration, are in a bigger scale, since, in reality, only in few places their amplitudes exceed the value of 30 mm.

For such a type of imperfections the limit value of **the negative pressure** is equal to $p_{\text{hom}} = 2.282$ kPa.

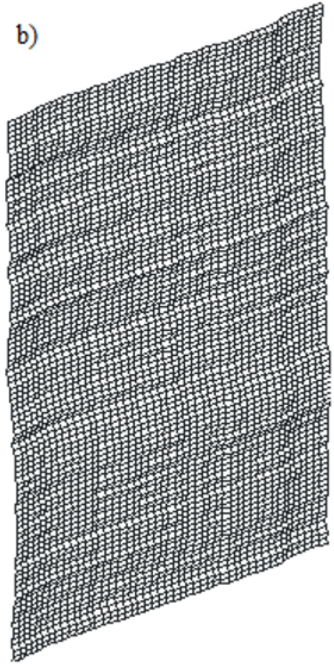


The second field analysed is a **non-homogenous field** described by the function

$$K(\Delta x_1, \Delta x_2) = \sigma \cos(\alpha \Delta x_1) \exp(-\beta \Delta x_1 - \gamma \Delta x_2) \qquad (10)$$

where:

$\sigma = 0.0144\,\mathrm{m}$, $\alpha = 0.2\,\mathrm{m}^{-1}$, $\beta = \gamma = 0.005\,\mathrm{m}^{-1}$.



A model of a field of imperfections is presented in Figure.

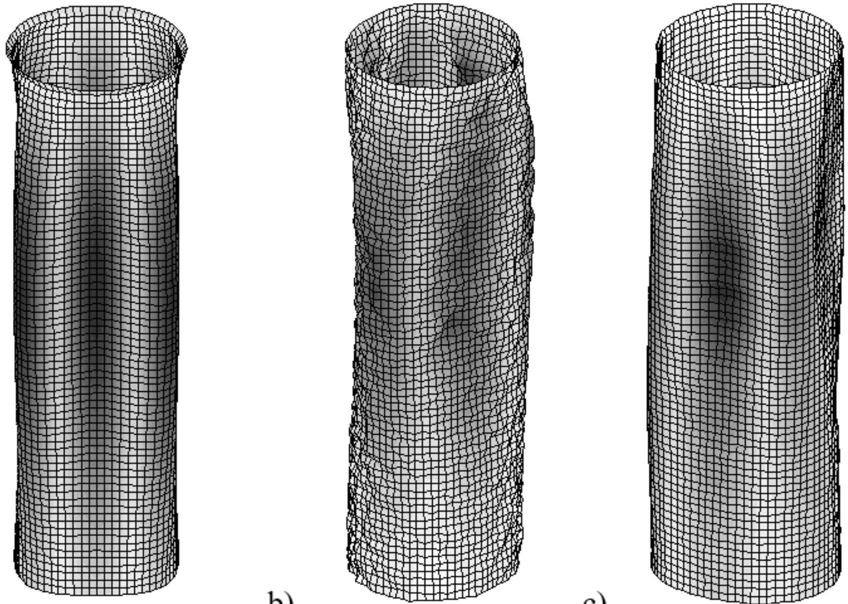
2000 samples of initial imperfections were simulated.

Next they were classified according to an average amplitude of the displacements.

Three fields were chosen for the calculations:
one with the lowest amplitude,
one with an average amplitude,
and one with the highest average amplitude.

The following values of the limit negative pressure were obtained: $p_{\mathrm{nonhom1}} = 2.542\,\mathrm{kPa}$, $p_{\mathrm{nonhom2}} = 2.543\,\mathrm{kPa}$, and $p_{\mathrm{nonhom3}} = 2.5445\,\mathrm{kPa}$.

Porównanie deformacji zbiorników



a) b) c)

Płaszcz zniszczonego silosu (fragmenty) w wyniku działania podciśnienia na: powłokę idealną (a), z imperfekcjami zdefiniowanymi białym szumem (b), z imerfekcjami zdefiniowanymi losowym, niejednorodnym polem losowym (c)

Silo loaded with wind

The calculations are performed for **two cases of shell deformation**.

In the first series, random deviations are once more described by means of a **non-correlated random field**.

A normal distribution is used.

The same value of standard deviation as for uniform distributions (7) is taken into consideration.

The geometry of the roof, hopper and the supportive structure of the silo is assumed without any initial deformations.

Defining the initial deformation as set out above, a limit load value being equal to $p_{wn} = 3.305\ \text{kPa}$ is studied, a value greater than that of a perfect silo.

The next stage of the analysis includes calculations for the case of shell deformation described by a **heterogeneous correlation function** (10).

The ultimate load value for such an initial deformation corresponds $p_{wc} = 2.331\ \text{kPa}$, a value slightly lower compared with a perfect silo.