OUTLINE ON STRUCTURAL LOADS

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Analysis of structural loads, prior to probabilistic analysis, covers load classification, mathematical model assessment and parameter estimation (characteristic values, safety measures) and analysis of effects of interaction of multi-source actions.

Classification of loads may be done, due to the following criteria:
(a) the origin of loads,
(b) their time variation,
(c) spatial variation of loads,
(d) a sort of structural response,
(e) inspection and limitation possibilities during operation.

The following sub-species of loads may be distinguished, due to the abovementioned criteria:

(a)–the origin of loads:
– **natural loads**: gravity, atmospheric phenomena, pressure, temperature and humidity variation, subsoil variation,
– **man-made loads**: room occupation, operation of machinery and devices, technological temperature variation.

(b)– timevariation:
– **permanent loads**: self-weight of structural elements, ground weight and pressure,
– **environmental and operational live loads,**
– **extraordinary loads**: vehicle impact, explosions, fires, hurricanes, catastrophic snowfalls, earthquakes.

(c)–space variation:
– **non-movable loads**(constant position),
– **movable loads**(arbitrary position).

(d)–type of structural response:
– **static loads**, generating neither accelerations nor inertia forces,
– **dynamic loads**– inertia forces may be significant, acting on the structural analysis,
– **repetitive (cyclic) loads**, which may lead to structural fatigue.

(e) - inspection and limitation possibilities during operation
– **controlled loads**: permanent and live loads,
– **non-controlled loads**, e.g. wind, temperature, vehicle impact, explosions, earthquake.

The mathematical background to model the loads, especially **random function theory** is a highly developed field.

Model simplification may be introduced at different stages of analysis, the difficulties in model calibration come from the limited statistical database access.The **engineering approach** is a simplified solution, intended to converge with the theoretical models, taking advantage of statistical data.

SELECTED RANDOM LOAD MODELS

The basic mathematical model is a random function -generalization of a random variable.
Every elementary event is mapped into a function, not a numerical value, which was relevant for a random variable model.

The theory of random loads states the **elementary event – taking a single structure** from a virtual or real population of structures of an identical design and operation conditions.

The non-random parameters of a considered random function are point coordinates(x, y, z) inside a structure and time $t \in \langle 0, T \rangle$.

The reference time T may be stated as the intended lifetime of a structure, $T = t_{\text{int}}$.

A general load model is a four-variable function $\underline{F} = \underline{F}(x, y, z, t)$ -
a spatial-temporal random field.

The distinct forms of $\underline{F}$ may be scalar, vector or tensor random fields.

The function $\underline{F}$ may serve for the load identification, taken directly from weather station measurements, like **wind speed** v_b, **water-snow equivalent** m or **the parameter obtained at the building site,** e.g. **floor slab thickness** t_s.

Random field, a key concept in engineering applications, may be assumed separate time and space variation, in terms of a single- or multivariable functions: $\underline{F}(t)$, $\underline{F}(x)$, $\underline{F}(x,y)$.

Four load models are applied in structural design standards.

The first and second models are discrete, the rest is continuous.

a) Model of a stream of random impulses

A stream of random impulses is a random function $\underline{N}(t)$ of a continuous time t, whose values are nonnegative integers, $N = n$.

The number of load impulses, without information on load values, occurring at a time interval is sufficient for extraordinary loads, damaging civil engineering structures, e.g. explosions, fires, plane collisions.

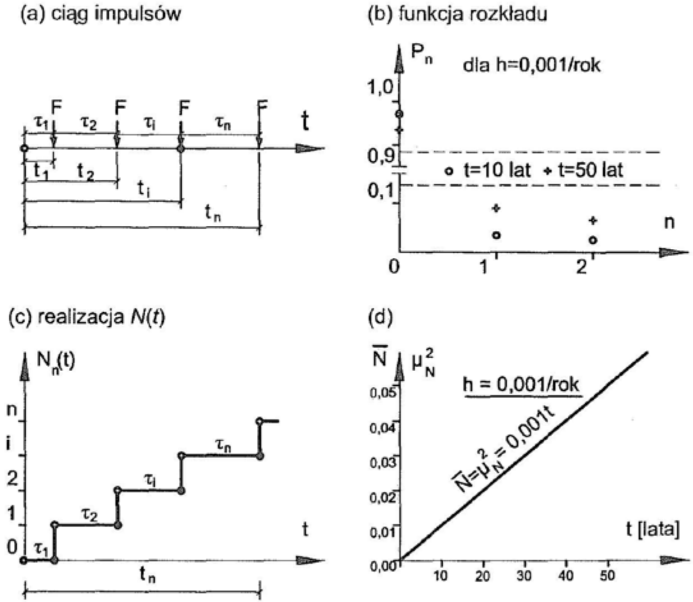
This impact load is a rare and immediate event, comparing to the intended lifetime (durability) of a structure.

The streams of random events (impulses), being random time functions, may be considered random processes too.

A Poisson process specifies the random function of a number of impulses $\underline{N}(t)$ at a time interval (0, *t*) being time-dependent, described by a single empirical parameter $h[t^{-1}]$:

$$P_n(t) = P_n(\underline{N} = n; t) = \frac{(ht)^n}{n!} e^{-ht} \qquad (1)$$

The formula(1)may be derived assuming independence of random intervals between impulses $\underline{\tau}_1$, $\underline{\tau}_2$, ... , $\underline{\tau}_2$ (Fig. 4.17a) and a time-in variant *h* value.



Rys. 4.17. Proces Poissona: losowy ciąg równych sił F (a), przykłady funkcji prawdopodobieństwa P_n w przekrojach: t = 10 lat i t = 50 lat (b), przykładowa realizacja $N(t)$ (c), przykładowy wykres średniej $\overline{N}(t)$ i wariancji μ_N^2 losowej liczby impulsów $\underline{N}(t)$ (d)

Fig. 4.17b shows a Poisson process section for*t* = const. Given a value *h* = 0,001/year it presents a discrete random variable whose single value is a number of forces *F*ata time interval (0, *t)* for a considered structure.

Fig. 4.17c shows a step function of a single realization *N*(*t*), depicting the time history of occurrence of forces *F* for a given structure.

The mean value and variance of a number of forces *F* in the time *t* are expressed by:

$$\bar{N}(t) = \sum_{n=0}^{\infty} n P_n(t) = ht, \qquad \mu_n^2(t) = \sum_{n=0}^{\infty} \left[n - \bar{N}(t) \right]^2 P_n(t) = ht \tag{2}$$

The formula (2) yields:

$$\bar{N}(t) = \mu_n^2(t) \tag{3}$$

$$h = \frac{\bar{N}(t)}{t} = const(t) \tag{4}$$

Equality of mean and variance, according to (3) is a check for Poisson process in the field of statistical data testing.

The formula (4) states that the Poisson force stream is uniform(constant *h)*, where *h* is a mean number of impulses per unit time.

The probability of survival of a structure, with respect to an impulse of a probability function(1) is the probability of non-occurrence of impulses *F* in time interval (0, *t*):

$$P\left(N=0;t\right)=e^{-ht} \tag{5}$$

The probability of collapse (cumulative distribution of durability), i.e. probability of at least one impulse *F* is equal:

$$P\left(N\geq 1;t\right)=P\left(T<t\right)=F\left(t\right)=1-e^{-ht} \tag{6}$$

Where *T* is a structural durability.

The time interval between two loadings *F* follows the probability density function:

$$f\left(t\right)=he^{-ht} \tag{7}$$

The h parameter of the Poisson process is the occurrence rate, equal to $h=1/\bar{T}$.

This process is uniform (constant *h)***and non-stationary**, while the mean and variance of random $\underline{N}(t)$ are time-variant,

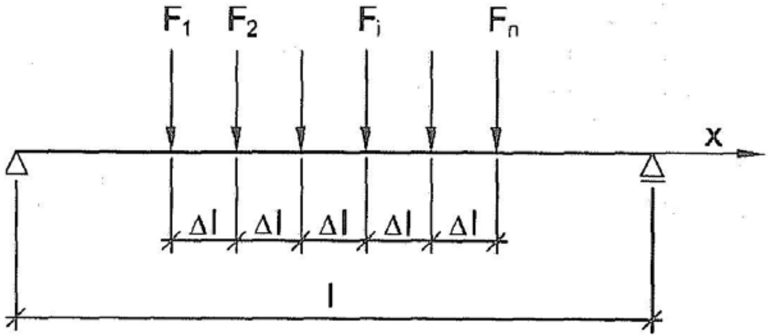
see Fig. 4.17(d).

The random process is stationary having a constant mean: $\bar{F}(t) = \bar{F} = const$ constant variance: $\mu_F^2(t) = \mu_F^2 = const$ and correlation moments of the process sections $\underline{F}(t)$ and $\underline{F}(t + \Delta t)$ dependent on the time interval (time lag) Δt, only, regardless of a specific time *t*: $K_F(t, t + \Delta t) \equiv K_F(\Delta t)$.

b) Regular sequence of random loads

This model is relevant for live loads, e.g. operational load on floors or snow load on roofs.

The interval ΔL between the point forces F_i (Fig. 4.18) may be stated arbitrarily, e.g. turning the load distributed along the length L into $n = L/\Delta L$ point forces F_i.



Rys. 4.18. Regularny ciąg sił losowych

This simple model is sufficient for stochastically independent, time-invariant and identically distributed F_i forces.

Equivalent approach of identical results states that random forces $\underline{F}_i$ move along the *x* axis with a constant velocity *v*, appearing in constant time intervals τ_0.

Thus $\Delta L = \tau_0 / v$ is a space interval of forces $\underline{F}_i$ (Fig. 4.18).

Linear system, due to structural mechanics, considers the total load effect E (**cross-sectional force, deflection**) a combination of component loads F_i, using non-random influence coefficients c_i:

$$\underline{E} = \sum_{i=1}^{n} c_i \underline{F}_i \tag{8}$$

The algebra of linear combination of random variables gives expressions for mean and variance of a random total load effect:

$$\bar{E} = \sum_{i=1}^{n} c_i \bar{F}_i = \bar{F}_1 \sum_{i=1}^{n} c_i \approx \bar{F}_1 \int_0^L c(x) \frac{dx}{\Delta L} \tag{9}$$

$$\mu_E^2 = \sum_{i=1}^{n} c_i^2 \mu_{Fi}^2 = \mu_{F1}^2 \sum_{i=1}^{n} c_i^2 \approx \mu_{F1}^2 \int_0^L c^2(x) \frac{dx}{\Delta L} \tag{10}$$

The equalities of mean values $\bar{F}_i = \bar{\underline{F}}_i$ and variances $\mu_{Fi}^2 = \mu_{F1}^2$ (fo ri = 1,2, …, n) come from the initial assumptions.

The approximate equations, on the right-hand sides of (9) and(10), result from replacing the influence coefficients c_i by an influence function $c(x)$.

In the case of surface loads the linear section ΔL turns into a surface element ΔA, integrals(9) and(10) turn into an area integral.

Other parameters are updated too, e.g. the number of forces acting on a floor equals $n = A / \Delta A$.

Assuming influence coefficients constant, say $c_i = 1$ (the case of a column axial force), the formulae (9) and(10) lead to a **coefficient of variation of a load effect**:

$$v_E = \frac{\mu_E}{\overline{\overline{E}}} = \frac{\sqrt{n}\mu_{F1}}{n\overline{\overline{F}}_1} = \frac{\mu_{F1}}{\sqrt{n}\overline{\overline{F}}_1} \to 0, \text{ gdy } n \to \infty \tag{11}$$

Formula (11) shows that large structures of a high number of independent loads (as civil engineering structures do) random loads stabilize, so static deterministic analysis is sufficient.

The disadvantage of this model is an independence assumption for the loads $\underline{F}_i$, sometimes divergent from the real conditions.

Given a partial reliability index value β_E the formula(9) and (10) make it possible to compute the design load effect, as follows:

$$E_d = \bar{E} + \beta_E \mu_E = \bar{F}_1 \sum_{i=1}^{n} c_i + \beta_E \mu_{Fi} \sqrt{\sum_{i=1}^{n} c_i^2} \tag{12}$$

The design values for each separate load, $F_{di} = \bar{F}_1 + \beta_E \mu_{F1}$ combined with (8) lead to a load effect:

$$E_d^* = \sum_{i=1}^{n} c_i F_{di} = \sum_{i=1}^{n} c_i \left(\bar{F} + \beta_E \mu_{F1} \right) \tag{13}$$

The ratio of (12) and (13) gives a reduction coefficient α_A ,assuming $c \geq 0$; i=1,2, ... , n it is **the measure of advantageous impact of statistical independence of loads** $\underline{F}_i$:

$$\alpha_A = \frac{E_d}{E_d^*} = \frac{1 + \varsigma \beta_E v_{F1}}{1 + \beta_E v_{F1}}, \quad \text{gdzie } v_{F1} = \frac{\mu_{F1}}{\bar{F}_1}, \ \varsigma = \sqrt{\frac{\sum_{i=1}^{n} c_i^2}{\sum_{i=1}^{n} c_i}} \tag{14}$$

The reduction coefficient for live loads is introduced in the standard PN-EN 1991-1-1, taking a form relevant for A ÷ E floor types:

$$\alpha_A = \frac{5}{7}\psi_0 + \frac{\Delta A}{A} \le 1{,}0 \tag{15}$$

limiting the C and D cases; $\alpha_A \ge 0{,}6$.

The reduction coefficient for live (operational) loads of walls and columns of multistorey buildings, α_m, according to the same standard:

$$\alpha_m = \frac{2 + (m^* - 2)\psi_0}{m^*} \tag{16}$$

The formulae (15) and (16) specify: $\Delta A = 10{,}0\ \mathrm{m}^2$ - floor area element, ψ_0 - **coincidence coefficient for live loads** (Table4.18) according to Polish standard PN-EN 1990 (having live loads dominant ψ_0 = 1,0), $m^* \ge 2$ - the number of storeys above the analysed one.

Reduction by means of a_m coefficient refers only to axial forces produced by live loads.

Recommended values of Ψ_j for buildings, Polish standard, 1990

Tablica 4.18

Zalecane wartości współczynników ψ_j dla budynków wg PN-EN 1990

Obciążenia zmienne, kategoria	ψ_0	ψ_1	ψ_2
(1)	(2)	(3)	(4)
Kategoria A: powierzchnie mieszkalne	0,7	0,5	0,3
Kategoria B: powierzchnie biurowe	0,7	0,5	0,3
Kategoria C: miejsca zebrań	0,7	0,7	0,6
Kategoria D: powierzchnie handlowe	0,7	0,7	0,6
Kategoria E: powierzchnie magazynowe	1,0	0,9	0,8
Kategoria F: powierzchnie ruchu pojazdów ≤ 30 kN	0,7	0,7	0,6
Kategoria G: powierzchnie ruchu pojazdów ≤ 160 kN	0,7	0,5	0,3
Obciążenie śniegiem ≤ 1000 m n.p.m.	0,5	0,2	0,2
Obciążenie wiatrem	0,6	0,2	0
Temperatura (nie pożarowa) w budynku	0,6	0,5	0

Area occupation: A – residential , B – office, C – meeting rooms, D – shops, markets,
E – warehouses, F – vehicles <= 30 kN, G – vehicles <= 160 kN,
last three rows: snow <= 1000 m above sea level, wind, non-fire room temperature

Discrete model of random forces, at equal intervals (having accepted all its assumptions)**may be used for the climatic load forecast** (wind and snow actions)

The focus of the weather station measurements is the **water-snow equivalent** – the mass *m* of water equivalent to a given batch of ground snow.

Wind load analysis is preceded by recording **the wind speed** v_{bo}**.**

The observations of a measurement period *t* are done continuously, the maximum values F_i of unit periods t_0 are chosen(finally the number of outcomes is $n = t/t_0$).

The unit observation period for climatic actions, due to their seasonal feature, equals t_0= 1 year, starting 1st of October
(like the academic year).
Statistical analysis of meteorological data usually neglects non-stationarities at multi-year periods, assuming that the annual maximum values are independent.

The **multi-year load** forecast for a single station, based on observation of annual maxima may be done **analytically or graphically**.

According to **graphical method** the annual maxima are formed in ascending order $F_1 < F_2 < \ldots < F_i < \ldots < F_n$, , being ordinates of points on a probability paper.

The following F_i values correspond to the ordinates (empirical CDF) $F_i^* = i/(n+1)$.
The forecast resembling the linear one is accepted on the probability paper. **Characteristic maximum values** F_k of the return period of an extreme load, t_{ret} is a fractile whose order is $F_1(F_k)$:

$$F_1\left(F_k\right) = 1 - \frac{t_0}{t_{\mathrm{ret}}} \tag{17}$$

where $F_1(\bullet)$ is a cumulative distribution function.

Probability P **of non-exceeding the** F_k **value in time period** $T = n \cdot t_0$ equals:

$$P\left(F < F_k\right) = \left[F_1\left(F_k\right)\right]^m = \left(1 - \frac{t_0}{t_{\mathrm{ret}}}\right)^{T/t_0} \tag{18}$$

Large T values lead to a formula:

$$P\left(F < F_k\right) = p = e^{-1} = e^{-T/t_{ret}} \quad (19)$$

Given $T = t_{\mathrm{ret}}$ the result is p =0,368, so the probability of taking loads less than F_k is $\omega \cong 0,368$.
The probability of exceeding the F_k value more than once equals
1 – 0,368 = 0,632.

The formula (19) gives the return period of loads F_k:

$$t_{ret} = \frac{T}{\ln\left(\frac{1}{p}\right)} \quad (20)$$

Assuming, after the PN-EN 1990 standard that characteristic loads are described by a probability p = 0,98 and an operational period
T = 50 years, the formula (20) leads to the return period of characteristic loads t_{ret}= 2575 years.

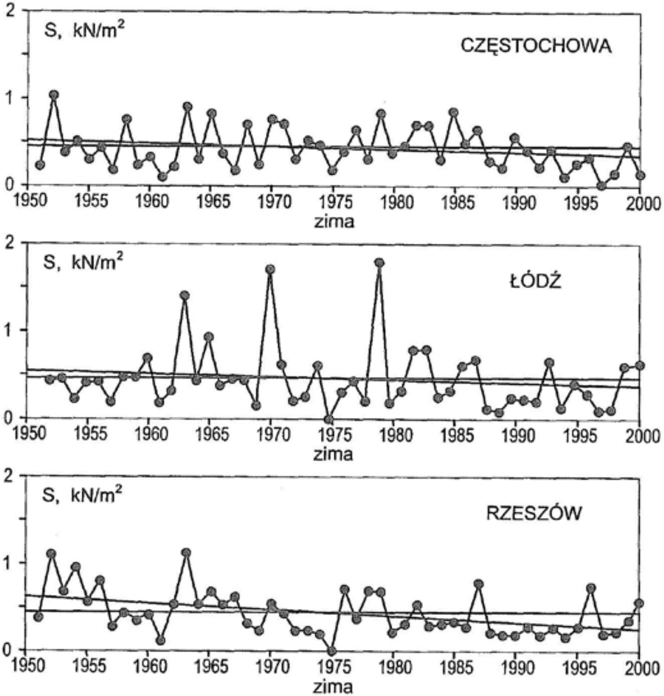
Note that the characteristic loads F_k are determined by (18) or the equivalent relation(19).

The analytical method investigates for parameters of an assumed probability distribution by the maximum likelihood method (the sample mean and sample variance are used to compute the assumed distribution parameters) and states characteristic maximum values for a prescribed operational period *T*.

In particular, characteristic maximum for a Gumbel distribution $\tilde{F}$ is stated by:

$$\tilde{F}_n = \tilde{F}_1 + \mu \ln\left(\frac{T}{t_0}\right) \qquad (21)$$

In order to illustrate the discrete model of random loads in equal time intervals, Fig. 4.19 presents maximum values of ground snow loads detected in the period 1950-2000 at three weather stations: Częstochowa(1), Łódź (2) and Rzeszów (3).
(note a 1979 "Winter of the Century" peak in Łódź)



Rys. 4.19. Przykładowe realizacje wartości maksymalnych rocznych ciężaru pokrywy śnieżnej na gruncie wg pracy [77]

The points in 4.19 led to a scanning-based recovery of annual maximum values s[kN/m^2], collected in Table4.12.

Tablica 4.12

Wartości maksymalnych rocznych ciężarów pokrywy śnieżnej na gruncie zarejestrowane w latach 1951–2000 na trzech stacjach pomiarowych wg rys. 4.20

Maksymalne ciężary pokrywy śnieżnej na gruncie s [kN/m^2]											
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Rok	St. 1	St. 2	St. 3	Rok	St. 1	St. 2	St. 3	Rok	St. 1	St. 2	St. 3
1951	0,23	–	0,39	1968	0,72	0,43	0,32	1985	0,91	0,34	0,34
1952	**1,04**	0,42	1,13	1969	0,26	0,15	0,24	1986	0,52	0,63	0,29
1953	0,40	0,45	0,69	1970	0,78	1,72	0,55	1987	0,68	0,70	0,80
1954	0,51	0,21	0,97	1971	0,73	0,63	0,43	1988	0,32	0,13	0,22
1955	0,30	0,41	0,57	1972	0,34	0,22	0,24	1989	0,24	0,09	0,20
1956	0,45	0,42	0,82	1973	0,54	0,26	0,23	1990	0,60	0,26	0,20
1957	0,18	0,18	0,29	1974	0,49	0,62	0,20	1991	0,46	0,24	0,27
1958	0,77	0,48	0,45	1975	0,20	0,00	0,00	1992	0,25	0,21	0,19
1959	0,26	0,47	0,36	1976	0,41	0,31	0,72	1993	0,45	0,68	0,28
1960	0,34	0,70	0,43	1977	0,68	0,44	0,38	1994	0,14	0,15	0,18
1961	0,11	0,19	0,12	1978	0,33	0,20	0,70	1995	0,30	0,42	0,31
1962	0,24	0,31	0,55	1979	0,86	**1,82**	0,69	1996	0,36	0,30	0,76
1963	0,92	1,42	**1,14**	1980	0,40	0,19	0,23	1997	0,06	0,13	0,22
1964	0,32	0,44	0,55	1981	0,49	0,33	0,32	1998	0,20	0,14	0,23
1965	0,84	0,94	0,69	1982	0,73	0,80	0,55	1999	0,51	0,62	0,38
1966	0,39	0,38	0,54	1983	0,72	0,80	0,30	2000	0,18	0,65	0,57
1967	0,20	0,46	0,63	1984	0,34	0,26	0,32	Σs	**22,70**	**22,75**	**22,18**
Wartość średnia								$\bar{s}$	**0,454**	**0,464**	**0,439**
Odchylenie standardowe								μ_s	**0,242**	**0,375**	**0,259**
Współczynnik zmienności								v_s	**0,532**	**0,808**	**0,589**
Wartość centralna (charakterystyczna) wg (1.41)								$\tilde{s}$	**0,369**	**0,332**	**0,348**
Miara zmienności wg (1.42)								u_x	**0,189**	**0,293**	**0,202**

Lower rows of the table present mean (lines parallel to time axis, Fig. 4.19), standard deviation, coefficient of variation for every realization of Σs (total 50-year snow weight).

Comparison of realizations in Fig. 4.19 shows, that the assumption of the stationary character is only partially fulfilled.

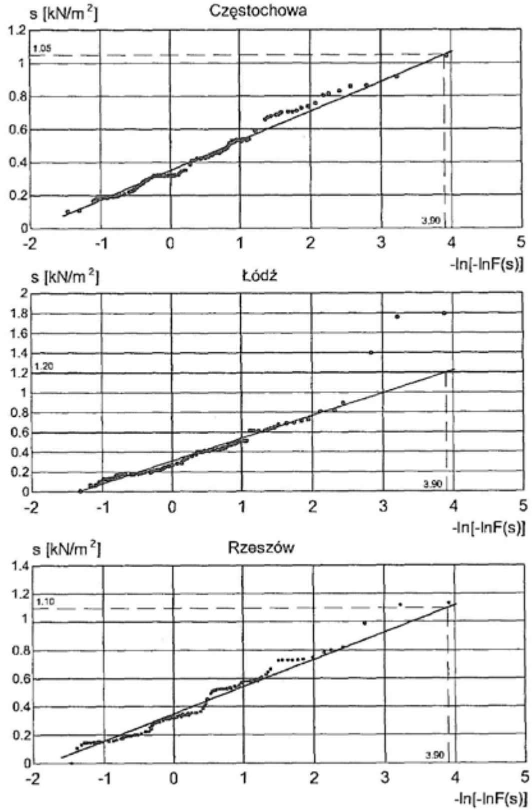
The mean values are identical, weak decreasing trend is observed at stations 1 and 2, stronger at station 3.
The tests of significance for trend detection were not performed.

An unexpected result came for the total 50-year snow load – almost constant for three realizations, Σs = 22 – 23 kN/m^2 = const.

The coefficient of variation is high, equal v_s = 81% for 2nd realization.

The probability distribution type for a snow load should be verified.

Fig. 4.20 shows a forecast for multi-year load at particular weather stations, presented on a Gumbel probability paper.



Rys. 4.20. Wyrównanie rozkładów empirycznych obciążenia śniegiem gruntu na siatce rozkładu Gumbela

The empirical points for three sequences of values fit the straight line well, so this sort of distribution may be accepted.

Given a return period if maximum load t_{ret}= 50 year, the fractile of $F(s_k)$ = 1 – 1/50 = 0,98 order corresponds to the ordinate of a point on an approximation straight line, according to Fig. 4.20 for the abscissa equal to –ln[–ln(0,98)] = 3,90.

Statistical data for three Polish weather stations lead to the corresponding characteristic values of snow load:
Częstochowa - s_k= 1,05 kN/m^2,
Łódź - s_k= 1,20 kN/m^2,
Rzeszów -s_k= 1,10 kN/m^2.

Two bottom rows of Table 4.12 show the Gaussian – Gumbel parameter conversion, by means of an analytical method.

The characteristic values are therefore stated, according to(21), for p = 0,98:

- station 1: s_k= 0,369 + 3,90·0,189 = 1,11 kN/m^2,
- station 2: s_k= 0,332 + 3,90·0,293 = 1,47 kN/m^2,
- station 3: s_k= 0,348 + 3,90·0,202 = 1,14 kN/m^2,

The multipliers of variability measure:–ln[–ln(0,98)] = 3,91.

Comparison of graphical and analytical methods is shown in Fig. 4.13, by means of characteristic snow ground load values, according to Polish standard PN-EN 1991-1-3.

The empirical forecast concerns the climatic zone 2 venues, so the analytical method proved to fit the standard recommendations more than graphical method for the cases considered.

Tablica 4.13

Wartości charakterystyczne obciążenia śniegiem gruntu w Polsce

Strefa klimatyczna	s_k [kN/m^2]
(1)	(2)
1	$0{,}007H - 1{,}4 \geq 0{,}70$
2	0,9
3	$0{,}006H - 0{,}6 \geq 1{,}2$
4	1,6
5	$0{,}93\exp(0{,}00134H) \geq 2{,}0$
H – wysokość nad poziomem morza [m]	

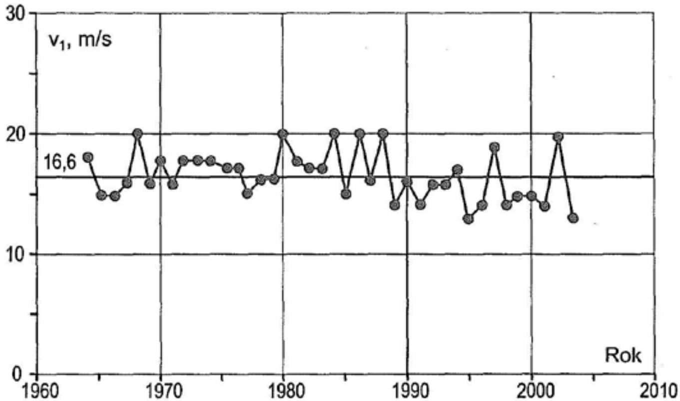
Statistical analysis of wind load is much more complicated than the analysis of snow load.

The following directions of wind speed measurement are valid at the weather stations:
1. The measurement time interval is 1 hour, recording is made every 10 min before the full hour of universal time,
2. Wind speed is registered with a1 m/s accuracy the wind direction with a 10^{o}accuracy.
3. The instantaneous wind speed is recorded in the case of gusts, if the 10 minute time interval shows the mean speed exceedance not less than 5 m/s.
The former and the present measurement requirements and procedures differ.
a. until the end of 1975 r. weather stations recorded mean 2-minute wind speeds, from the beginning of1976 the 10-minute speeds,
b. until 2000 the weather station workers assessed wind speed and direction without any equipment, from 2001 r. these operations are done automatically.

The statistical database of the Institute for Meteorology and Hydraulic Management is vast but **its homogeneity is not assured**.

The basic methods of data acquiring for the wind speed cover:
- maximum annual values are used to assess the characteristic wind speed v_{bO},
- mean 10-minute wind speeds used to estimate the parameters and verify the type of empirical distribution

Fig. 4.21 shows maximum annual wind speeds for the years 1964-2003 at Warszawa-Okęcie weather station.
The points in Fig. 4.21led to a recovery of annual maximum values wind speed values v_1 [m/s], shown in Table 4.14.



Rys. 4.21. Przykładowa realizacja wartości maksymalnych rocznych prędkości wiatru ze stacji Warszawa Okęcie z lat 1964–2003 wg pracy [76]

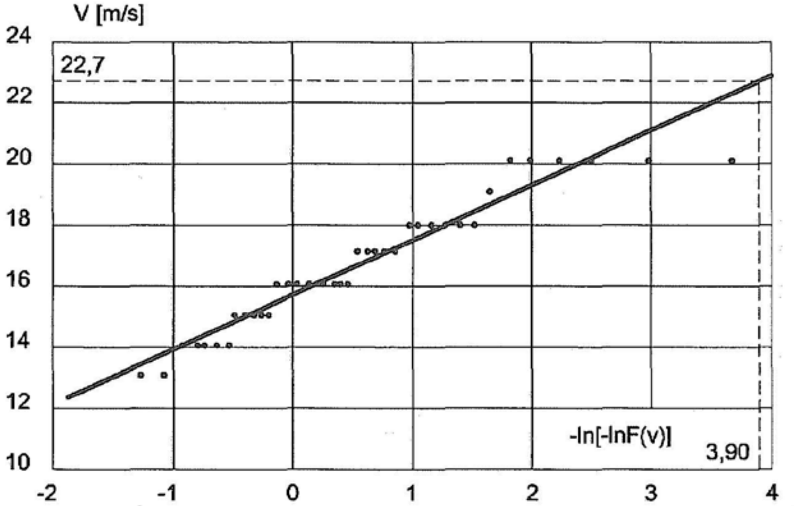
Tablica 4.14

Wartości maksymalne rocznych prędkości wiatru zarejestrowane w latach 1964–2003 na stacji meteorologicznej Warszawa Okęcie wg rys. 4.21

Maksymalne roczne prędkości wiatru v_1 [m/s]											
Rok	1964	1965	1966	1967	1968	1969		1970	1971	1972	1973
v_1	18	15	15	16	20	16		18	16	18	18
Rok	1974	1975	1976	1977	1978	1979	1980	1981	1982		1983
v_1	18	17	17	15	16	16	20	18	17		17
Rok	1984	1985	1986	1987	1988	1989	1990	1991	1992		1993
v_1	20	15	20	16	20	14	16	14	16		16
Rok	1994	1995	1996	1997	1998	1999	2000	2001	2002		2003
v_1	17	13	14	19	14	15	15	14	20		13
Wartość średnia									$\overline{v}_1$		**16,6**
Odchylenie standardowe									μ_v		**2,06**
Współczynnik zmienności									v_v		**0,124**
Wartość centralna (charakterystyczna) wg (1.41)									$\tilde{v}_1$		**15,9**
Miara zmienności wg (1.42)									u_v		**1,61**

Lower rows of the table present mean wind speed (a line parallel to time axis, Fig. 4.21), standard deviation and coefficient of variation.

Fig. 4.22 presents graphical equalization of measured annual maximum wind speeds on a Gumbel probability paper.



Rys. 4.22. Wyrównanie pomierzonych na stacji meteorologicznej Warszawa Okęcie maksymalnych prędkości wiatru

The empirical point sequence of wind speed $v_1$fit the straight line well, thus Gumbel distribution may be accepted for wind speed.

The fractile of the $F(v_k)$ = 1 – 1/50 = 0,98 order, related to the return period of maximum load, t_{ret} = 50 years, corresponds to an ordinate of a point lying on the approximation line (Fig. 4.22) whose abscissa equals –ln[–ln(0,98)] = 3,90.

The characteristic value of maximum annual wind speed v_k = 22,7 m/s is obtained from the diagram.

Two bottom rows of Table 4.14 present the Gaussian – Gumbel conversion of parameters, by means of analytical method, see (21).

The characteristic wind speed value, i.e. 89% fractile is therefore obtained :v_k= 15,9 + 3,90 (1,61) = 22,2 m/s.

Comparison of graphical and analytical methods is shown in Table 4.15, by means of characteristic wind speed values, according to Polish standard PN-EN 1991-1-4.

Tablica 4.15

Wartości charakterystyczne prędkości wiatru i ciśnienia prędkości wiatru w strefach klimatycznych w Polsce wg PN-EN-1991-1-4

Strefa	$v_k \equiv v_{b0}$ [m/s]		q_k [kN/m^2]	
	$H \leq 300$ m	$H > 300$ m	$H \leq 300$ m	$H > 300$ m
(1)	(2)	(3)	(4)	(5)
1	22	$22[1 + 0{,}0006(H - 300)]$	0,30	$0{,}30[1 + 0{,}0006(H - 300)]^2$
2	26	26	0,42	0,42
3	22	$22[1 + 0{,}0006(H - 300)]$	0,30	$0{,}30k[1 + 0{,}0006(H - 300)]^2$

$$k = \frac{20000 - H}{20000 + H}$$

H – wysokość nad poziomem morza w [m]

The empirical forecast concerns the climatic zone 1, so the graphical method fits the standard recommendations more than the analytical method. Columns (4) and (5) of the Table 4.15 present characteristic values of wind pressure, by the formula

$$q_k = \frac{\rho v_{b0}^2}{2} \qquad (22)$$

where ρ - mass density of air, dependent on the elevation above sea level, temperature and atmospheric pressure (Table 4.15 states ρ = 1,23 kg/m^3.