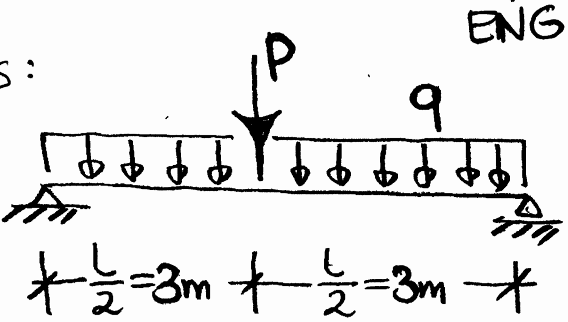
* A simply supported beam (figure) is subjected to a set of random loads:

- uniform q, normal distribution $N(20,1)\,\left[\frac{kN}{m}\right]$
- point load P, normal distribution $N(40,3)\,[kN]$



Random yield stress of steel normally distributed, $f_y \equiv f$ $N(300,5)\cdot 10^3\,[kPa]$

Plastic section modulus of a cross-section, deterministic, $W = 600\,cm^3 = 6\cdot 10^{-4}\,m^3$.

Assess the failure probability – exceeding the ultimate limit state (bending only)

The limit state function:

$$G(q,P,f) = fW - \frac{qL^2}{8} - \frac{PL}{4} = 6\cdot 10^{-4} f - 4{,}5q - 1{,}5P \quad [kNm]$$

It is a linear combination of normal (Gaussian) random variables, therefore it is a normal (Gaussian) variable.

Mean value of G: $\bar{G} = 6\cdot 10^{-4}\bar{f} - 4{,}5\bar{q} - 1{,}5\bar{P} = 6\cdot 10^{-4}\cdot 3\cdot 10^{5} - 4{,}5\cdot 20 - 1{,}5\cdot 40 = 30\,kNm$

(it is a deterministic, mean value of the limit state function, its positive value is a reliability check for the mean values)

Variance of G: $\sigma_G^2 = (6\cdot 10^{-4}\sigma_f)^2 + (4{,}5\sigma_q)^2 + (1{,}5\sigma_P)^2 = (6\cdot 10^{-4}\cdot 5\cdot 10^3)^2 + (4{,}5\cdot 1)^2 + (1{,}5\cdot 3)^2 = 49{,}5\,[(kNm)^2]$

Standard deviation of G: $\sigma_G = 7{,}036\,kNm$

Failure probability (probability of exceeding ultimate limit state of a beam)

$$P_f = P(G<0) = F\left(\frac{0-30}{7{,}036}\right) = F(-4{,}264) = 1 - F(4{,}264) = 1{,}02\cdot 10^{-5}$$

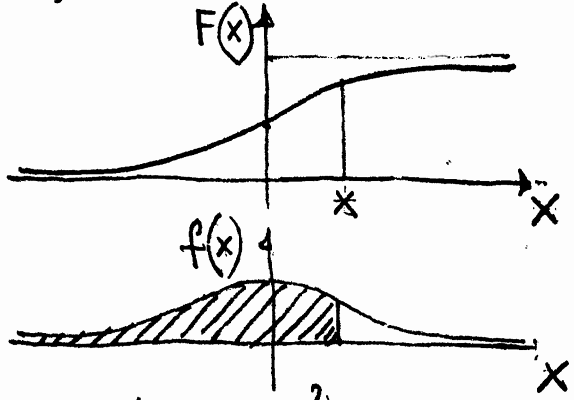
NOTE: A function $G(\underset{\sim}{X})$, normally distributed, $\underset{\sim}{X} = \{X_1 \ldots X_n\}$ – basic variable vector is a limit state function:

$G(X) > 0 \rightarrow$ safe state (region)
$G(X) < 0 \rightarrow$ failed state (failure region)
$G(X) = 0 \rightarrow$ limit state

$\beta = \frac{\bar{G}}{\sigma_G}$ – Cornell's reliability index

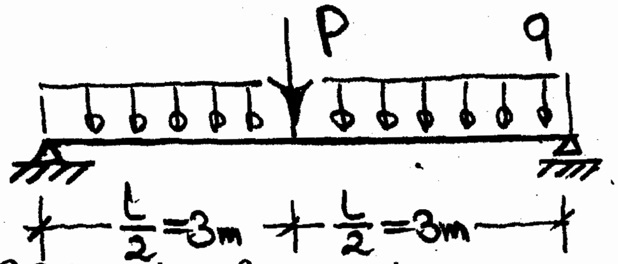
$$P(G<0) = F\left(\frac{0-\bar{G}}{\sigma_G}\right) = F(-\beta)$$

It is a second-level method reliability assessment (check).



$$f(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)$$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{t^2}{2}\right)dt$$

* 

$EI = 20000\ kNm^2 = const$

The beam shown in figure is subjected to random loads
- uniform q – normal, $N(20, 1)\left[\frac{kN}{m}\right]$
- point load P – normal $N(40, 3)\,[kN]$

Estimate the probability of exceeding allowable deflections (serviceability limit state):

a. $V_{all} = 2{,}8$ cm b. $V_{all} = 3{,}0$ cm.

c. what is the mid-span deflection V_0, that may be exceeded with the probability of 10% (90% reliability)?

Random mid-span deflection of a beam – normally distributed random variable

$$V(q,P) = \frac{5qL^4}{384EI} + \frac{PL^3}{48EI} = \frac{q\cdot 5\cdot 6^4}{384\cdot 2\cdot 10^4} + \frac{P\cdot 216}{48\cdot 2\cdot 10^4} = (8{,}4375q + 2{,}25P)\cdot 10^{-4}\,[m]$$

Parameters of random deflection V:

mean value $\overline{V} = 10^{-4}(8{,}4375\bar{q} + 2{,}25\bar{P}) = 10^{-4}(8{,}4375\cdot 20 + 2{,}25\cdot 40) =$
$= 258{,}75\cdot 10^{-4}\,m = 2{,}588\,cm$

variance $\sigma_V^2 = 10^{-8}\left[(8{,}4375\cdot 1)^2 + (2{,}25\cdot 3)^2\right] = 116{,}754\cdot 10^{-8}\ m^2$

standard deviation $\sigma_V = 10{,}805\cdot 10^{-4}\,m = 0{,}108\,cm$

a. $P(v > 2{,}8cm) = 1 - P(V < 2{,}8cm) = 1 - F\left(\frac{2{,}8 - 2{,}588}{0{,}108}\right) = 1 - F(1{,}97) =$
$= 1 - 0{,}9756 = 0{,}0244$

OTHER VARIANT:
serviceability limit state function

$$G(P,q) = V_{dop} - \frac{5qL^4}{384EI} - \frac{PL^3}{48EI}, \quad \overline{G} = 2{,}8 - 2{,}588 = 0{,}212\,cm$$
$$\sigma_G = 0{,}108\,cm$$

$P(v > 2{,}8) = P(G < 0) = F\left(\frac{0 - 0{,}212}{0{,}108}\right) = F(-1{,}97) = 0{,}0244$

b. $P(v > 3{,}0cm) = 1 - P(v < 3{,}0cm) = 1 - F\left(\frac{3{,}0 - 2{,}588}{0{,}108}\right) = 1 - F(3{,}818) =$
$= 1 - 0{,}999933 = 6{,}67\cdot 10^{-5}$

c. $P(V > V_0) = 1 - P(v < V_0) = 1 - F\left(\frac{v_0 - 2{,}588}{0{,}108}\right) = 0{,}1$

thus $F\left(\frac{V_0 - 2{,}588}{0{,}108}\right) = 0{,}9 \Rightarrow \frac{V_0 - 2{,}588}{0{,}108} = 1{,}28$

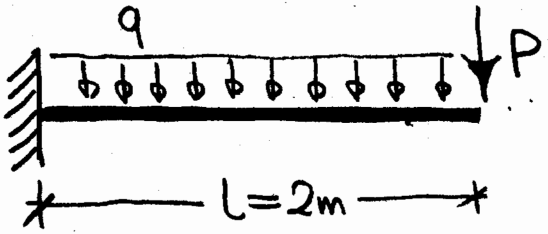
$V_0 = 2{,}588 + 0{,}108\cdot 1{,}28 = 2{,}73\,cm$

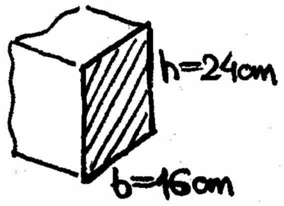
It is a 0,9 – fractile of variable v – value of v which is not exceeded with the probability of 0.9.

the value $F(t) = 0{,}90$ corresponds to $t = 1{,}28 \Rightarrow \frac{V_0 - \overline{V}}{\sigma_V} = t$

so $V_0 = \overline{V} + t\cdot\sigma_V = 2{,}588 + 1{,}28\cdot 0{,}108 = 2{,}73\,cm$

* D1





3
ENG

$E = 80\,GPa$
$f_y = 55\,MPa$

A cantilever beam of deterministic cross-sectional dimensions and material is subjected to random actions:

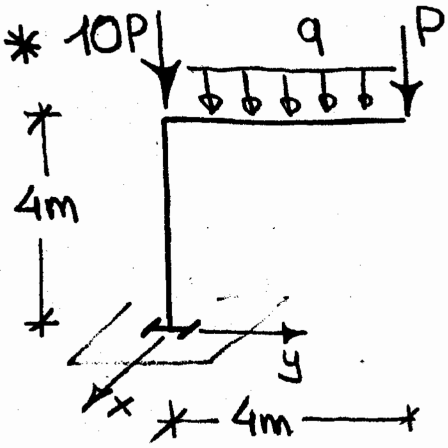
- point load P, normal $N(45;\ 2)$ [kN]
- uniform load q, normal $N(10,\ 1)$ $\left[\frac{kN}{m}\right]$

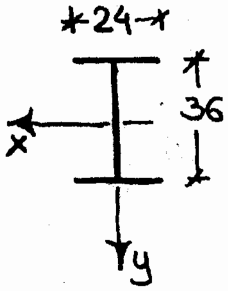
a. estimate the probability of exceeding the ultimate limit state due to bending

b. estimate the probability of exceeding the allowable deflection equal to 1 cm.

c. what is the deflection value to be exceeded with the probability of 0,01 ?

D2 *





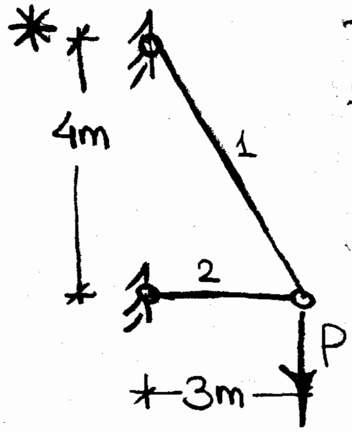
$t = 1{,}0\,cm$
thin-walled cross-section

material → $f_y = 270\,MPa = const$

random loads

$q \rightarrow N(7,\ 1)$ $\left[\frac{kN}{m}\right]$
$P \rightarrow N(40;\ 3)$ [kN]

Estimate the probability of the ultimate limit state of the structure,
consider bending-compression interaction
(idealized I-section interaction formula)



Two-element truss is subjected to a random point load $P \to N(180,3)$ [kN].
The yield stress of a material is a random variable $f_y \longrightarrow N(200,5)\cdot 10^3$ [kPa].
Cross-section of truss elements, $A \to N(16,2)\cdot 10^{-4}$ [m^2]
Estimate the probability of exceeding the ultimate limit state of a structure, neglecting stability problems involved.

Axial forces in truss members $S_1 = \frac{5}{4}P$; $S_2 = -\frac{3}{4}P$.

Limit state function : $G(P,f,A) = Af - S_1 = Af - \frac{5}{4}P$

It is a nonlinear function, of a simple-product form

Gaussian distribution – mean value and variance formulae may be applied

mean value $\quad \bar{G} = \bar{A}\bar{f} - \frac{5}{4}\bar{P} = 16\cdot10^{-4}\cdot 200\cdot10^{3} - \frac{5}{4}\cdot 180 = 95 \text{ kN}$

variance $\quad \sigma_G^2 = \bar{A}\sigma_f^2 + \sigma_A^2\sigma_f^2 + \bar{f}\sigma_A^2 + \left(\frac{5}{4}\sigma_P\right)^2 =$

$$= \left(1{,}6\cdot10^{-4}\cdot 5\cdot10^{3}\right)^2 + \left(2\cdot10^{-4}\cdot 5\cdot10^{3}\right)^2 + \left(200\cdot10^{3}\cdot 2\cdot10^{-4}\right)^2 + \left(\frac{5}{4}\cdot 3\right)^2 = 1679{,}06 \text{ kN}^2$$

standard deviation $\quad \sigma_G = 40{,}97 \text{ kN}$

probability of the limit state $\quad P(G<0) = F\left(\frac{0-95}{40{,}97}\right) = F(-2{,}31) = 0{,}0104$

NOTE: In general case the limit state function may be linearised with respect to mean values (Taylor series expansion, linear terms only)

Formulae: X and Y – Gaussian-distributed variables $\quad X(\bar{X}, \sigma_X)$, $Y(\bar{Y}, \sigma_Y)$

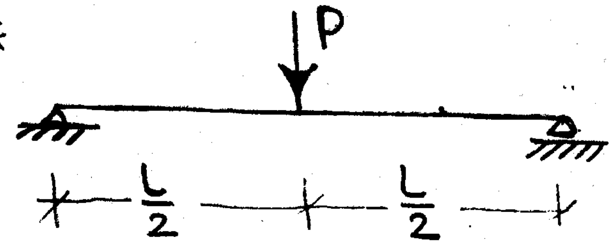
sum $S = X + Y$: $\bar{S} = \bar{X} + \bar{Y}$, $\sigma_S^2 = \sigma_X^2 + \sigma_Y^2$ $\left[Var(S) = Var(X) + Var(Y)\right]$

difference $R = X - Y$: $\bar{R} = \bar{X} - \bar{Y}$, $\sigma_R^2 = \sigma_X^2 + \sigma_Y^2$ $\left[Var(R) = Var(X) + Var(Y)\right]$

product $P = X\cdot Y$: $\bar{P} = \bar{X}\cdot\bar{Y}$, $\sigma_P^2 = \bar{X}^2\sigma_Y^2 + \sigma_X^2\sigma_Y^2 + \bar{Y}^2\sigma_X^2$

$$\left[Var(P) = \bar{X}^2 Var(Y) + Var(X)\,Var(Y) + \bar{Y}^2 Var(X)\right]$$

*



simply supported beam of a rectangular cross-section is subjected to a random load. Parameters:

- load $P \to N(120, 2)$ [kN]
- dimensions of a cross-section: width $b \to N(12; 0,2)\cdot 10^{-2}$ [m], depth $h \to N(20; 0,5)\cdot 10^{-2}$ [m]
- length of a beam $L \to N(8; 0,1)$ [m]
- yield stress of material: $f \to N(250, 3)\cdot 10^{3}$ [kPa]
- Young's modulus $E \to N(180, 2)\cdot 10^{6}$ [kPa]

a. estimate the probability of the ultimate limit state of the beam

b. estimate the probability of the serviceability limit state, given $V_{all} = 0,1$ m

---

a. The limit state function $G(P, b, h, L, f) = f\frac{bh^2}{4} - \frac{PL}{4} \to$ nonlinear

Taylor series expansion with respect to the mean value vector (*):

$$G \approx G|_* + \frac{\partial G}{\partial f}\Big|_*(f-\bar f) + \frac{\partial G}{\partial b}\Big|_*(b-\bar b) + \frac{\partial G}{\partial h}\Big|_*(h-\bar h) + \frac{\partial G}{\partial P}\Big|_*(P-\bar P) + \frac{\partial G}{\partial L}\Big|_*(L-\bar L)$$

$$G|_* = \frac{1}{4}\bar f \bar b \bar h^2 - \frac{1}{4}\bar P \bar L = \frac{1}{4}(250\cdot 10^3\cdot 0,12\cdot 0,2^2 - 120\cdot 8) = 60$$

$$\frac{\partial G}{\partial f}\Big|_* = \frac{bh^2}{4}\Big|_* = \frac{1}{4}\bar b \bar h^2 = \frac{1}{4}\cdot 0,12\cdot 0,2^2 = 1,2\cdot 10^{-3}$$

$$\frac{\partial G}{\partial b}\Big|_* = \frac{fh^2}{4}\Big|_* = \frac{1}{4}\bar f \bar h^2 = \frac{1}{4}\cdot 250\cdot 10^3\cdot 0,2^2 = 2500$$

$$\frac{\partial G}{\partial h}\Big|_* = \frac{fbh}{2}\Big|_* = \frac{1}{2}\bar f \bar b \bar h = \frac{1}{2}\cdot 250\cdot 10^3\cdot 0,12\cdot 0,2 = 3000$$

$$\frac{\partial G}{\partial P}\Big|_* = -0,25 L|_* = -0,25\bar L = -2 \;;\; \frac{\partial G}{\partial L}\Big|_* = -0,25P|_* = -0,25\bar P = -30$$

thus $G \approx 60 + 1,2\cdot 10^{-3}(f - 250\cdot 10^3) + 2500(b-0,12) + 3000(h-0,2) - 2(P-120) - 30(L-8) =$
$= 1,2\cdot 10^{-3} f + 2500b + 3000h - 2P - 30L - 660$

Parameters of G: $\bar G = G|_* = 60$

$$\sigma_G^2 = (1,2\cdot 10^{-3}\cdot 3\cdot 10^3)^2 + (2500\cdot 0,002)^2 + (3000\cdot 0,005)^2 + (2\cdot 2)^2 + (30\cdot 0,1)^2 = 287,96$$

$\sigma_G = 16,97$

$$P_f = P(G<0) = F\left(\frac{0-60}{16,97}\right) = F(-3,54) = 2\cdot 10^{-4}$$

---

b. serviceability limit state $G(P, b, h, L, E) = V_{dop} - \frac{PL^3}{48EI} = 0,1 - \frac{PL^3}{4Ebh^3}$

derivation: $\frac{\partial G}{\partial P}\Big|_* = -\frac{\bar L^3}{4\bar E \bar b \bar h^3} = -\frac{8^3}{4\cdot 180\cdot 10^6\cdot 0,12\cdot 0,2^3} = -7,407\cdot 10^{-4}$

$\frac{\partial G}{\partial L}\Big|_* = -\frac{3\bar P \bar L^2}{4\bar E \bar b \bar h^3} = -\frac{3\cdot 120\cdot 64}{4\cdot 180\cdot 10^6\cdot 0,12\cdot 0,2^3} = -0,0333$; $\frac{\partial G}{\partial E}\Big|_* = \frac{\bar P \bar L^3}{4E^2 bh^3} = \frac{120\cdot 8^3}{4\cdot 180^2\cdot 10^{12}\cdot 0,12\cdot 0,2^3} = 4,94\cdot 10^{-10}$

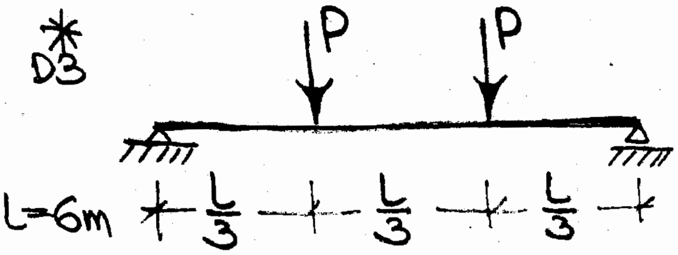
$\frac{\partial G}{\partial b}\Big|_* = \frac{\bar P \bar L^3}{4\bar E \bar b^2 \bar h^3} = \frac{120\cdot 8^3}{4\cdot 180\cdot 10^6\cdot 0,12^2\cdot 0,2^3} = 0,741$; $\frac{\partial G}{\partial h}\Big|_* = \frac{3\bar P \bar L^3}{4\bar E \bar b \bar h^4} = \frac{3\cdot 120\cdot 8^3}{4\cdot 180\cdot 10^6\cdot 0,12\cdot 0,2^4} = 1,333$

$G \approx 0,0111 - 7,407\cdot 10^{-4}(P-120) - 0,0333(L-8) + 4,94\cdot 10^{-10}(E - 180\cdot 10^6) +$
$+ 0,741(b-0,12) + 1,33(h-0,2) = -7,407\cdot 10^{-4}P - 0,0333L + 4,94\cdot 10^{-10}E + 0,741b + 1,33h - 0,0778$

$\bar G = G|_* = 0,0111$; $\sigma_G^2 = (7,407\cdot 10^{-4}\cdot 2)^2 + (0,033\cdot 0,1)^2 + (4,94\cdot 2\cdot 10^{-4})^2 + (0,741\cdot 0,002)^2 + (1,33\cdot 0,005)^2 =$
$= 1,4714\cdot 10^{-5}$

$\sigma_G = 0,003836$

$$P_f = P(G<0) = F\left(\frac{0 - 0,0111}{0,003836}\right) = F(-2,90) = 0,00187$$



A simply supported beam of random material parameters is randomly loaded, as shown.
Basic variables:

- dimension of a square cross-section $a \to N(20; 0{,}2) \cdot 10^{-2}$ [m]
- point load $P \to N(18; 0{,}5)$ [kN]
- yield stress of the material of the beam $f \to N(30,3) \cdot 10^3$ [kPa]
- Young's modulus of the material $E \to N(80,1) \cdot 10^6$ [kPa]

a. assess the probability of the ultimate limit state of the beam

b. assess the probability of exceeding allowable deflections, $v_{all} = 1{,}5$ cm

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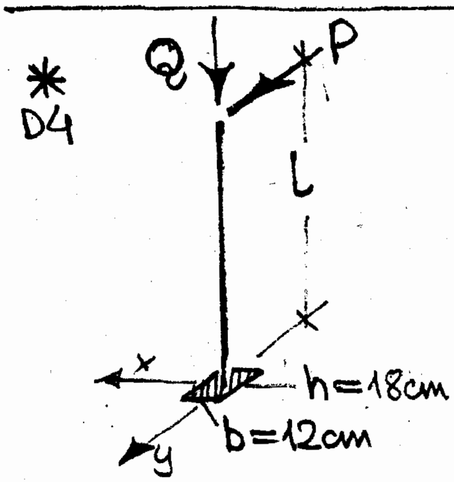


A cantilever column is subjected to a combination of random loads.
Basic variables:

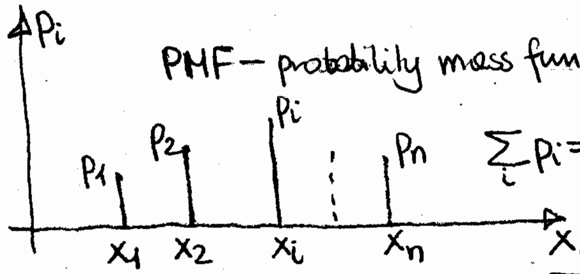
$Q \to N(200, 2)$ [kN]
$P \to N(30, 1)$ [kN]
$f \to N(100, 1) \cdot 10^3$ [kPa] yield stress
$L \to N(3; 0{,}02)$ [m]

Assess the probability of the ultimate limit state of the system, considering bending–compression interaction.

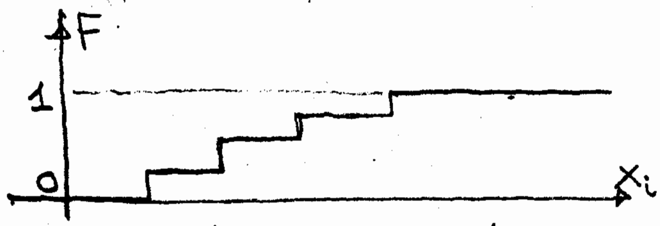
REVIEW – probabilistic models – random variables

DISCRETE RANDOM VARIABLES

PMF – probability mass function $\quad \sum_i p_i = 1$



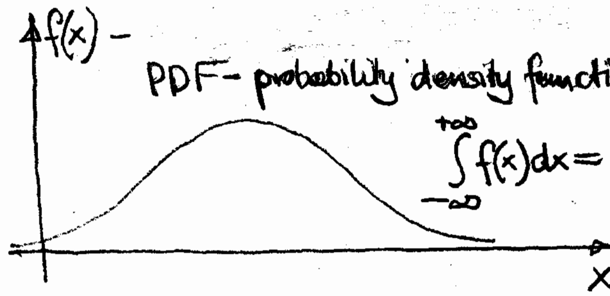
x_i	x_1	x_2	...	x_n
$p_i = P(x_i)$	p_1	p_2		p_n



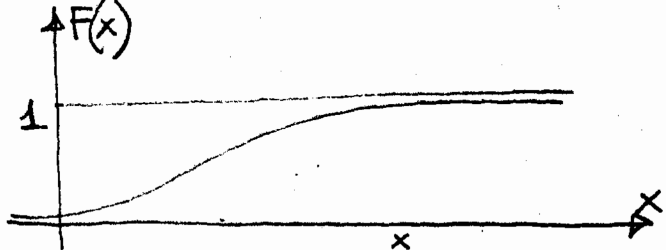
$$F(x) = P(X \leq x_i) = \sum_{x_i \leq x} P(X = x_i)$$

CONTINUOUS RANDOM VARIABLES

PDF – probability density function $\quad \int_{-\infty}^{+\infty} f(x)\,dx = 1$



CDF – cumulative distribution function



$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt \ ; \ f(x) = \frac{dF}{dx}$$

Parameters of random variables – moments

N-th order moment with respect to c (a constant value)

$$M_n(X) = \sum_i (x_i - c)^n p(x_i) \qquad M_n(X) = \int_{-\infty}^{+\infty} (x-c)^n f(x)\,dx$$

The case $c = 0 \rightarrow$ ordinary moments

$$M_n(X) = \sum_i x_i^n p(x_i) \qquad M_n(X) = \int_{-\infty}^{+\infty} x^n f(x)\,dx$$

Expected value (mean value) of a random variable

$$\overline{X} \equiv E(X) = \sum_i x_i\, p(x_i) \qquad \overline{X} \equiv E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx$$

$E(\cdot)$ – expected value operator (expectation operator)

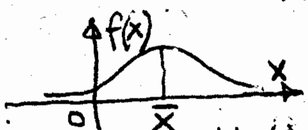
Moments of a random variable with respect to its mean value – central moment.

Variance – second central moment of a random variable

$$Var(X) = E(X - \overline{X})^2 \equiv \sigma_X^2 = \sum_i (x_i - \overline{x})^2 p_i \qquad Var(X) = E(X-\overline{X})^2 \equiv \sigma_X^2 = \int_{-\infty}^{+\infty} (x-\overline{x})^2 f(x)\,dx$$

standard deviation $\quad \sigma_X = \sqrt{Var(X)}$; coefficient of variation $\quad \gamma_X = \frac{\sigma_X}{\overline{X}}$

graphically



$\overline{X}$ – position of a centroid of a unit f(x) area

$E(X^2)$ – moment of inertia of f(x) area with respect to x=0

$Var(X) = E(X-\overline{X})^2$ – central moment of inertia of f(x) area

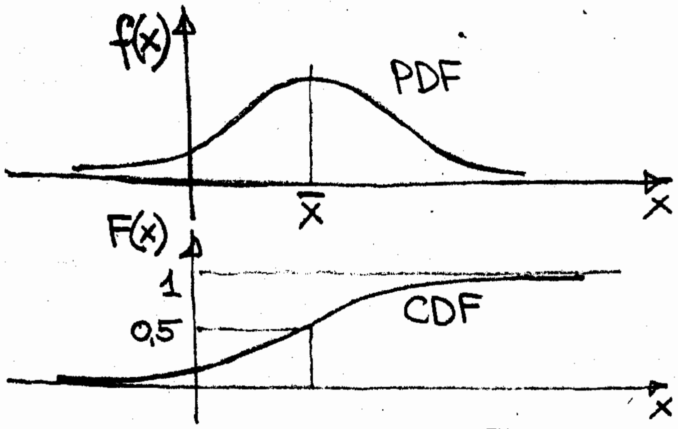
identity $\quad Var(X) = E(X^2) - \overline{X}^2 \rightarrow$ Steiner's law

Central moment of the third order of a random variable – measure of assymetry

$$E(X-\overline{X})^3 = \sum_i (x_i - \overline{x})^3 p_i \qquad E(X-\overline{X})^3 = \int_{-\infty}^{+\infty} (x-\overline{x})^3 f(x)\,dx$$

skewness (skewness coefficient) $\quad \theta = \frac{E(X-\overline{X})^3}{\sigma^3} \rightarrow$ nondimensional $\qquad$ symmetric random variable $\rightarrow \theta = 0$

CONTINUOUS RANDOM VARIABLE — NORMAL (GAUSSIAN) DISTRIBUTION



domain : $x \in R$

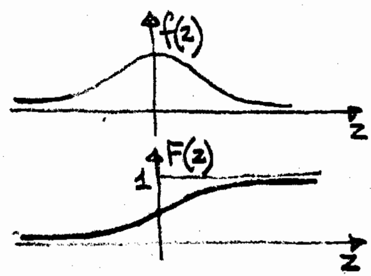
$$f(x)=\frac{1}{\sigma_x\sqrt{2\pi}}\exp\left[-\frac{(x-\bar{x})^2}{2\sigma_x^2}\right]$$

symmetric with respect to $x=\bar{x}$

$$F(x)=\frac{1}{\sigma_x\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left[-\frac{(t-\bar{x})^2}{2\sigma_x^2}\right]dt$$

Standard form : $z=\frac{x-\bar{x}}{\sigma_x} \rightarrow f(z)=\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{z^2}{2}\right]$

$$F(z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z}\exp\left[-\frac{t^2}{2}\right]dt$$

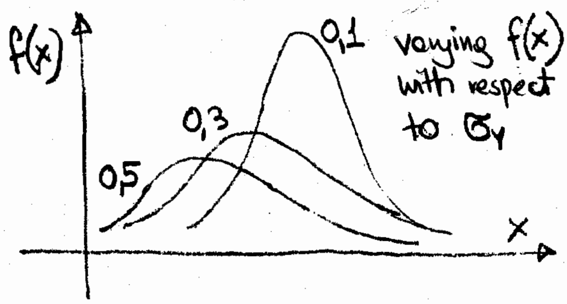


$F(z) \rightarrow$ non-elementary functions, tables : $z \leqslant 0 \Rightarrow F(z) \in (0;0{,}5\rangle$

$z \geqslant 0 \Rightarrow F(z) \in \langle 0{,}5;1)$

CONTINUOUS RANDOM VARIABLE – LOGARITHMIC-NORMAL (LOGNORMAL) DISTRIBUTION

– random variable X, related : $Y=\ln(X)$ is a Gaussian random variable



$$f(x)=\frac{1}{x\sigma_Y\sqrt{2\pi}}\exp\left[-\frac{(y-\bar{Y})^2}{2\sigma_Y^2}\right]=$$

$$=\frac{1}{x\sigma_Y\sqrt{2\pi}}\exp\left[-\frac{(\ln x-\bar{Y})^2}{2\sigma_Y^2}\right]$$

Relations between $(\bar{X},\sigma_X) \longleftrightarrow (\bar{Y},\sigma_Y)$

$$\begin{cases} Var(Y)=\sigma_Y^2=\ln\left(\frac{\sigma_X^2}{\bar{X}^2}+1\right)=\ln\left(\nu_X^2+1\right) \\ \bar{Y}=\ln(\bar{X})-\frac{1}{2}\sigma_Y^2 \end{cases}$$

$$\begin{cases} \bar{X}=\exp\left(\bar{Y}+\frac{1}{2}\sigma_Y^2\right) \\ Var(X)=\sigma_X^2=\bar{X}^2\left[\exp\left(\sigma_Y^2\right)-1\right] \end{cases}$$

* Intensity of a sewage system of a city is lognormally distributed, its parameters are:
$(\bar{X} = 1{,}2;\ \sigma_x = 0{,}4) \cdot 10^6 \frac{m^3}{day}$. Limit intensity of a system is $1{,}5 \cdot 10^6 \frac{m^3}{day}$.
What is the probability of submergence?

Parameters of Y – corresponding normal random variable $Y = \ln(X)$

$$\sigma_Y^2 = \ln(1+\nu_X^2) = \ln\left(1+\frac{\sigma_X^2}{\bar{X}^2}\right) = \ln\left(1+\frac{1}{9}\right) = \ln 1{,}111 = 0{,}105$$

$$\sigma_Y = 0{,}324 \quad \left[\cdot 10^6 \frac{m^3}{day}\right]$$

$$\bar{Y} = \ln(\bar{X}) - \frac{1}{2}\sigma_Y^2 = \ln 1{,}2 - \frac{1}{2}\cdot 0{,}105 = 0{,}130 \left[\cdot 10^6 \frac{m^3}{day}\right]$$

Y – Gaussian distributed random variable $N(0{,}130;\ 0{,}324)$, stgd

$$P(X>1{,}50) = 1 - P(X<1{,}50) = 1 - P(Y<\ln 1{,}50) = 1 - F\left(\frac{\ln 1{,}5 - 0{,}130}{0{,}324}\right)$$
$$= 1 - 0{,}8023 = 0{,}1977$$

---

* The time between earthquakes at a given geographical position is a lognormally distributed random variable, whose mean value is 80 years and standard deviation equals 32 years (coefficient of variation equal 40%).

a. estimate the parameters of the corresponding Gaussian variable $Y = \ln X$
b. determine the probability of a 20-year return period of a given earthquake.
c. assuming no earthquake in the past 100 years what is the probability it will happen in the upcoming year?

a. $Var(Y) = \sigma_Y^2 = \ln(\nu_X^2 + 1) = \ln 1{,}16 = 0{,}1484 \Rightarrow \sigma_Y = 0{,}3852$

$\bar{Y} = \ln(\bar{X}) - \frac{1}{2}\sigma_Y^2 = \ln 80 - \frac{1}{2}\cdot 0{,}1484 = 4{,}308$

b. $P(X<20) = P(Y<\ln 20) = F\left(\frac{\ln 20 - 4{,}308}{0{,}3852}\right) = F(-3{,}407) = 3{,}14\cdot 10^{-4}$

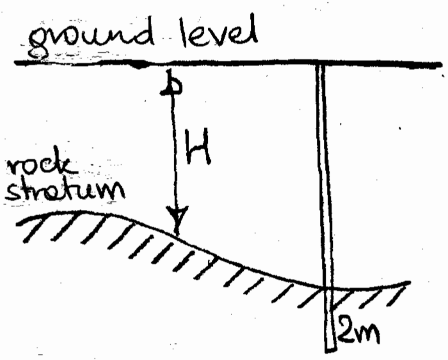
c. conditional probability $\quad p = \frac{P(100<X<101)}{P(X>100)} = \frac{P_1}{P_2}$

$$P_1 = P(100<X<101) = P(X<101) - P(X<100) = P(Y<\ln 101) - P(Y<\ln 100) =$$
$$= F\left(\frac{\ln 101 - 4{,}308}{0{,}3852}\right) - F\left(\frac{\ln 100 - 4{,}308}{0{,}3852}\right) = F(0{,}797) - F(0{,}7714) =$$
$$= 0{,}7875 - 0{,}7799 = 0{,}0076$$

$$P_2 = P(X>100) = 1 - P(X<100) = 1 - 0{,}7799 = 0{,}2201$$

so $\quad p = \frac{P_1}{P_2} = \frac{0{,}0076}{0{,}2201} = 0{,}0345$

D5 The depth H from the ground level to the rock stratum is lognormally distributed, with the mean value of **20m** and standard deviation of **6m**.



In order to provide a satisfactory support a pile must be embedded 2m into the rock. What is the probability, that a 25m pile is correctly anchored in the rock stratum?

BASED ON: A. Ang, W. Tang Probability concepts in engineering, Wiley 2007

D6 The load-carrying capacity of a transmission tower column is lognormally distributed with the mean value of 100 kN and standard deviation equal to 20 kN. Determine the probability that a column will resist a load of 100 kN? What is the answer, assuming normal distribution of equal parameters? Comment the solution.

D7 The maximum velocity of a tornado wind at a given city follows a lognormal distribution with a mean value $90\ \frac{m}{s}$ and standard deviation equal $18\ \frac{m}{s}$.

a. determine the probability that the maximum wind velocity will exceed $120\ \frac{m}{s}$ during the next tornado.

* b. determine the wind velocity not to be exceeded during the next 100 years (return period equal to 100 years). Assume one tornado will strike this city each year.

BASED ON: A. Ang, W. Tang Probability concepts in engineering, Wiley 2007

DISCRETE RANDOM VARIABLE – BINOMIAL (BERNOULLI) DISTRIBUTION

Bernoulli trials – independent, repetitive ; identical conditions,
a single trial – distinguished event (success) of probability p,
its complement of probability $q = 1 - p$ (failure, loss)
The values p and q constant throughout the trials.
Engineering examples of Bernoulli trials :
- determination of tensile strength of steel, identical specimens,
- determination of compressive strength of concrete, identical recipes,
- series of manufactured hot-rolled elements (e.g. I-beams) identical technology
- identically designed structural elements or substructures

The probability of k successes out of n Bernoulli trials defines a binomial (Bernoulli) probability distribution

$$P(X) \equiv P_n(k) = \binom{n}{k} p^k q^{n-k} = \frac{n!}{k!(n-k)!} p^k q^{n-k}$$

$$E(X) = np, \qquad Var(X) = npq$$

In case of a large number of Bernoulli trials (large n) and a small probability p, the product $\lambda = np$ is assumed, then the probability of k successes in n trials may be approximated by a Poisson distribution:

$$P(X) = P_n(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

It is a discrete probability distribution of wide application, including time-variant problems (Poisson process).
Assuming $\lambda = \nu t$ we obtain

$$P(X=k, t) = \frac{(\nu t)^k}{k!} e^{-\nu t}$$

This is a probability of k occurrences of an event in time t.
The parameter $\nu \left[\frac{1}{s}, \frac{1}{h}\right]$ is the occurrence rate.

* The prototype fasteners of timber elements are manufactured with a 65% efficiency.

a. let X be a number of efficient element in a 6-item pack. Show the probability distribution of X, its coefficient of variation.

b. determine the probability, that the number of inefficient element will be less than half the number of efficient elements.

---

a. $p=0{,}65 \quad q=0{,}35 \quad n=6$

$P(x=0)=\binom{6}{0}\cdot 0{,}65^0\cdot 0{,}35^6=0{,}00184$

$P(x=1)=\binom{6}{1}0{,}65^1\cdot 0{,}35^5=0{,}02048$

$P(x=2)=\binom{6}{2}0{,}65^2\cdot 0{,}35^4=0{,}09510$

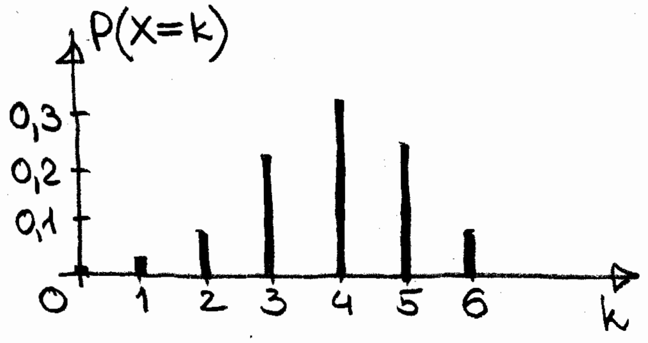
$P(x=3)=\binom{6}{3}0{,}65^3\cdot 0{,}35^3=0{,}23549$

$P(x=4)=\binom{6}{4}0{,}65^4\cdot 0{,}35^2=0{,}32801$

$P(x=5)=\binom{6}{5}0{,}65^5\cdot 0{,}35^1=0{,}24366$

$P(x=6)=\binom{6}{6}0{,}65^6\cdot 0{,}35^0=0{,}07542$

$\Sigma=1{,}0$



$\overline{X}=E(X)=np=3{,}9$

$Var(X)=npq=1{,}365$

$\sigma_X=1{,}1683$

$\nu_X=\frac{\sigma_X}{\overline{X}}=0{,}2996$

b. the cases $k=5$ or $k=6$, $P(A)=P(k=5)+P(k=6)=0{,}31908$
A – the event described in the text.

---

* The population of 600 structural element was produced, 3% of the elements were inefficient.

a. what is the probability of 1% inefficiency in a batch of 200 elements?

b. assess the relative error of two solutions of a problem, using binomial and Poisson distributions.

$n=200, \quad p=0{,}03$

a. Bernoulli distribution: $P_{200}(X=2)=\binom{200}{2}\cdot 0{,}03^2\cdot 0{,}97^{198}=$

$=199\cdot 100\cdot 0{,}03^2\cdot 0{,}97^{198}=0{,}04304$

b. Poisson distribution: $\lambda=np=6$

$P(x=2)=\frac{6^2}{2!}e^{-6}=0{,}04462$

relative error $\eta=\frac{|0{,}04304-0{,}04462|}{0{,}04304}\cdot 100\%=3{,}6\%$

D8 Manufacturing process of steel elements results in a 20% population inefficient.

a. let X be a number of efficient element in a 5-item batch. Describe analytically and illustrate the distribution of X, determine the mean value, standard deviation, coefficient of variation and the most probable value of X,

b. determine the probability of two elements or more efficient in a batch.

D9. Ten identically structures were erected, with 75% efficient ventilation systems. What is the probability of its proper function
a. in every second structure b. not less than 3 structures?

D10 Concrete cylinder specimens are compressed in a laboratory. The check for a test is the compressive strength R_c exceeding 30 MPa. The total number of 250 tests rejected 75 cases. Determine the probability of events:
a. the 20-piece batch will induce at least 90% efficiency
b. the whole 20-piece batch will be efficient

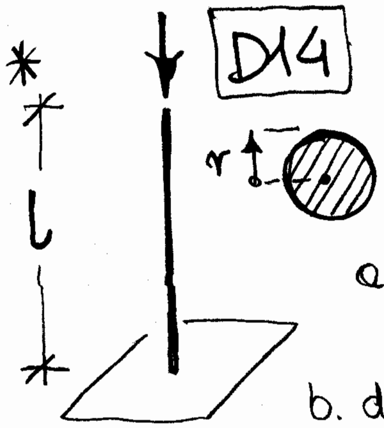
D11 The daily temperature distribution at Gdansk seaside in July follows the uniform distribution bounded by (15, 35) [°C].

a. determine, analytically and graphically, PDF and CDF of temperature
b. what is the probability that the July temperature will not reach 20, 30, 35°C, respectively?
c. determine the July temperature not to be exceeded with a probability of 0,7.

D12 The daily traffic intensity follows the distribution N(1000, 120). What is the probability that no more than 900 vehicles a day will come in two days of the week?

D13 The population of 80 transmission towers reflects allowable imperfection exceedance in four cases only. What is the probability of three exceedance cases in the entirety of 150 structures?

D14



A cantilever cylindrical column is axially loaded. The basic variables are:
$L \rightarrow N(4;0{,}04)[m]$; $E \rightarrow N(200,2)[GPa]$, $r \rightarrow N(6;0{,}08)[m]$

a. determine the parameters of a random buckling load N_{cr} – its mean value $\overline{N}_{cr}$ and standard deviation $\sigma_{N_{cr}}$.

b. determine the N_x value, that the buckling load will not exceed, with a probability of 10%. What is the order of this fractile?

c. considering the critical force (buckling load) of a column its load-carrying capacity ($R \equiv N_{cr}$), determine the probability of its exceedance by a random load $P \rightarrow N(280,10)$ [kN]. What is the corresponding Cornell's reliability index?

d. determine the mean value $\overline{Q}$ of a random axial load Q of a standard deviation $\sigma_Q = \sigma_P$, to make the column collapse with a probability of 4%?

A nonlinear function $T(x_1, x_2, \ldots, x_n)$ of basic variables $\underset{\sim}{X} = \{X_1, X_2, \ldots, X_n\}$ may be linearly approximated by a Taylor series around the mean values $\underset{\sim}{\overline{X}} = \{\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n\}$: $T \approx \overline{T} + \sum_{i=1}^{n} \frac{\partial T}{\partial x_i}(X_i - \overline{X}_i)$ – derivatives at mean values,
Mean value $\overline{T}$ and standard deviation σ_T may be estimated by

$$\overline{T} \approx T(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n) = T(\underset{\sim}{\overline{x}}) \; ; \; \sigma_T^2 = \left(\frac{\partial T}{\partial x_1}\bigg|_{\underset{\sim}{\overline{x}}} \sigma_{x_1}\right)^2 + \ldots + \left(\frac{\partial T}{\partial x_n}\bigg|_{\underset{\sim}{\overline{x}}} \sigma_{x_n}\right)^2 = \sum_{i=1}^{n} \left(\frac{\partial T}{\partial x_i}\bigg|_{\underset{\sim}{\overline{x}}} \sigma_{x_i}\right)^2$$

The current problem: $N_{cr} = \frac{\pi^2 E I}{L_{eff}^2} = \frac{\pi^3 E r^4}{16 L^2}$ $\left(L_{eff} = 2L, \; I = \frac{\pi r^4}{4}\right)$

$\underset{\sim}{X} = \{X_1, X_2, X_3\} = \{E, r, L\}$

a. $\overline{N}_{cr} = N_{cr}(\underset{\sim}{\overline{x}}) = \frac{\pi^3 \overline{E}\, \overline{r}^4}{16 \overline{L}^2} =$

$\frac{\partial N_{cr}}{\partial E}\bigg|_{\underset{\sim}{\overline{x}}} = \frac{\pi^3 \overline{r}^4}{16 \overline{L}^2} = \ldots$, $\frac{\partial N_{cr}}{\partial r}\bigg|_{\underset{\sim}{\overline{x}}} = \frac{\pi^3 \overline{E}\, \overline{r}^3}{4 \overline{L}^2} = \ldots$, $\frac{\partial N_{cr}}{\partial L}\bigg|_{\underset{\sim}{\overline{x}}} = -\frac{\pi^3 \overline{E}\, \overline{r}^4}{8 \overline{L}^3} = \ldots$

the partial derivatives → sensitivity factors of N_{cr}, with respect to E, r, L, respectively.

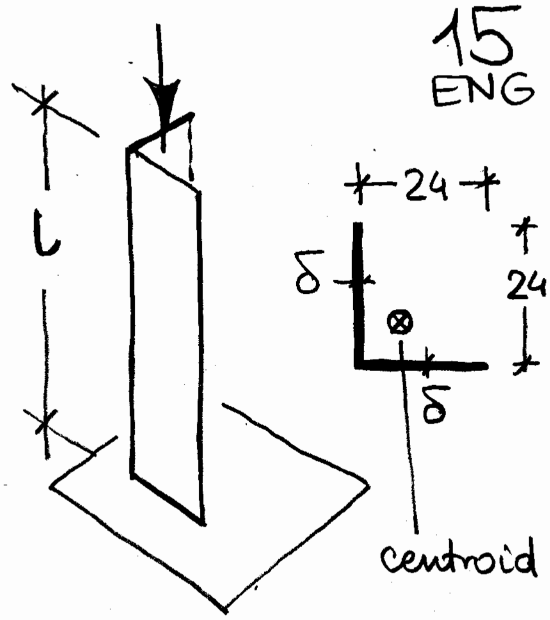
b. fractile of N_{cr} of the 0,1-order (10% order) → $F^{-1}(0{,}1)$, $t = 1{,}28$

c. define the limit state function, procedure based on the initial homework problems

d. updated limit state function, follow the procedure above

D15 A L-shaped thin-walled cantilever column is ideally axially compressed.
The basic random variables are:

$$L \to N(3;0{,}02)[m] \quad , E \to N(180,2)[GPa]$$
$$\delta \to N(1;0{,}03)[cm]$$



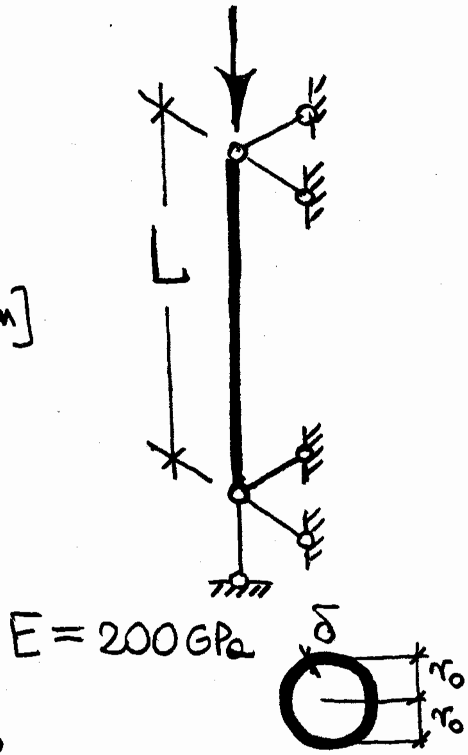
a. determine the mean value and standard deviation of the random critical force (buckling load) N_{cr}

b. determine the N_x value that will not be exceeded with a probability of 4%.

c. determine the standard deviation σ_P of a random load P whose mean is $\overline{P}=500\,kN$ to achieve the failure probability equal to 10^{-3}.

D16 Pin-supported tubular column is axially loaded. The basic random variables are:

$$r_0 \to N(10;0{,}06)[cm]\,;\ \delta \to N(0{,}8;0{,}006)[cm]$$
$$L \to N(4,0)\,[m].$$



a. determine the parameters of the random critical force N_{cr}.

b. determine the N_x value that will not be exceeded on the level $t=3$. What is the reliability?

c. determine the mean value of the load P to achieve the load-carrying capacity of the column with a probability of 99,8%. Assume identical standard deviations for the critical force N_{cr} and the load P.

SYSTEM RELIABILITY

Each structural system is an assembly of decisive elements – real system components (e.g. truss members), critical cross-sections of framed structures or, in general, failure modes (global or local collapse, excessive deflections or vibration amplitudes, etc.). Reliability of the system is highly affected by the connection of the decisive elements.

* SERIES SYSTEM

examples: statically determinate truss – member failure causes overall system failure; statically determinate frame – plastic hinges (while bending action taken only) – one is enough to make the entire system fail.

Reliability of a series system = the probability of a <u>simultaneous</u> action of all its decisive elements

Assuming n elements, reliability of the i-th element equal to R_i $\Rightarrow$ system reliability $R = \prod_{i=1}^{n} R_i$

$R_i = \text{const} \Rightarrow R = (R_i)^n$

Failure of a series system is equivalent to failure of at least one element. Failure probability refers therefore to the complement of simultaneous action of all elements.

Assuming failure probability of the i-th element equal to p_{fi} $\Rightarrow$ system probability of failure $p_f = 1 - \prod_{i=1}^{n}(1-p_{fi}) = 1 - \prod_{i=1}^{n} R_i$

$p_{fi} = \text{const} \Rightarrow p_f = 1 - (1-p_{fi})^n$

The series system implies $R < \min(R_1, R_2, \dots, R_n) = \min R_i$

e.g. a. $R_1 = 0{,}7$; $R_2 = 0{,}8$; $R_3 = 0{,}9$ $\qquad R = 0{,}7 \cdot 0{,}8 \cdot 0{,}9 = 0{,}504$

b. $R_1 = R_2 = R_3 = 0{,}8$ $\qquad R = (0{,}8)^3 = 0{,}512$

also $p_f > \max(p_{f1}, p_{f2}, \dots, p_{fn}) = \max p_{fi}$

a. $p_{f1} = 0{,}3$; $p_{f2} = 0{,}2$; $p_{f3} = 0{,}1$ $\qquad p_f = 1 - 0{,}7 \cdot 0{,}8 \cdot 0{,}9 = 0{,}496$

b. $p_{f1} = p_{f2} = p_{f3} = 0{,}2$ $\qquad p_f = 1 - (0{,}8)^3 = 0{,}488$

* PARALLEL SYSTEM

examples : the structure of independent supports, different supporting systems – each one has to fail to cause a structural collapse ; internally statically indeterminate truss – more than one element must fail in order to disable the entire system ; framed structure of statical indeterminacy, bending action only – a number of plastic hinges introduced to make the system collapse.

Failure of a system = <u>simultaneous</u> failure of all its parts.

Assume p_{fi} is a failure probability of the i-th element, the total number of elements is n. ⇒ system probability of failure $p_f = \prod_{i=1} p_{fi}$

$p_{fi} = \text{const} \Rightarrow p_f = (p_{fi})^n$

Reliability of a parallel system corresponds to a proper function of at least one element, so its complement is failure of all elements.

Assume R_i is a reliability of the i-th element out of n ⇒ system reliability $R = 1 - \prod_{i=1}^{n}(1-R_i) = 1 - \prod_{i=1}^{n} p_{fi}$

$R_i = \text{const} \Rightarrow R = 1 - (R_i)^n$

The parallel system implies $p_f < \min(p_{f1}, p_{f2}, \dots, p_{fn}) = \min p_{fi}$

e.g. a. $p_{f1} = 0{,}3$; $p_{f2} = 0{,}2$; $p_{f1} = 0{,}1$ $\quad p_f = 0{,}3 \cdot 0{,}2 \cdot 0{,}1 = 0{,}006$

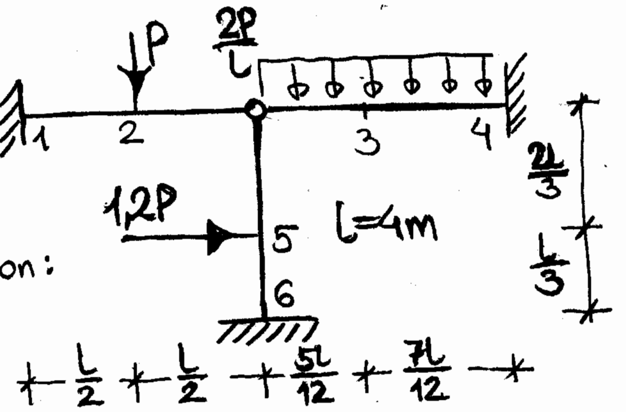
b. $p_{f1} = p_{f2} = p_{f3} = 0{,}2$ $\quad p_f = (0{,}2)^3 = 0{,}008$

also $R > \max(R_1, R_2, \dots, R_n) = \max R_i$

a. $R_1 = 0{,}7$; $R_2 = 0{,}8$; $R_3 = 0{,}9$ $\quad R = 1 - 0{,}1 \cdot 0{,}2 \cdot 0{,}3 = 0{,}994$

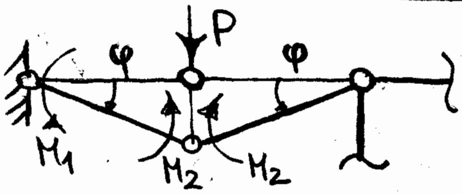
b. $R_1 = R_2 = R_3 = 0{,}8$ $\quad R = 1 - (0{,}2)^3 = 0{,}992$

* Assess the reliability and the failure probability of a framed system, failure modes due to bending.
Limit moments of sections $1 \div 6$ – identical distribution:
$\overline{M}_{lim} = 81{,}0$ kNm, $\sigma_{M_{lim}} = 9{,}26$ kNm.
Random load: $\overline{P} = 80$ kN; $\sigma_P = 8$ kN



Fundamental reliability problem: $G = R - P$ (R – resistance, P – load effect)
the Cornell's reliability index $\beta = \frac{\overline{G}}{\sigma_G} = \frac{\overline{R} - \overline{P}}{\sqrt{\sigma_R^2 + \sigma_P^2}}$; $P_f = F(-\beta)$, $R = 1 - P_f = F(\beta)$

Failure mode (I) virtual work theorem for a rigid-plastic system:



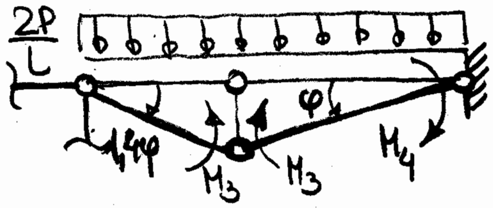
$L_{ext} = P\varphi\frac{L}{2}$, $L_{int} = M_1\varphi + 2M_2\varphi$, $L_{ext} = L_{int}$

$P_I = \frac{2M_1}{L} + \frac{4M_2}{L} = \frac{M_1}{2} + M_2$ [kN] $\quad \sigma_{P_I}^2 = \left(\frac{1}{2}\sigma_{M_{lim}}\right)^2 + \sigma_{M_{lim}}^2 = 9{,}26^2\sqrt{\left(\frac{1}{2}\right)^2 + 1^2}$

parameters of P_I: $\overline{P}_I = \frac{3}{2}\overline{M}_{lim} = 121{,}5$ kN; $\sigma_{P_I} = 9{,}26\sqrt{\left(\frac{1}{2}\right)^2 + 1^2} = 10{,}35$ kN

reliability index $\beta_1 = \frac{\overline{P}_I - \overline{P}}{\sqrt{\sigma_{P_I}^2 + \sigma_P^2}} = \frac{121{,}5 - 80}{\sqrt{10{,}35^2 + 8^2}} = 3{,}17$, $R_1 = F(\beta_1) = 0{,}999238$

Failure mode (II) $L_{ext} = L_{int}$



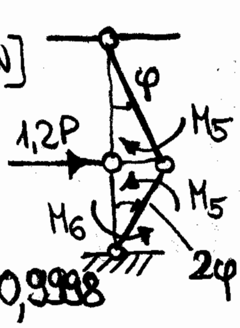
$L_{ext} = \frac{2P}{L} \cdot L \cdot \frac{1}{2} \cdot \frac{7}{12}\varphi L = \frac{7}{12}P\varphi L$; $L_{int} = 2{,}4M_3\varphi + M_4\varphi$

$P_{II} = \frac{36}{35}M_3 + \frac{3}{7}M_4$ [kN]

parameters of P_{II}: $\overline{P}_{II} = \left(\frac{36}{35} + \frac{3}{7}\right)\overline{M}_{lim} = 118{,}03$ kN; $\sigma_{P_{II}} = 9{,}26\sqrt{\left(\frac{36}{35}\right)^2 + \left(\frac{3}{7}\right)^2} = 10{,}32$ kN

reliability index $\beta_2 = \frac{\overline{P}_{II} - \overline{P}}{\sqrt{\sigma_{P_{II}}^2 + \sigma_P^2}} = \frac{118{,}03 - 80}{\sqrt{10{,}32^2 + 8^2}} = 2{,}91$; $R_2 = F(\beta_2) = 0{,}99819$

Failure mode (III)



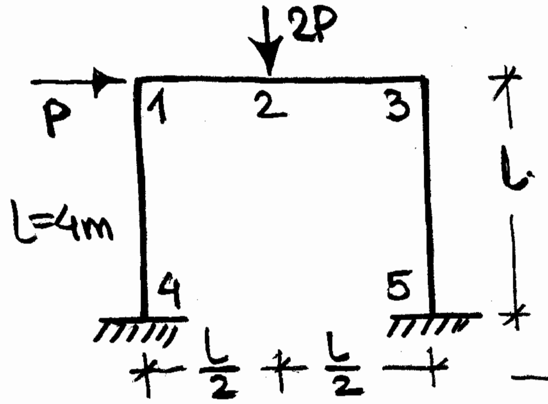
$$\left.\begin{aligned} L_{ext} &= 1{,}2P \cdot \tfrac{2}{3}\varphi L = 0{,}8\,P\varphi L \\ L_{int} &= 3M_5\varphi + 2M_6\varphi \end{aligned}\right\} P_{III} = \frac{15}{16}M_5 + \frac{5}{8}M_6 \text{ [kN]}$$

$\overline{P}_{III} = \left(\frac{15}{16} + \frac{5}{8}\right)\overline{M}_{lim} = 126{,}56$ kN; $\sigma_{P_{III}} = 9{,}26\sqrt{\left(\frac{15}{16}\right)^2 + \left(\frac{5}{8}\right)^2} = 10{,}43$ kN

reliability index $\beta_3 = \frac{\overline{P}_{III} - \overline{P}}{\sqrt{\sigma_{P_{III}}^2 + \sigma_P^2}} = \frac{126{,}56 - 80}{\sqrt{10{,}43^2 + 8^2}} = 3{,}54$; $R_3 = F(\beta_3) = 0{,}9998$

This is a series system, component reliabilities of the failure modes → R_1, R_2, R_3
there are no common critical cross-sections $1 \div 6$ in the three failure modes.
System reliability $R = \prod_{i=1}^{3} R_i = 0{,}9972 < \min R_i$
the corresponding reliability index $\beta = 2{,}77 < \min \beta_i$

19 ENG

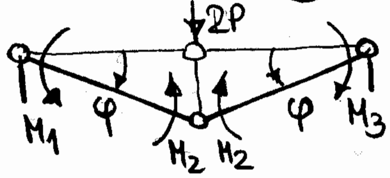


Determine the reliability and the probability of failure of a system design load-carrying capacity (limit load) corresponding to level $t=3$.

Random load $P \to N(40, 4)$ [kN]

Random limit bending moment $M_{lim} \to N(81; 9{,}26)$ [kNm]

Failure mode (I)

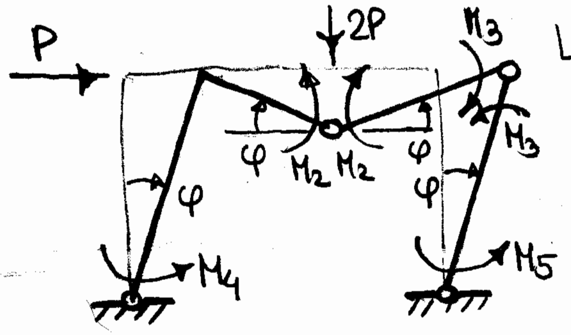


$$\left.\begin{aligned} L_{ext} &= 2P\cdot\varphi\cdot\tfrac{L}{2} = P\cdot\varphi\cdot L \\ L_{int} &= M_1\varphi + 2M_2\varphi + M_3\varphi \end{aligned}\right\} P_I = \frac{M_1}{4} + \frac{M_2}{2} + \frac{M_3}{4} \text{ [kN]}$$

$$\overline{P}_I = \left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)\overline{M}_{lim} = 81 \text{ kN} \; ; \; \sigma_{P_I} = 9{,}26\sqrt{2\left(\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)^2} = 5{,}67 \text{ kN}$$

$$\beta_1 = \frac{\overline{P}_I - \overline{P}}{\sqrt{\sigma_{P_I}^2+\sigma_P^2}} = \frac{81-40}{\sqrt{5{,}67^2+4^2}} = 5{,}90$$

Failure mode (II)



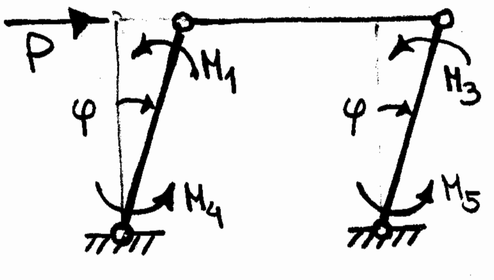
$$\left.\begin{aligned} L_{ext} &= 2P\varphi\tfrac{L}{2} + P\varphi L = 2P\varphi L \\ L_{int} &= 2M_2\varphi + 2M_3\varphi + M_4\varphi + M_5\varphi \end{aligned}\right\} P_{II} = \frac{M_2}{4} + \frac{M_3}{4} + \frac{M_4}{8} + \frac{M_5}{8} \text{ [kN]}$$

$$\overline{P}_{II} = \left(2\cdot\frac{1}{4}+2\cdot\frac{1}{8}\right)\overline{M}_{lim} = 60{,}75 \text{ kN}$$

$$\sigma_{P_{II}} = 9{,}26\sqrt{2\cdot\left(\frac{1}{4}\right)^2+2\cdot\left(\frac{1}{8}\right)^2} = 3{,}66 \text{ kN}$$

$$\beta_2 = \frac{\overline{P}_{II} - \overline{P}}{\sqrt{\sigma_{P_{II}}^2+\sigma_P^2}} = \frac{60{,}75-40}{\sqrt{3{,}66^2+4^2}} = 3{,}83$$

Failure mode (III)



$$\left.\begin{aligned} L_{ext} &= P\varphi L \\ L_{int} &= M_1\varphi + M_3\varphi + M_4\varphi + M_5\varphi \end{aligned}\right\} P_{III} = \frac{M_1}{4} + \frac{M_3}{4} + \frac{M_4}{4} + \frac{M_5}{4} \text{ [kN]}$$

$$\overline{P}_{III} = 4\cdot\frac{1}{4}\overline{M}_{lim} = 81{,}0 \text{ kN} \; ; \; \sigma_{P_{III}} = 9{,}26\sqrt{4\cdot\left(\frac{1}{4}\right)^2} = 4{,}63 \text{ kN}$$

$$\beta_3 = \frac{\overline{P}_{III} - \overline{P}}{\sqrt{\sigma_{P_{III}}^2+\sigma_P^2}} = \frac{81{,}0-40}{\sqrt{4{,}63^2+4^2}} = 6{,}70$$

There are common critical cross-sections in the failure modes above, The approximate solution – looking for the minimum reliability index (and the minimum mean value) of the load – for the problem considered failure mode (II) $\longrightarrow$

$$\overline{P}_{limit} = \overline{P}_{II} = 60{,}75 \text{ kN}$$

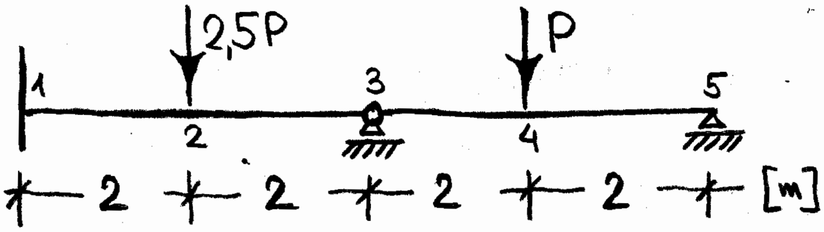
$$\sigma_{P_{limit}} = \sigma_{P_{II}} = 3{,}66 \text{ kN}$$

Reliability index of a system $\beta = 3{,}83 \to$ reliability $R = F(\beta) = 0{,}99993$

Design limit load of the system corresponding to $t=3$

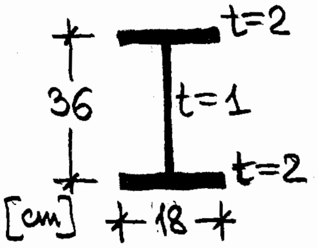
$$P_0 = \overline{P}_{limit} - t\cdot\sigma_{P_{limit}} = 60{,}75 - 3\cdot 3{,}66 = 49{,}77 \text{ kN}$$

D17

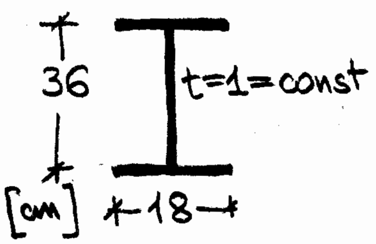


Determine the reliability and the reliability index of the beam

1-3



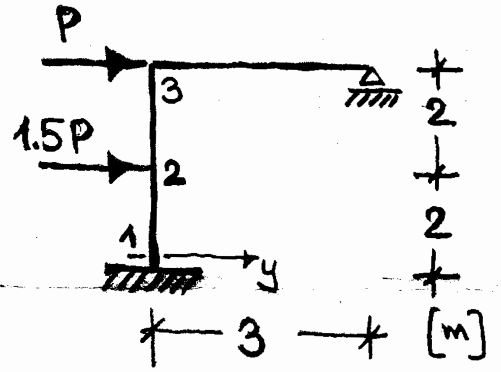
3-5

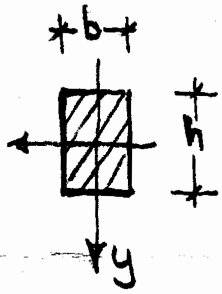


Yield stress of steel $f_y \to N(200{,}6)$ [MPa]

Random load $P \to N(180{,}5)$ [MPa]

D18





Determine the reliability of the system under deterministic load $P = 15$ kN.

Dimensions of the cross-section

$b \to N(0{,}1\,;\ 0{,}01)$ [m] ; $h = 0{,}18$ m

Yield stress $f_y = 80$ MPa.

Determine the parameters of the random limit load of the structure,

determine the design limit load → 1% fractile.