Introduction to Laboratory 7

Laboratory 7 concerns the implementation of the **natural cubic spline interpolation method**. Similarly to Laboratory 5, we use the values of f at *n* points x_j (separated by a step size *h*), and we want to approximate $f(x)$ by a polynomial $p(x)$ (see Figure 1). In the **cubic spline interpolation method**, we use a different cubic function in each interval $x_j \leq x \leq x_{j+1}$:

$$p(x) = a_j(x - x_j)^3 + b_j(x - x_j)^2 + c_j(x - x_j) + d_j$$

where the coefficients a_j, b_j, c_j and d_j define the cubic function in the interval $[x_j, x_{j+1}]$.

Then, by imposing that $p(x)$ and $p'(x)$ are continuous functions, we can express the coefficients a_j, b_j, c_j and d_j in terms of the coefficients $p_j \equiv p(x_j) = f(x_j)$ and $p_j'' \equiv p''(x_j)$, and show that (see main lecture):

$$p(x) = \frac{p_{j+1}'' - p_j''}{6h}(x - x_j)^3 + \frac{p_j''}{2}(x - x_j)^2 + \left[\frac{p_{j+1} - p_j}{h} - \frac{1}{6}hp_{j+1}'' - \frac{1}{3}hp_j''\right](x - x_j) + p_j$$

where the p_j are easily calculated from $p_j = f(x_j)$ and the p_j'' can be obtained by solving a tridiagonal system of equations (see Laboratory 7). In addition, in the **natural cubic spline interpolation method** the coefficients p_1'' and p_n'' are assumed to be zero.

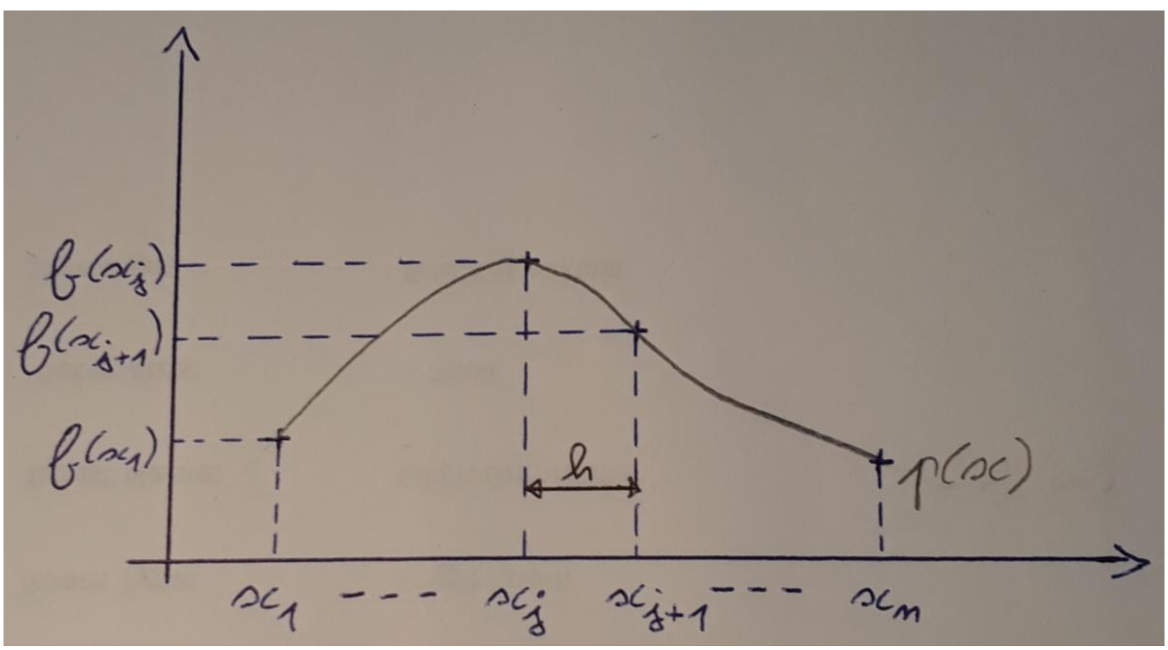

Figure 1: Example illustrating the interpolation of a function $f(x)$ using *n* points x_j. The interpolating polynomial is $p(x)$.

In Laboratory 7 you have to calculate the **natural cubic spline** polynomial $p(x)$ in the interval $[-5,5]$ for the function $f(x) = 1/(1 + x^2)$ (using 11 points). The coefficients p_j'' can be calculated using the **Gaussian elimination method** (see Laboratory 6).