Introduction to Laboratory 9

Laboratory 9 concerns the implementation of the **Trapezoid, Simpson, Boole** and **Euler-Maclaurin methods** to calculate the integral of a function $f(x)$ from x_0 to x_N, i.e. $\int_{x_0}^{x_N} f(x)dx$. The integration interval $[x_0, x_N]$ is divided into N segments of size h (see Figure 1).

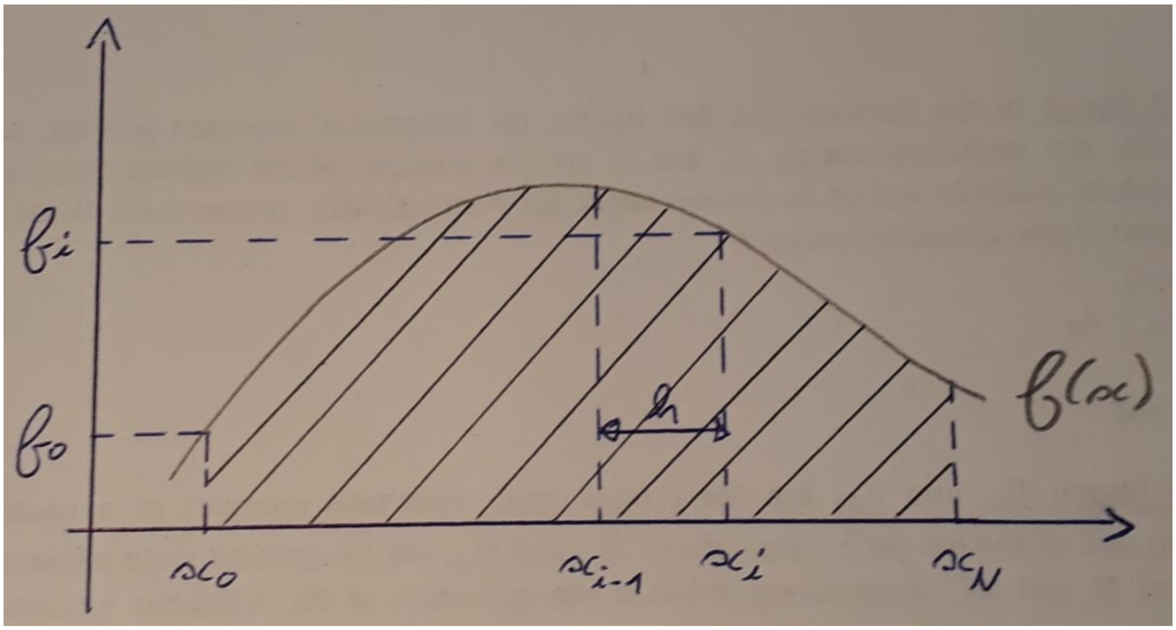

Figure 1: The integral $\int_{x_0}^{x_N} f(x)dx$ is equal to the surface under the curve of $f(x)$ (black hatches).

In the **Trapezoid method** the function $f(x)$ in the interval $[x_{i-1}, x_i]$ is approximated by the straight line (linear approximation interpolating two points) connecting the points (x_{i-1}, f_{i-1}) and (x_i, f_i) (see Figure 2). The surface under the line is then given by $\frac{h}{2}(f_{i-1} + f_i)$. By summing the surfaces of the N segments, one obtains the **Trapezoid method**:

$$\int_{x_0}^{x_N} f(x)dx = \sum_{i=1}^{N} \frac{h}{2}(f_{i-1} + f_i) = h\left(\frac{f_0}{2} + f_1 + f_2 + \cdots + f_{N-1} + \frac{f_N}{2}\right)$$

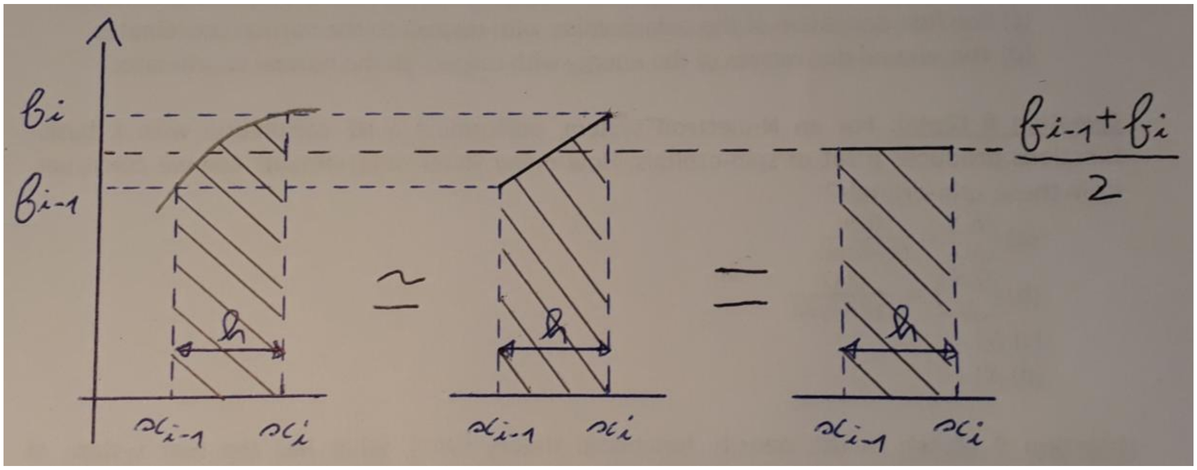

Figure 2: Illustration of the **Trapezoid method**.

Higher-order methods can be obtained by interpolating additional points. For example, the **Simpson method** uses a quadratic polynomial to interpolate three points, and the **Boole method** uses a quartic polynomial to interpolate five points (see Laboratory 9). Moreover, a correction to the **Trapezoid method** can be calculated from the first and third derivatives of $f(x)$ at the end points x_0 and x_N (i.e. **Euler-Maclaurin method**).

In Laboratory 9 you have to calculate the integral $\int_0^{\pi} \sin x \, dx$ using the **Trapezoid**, **Simpson**, **Boole** and **Euler-Maclaurin methods** for different number of segments N.