5.3 Material Models

The response of real materials to various loading conditions was discussed in the previous section. Now comes the task of creating mathematical models which can predict this response. To this end, it is helpful to categorise the material responses into ideal models. There are four broad **material models** which are used for this purpose: (1) the **elastic model**, (2) the **viscoelastic model**, (3) the **plastic model**, and (4) the **viscoplastic model**. These models will be discussed briefly in what follows, and in more depth throughout the rest of this book.

5.3.1 The Elastic Model

An ideal elastic material has the following characteristics:

(i) the unloading stress-strain path is the same as the loading path
(ii) there is no dependence on the rate of loading or straining
(iii) it does not undergo permanent deformation; it returns to its precise original shape when the loads are removed

Typical stress-strain curves for an ideal elastic model subjected to a tension (or (compression) test are shown in Fig. 5.3.1. The response of a **linear elastic material**, where the stress is *proportional* to the strain, is shown in Fig. 5.3.1a and that for a **non-linear elastic material** is shown in Fig. 5.3.1b.

From the discussion in the previous section, the linear elastic model will well represent the engineering materials up to their elastic limit (see, for example, Figs. 5.2.2-4). It will also represent the complete stress-strain response up to the point of fracture of many very brittle materials. The model can also be used to represent the response of almost any material, provided the stresses are sufficiently small.

The non-linear elastic model is useful for predicting the response of soft materials like rubber and biological soft tissue (see, for example Fig. 5.2.9).

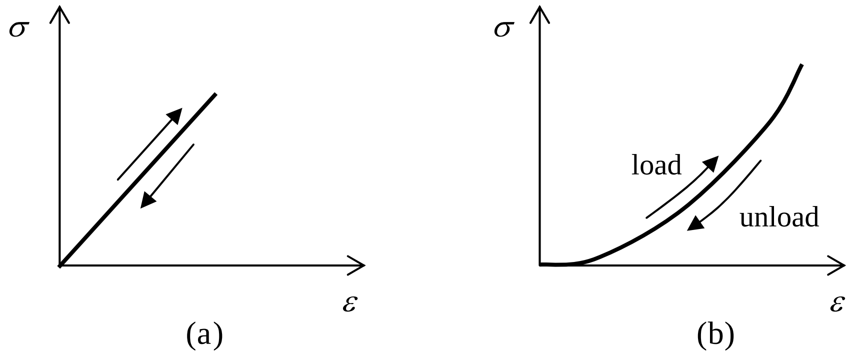


Figure 5.3.1: The Elastic Model; (a) linear elastic, (b) non-linear elastic

It goes without saying that there is no such thing as a purely elastic material. All materials will undergo at least some permanent deformations, even at low loads; no material's response will be exactly the same when stretched at different speeds, and so on.

However, if these occurrences and differences are small enough to be neglected, the ideal elastic model will be useful.

Note also that a prediction of a material's response may be made with accuracy using the elastic model in some circumstances, but not in others. An example would be metal; the elastic model might be able to predict the response right up to high stress levels when the metal is cold, but not so well when the temperature is high, when inelastic effects may not be so easily disregarded (see below).

5.3.2 Viscoelasticity

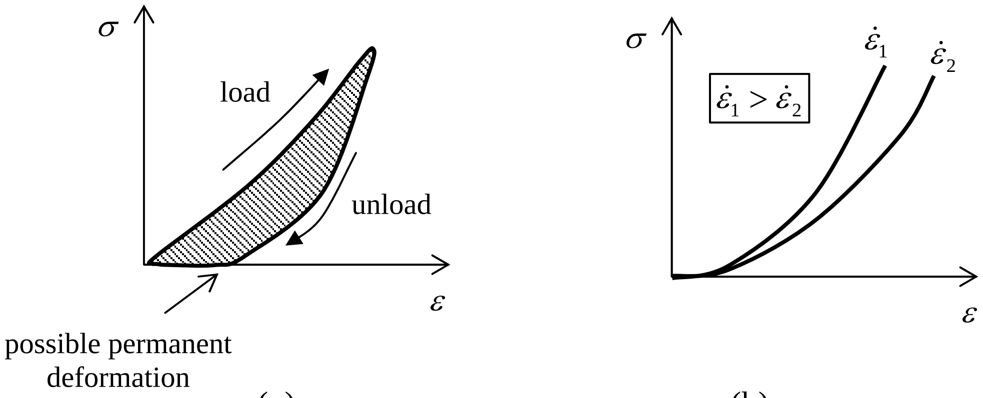
When solid materials have some "fluid-like" characteristics, they are said to be viscoelastic. A fluid is something which flows easily when subjected to loading – it cannot keep to any particular shape. If a fluid is one (the "viscous") extreme and the elastic solid is at the other extreme, then the viscoelastic material is somewhere in between.

The typical response of a viscoelastic material is sketched in Fig. 5.3.2. The following will be noted:

(i) the loading and unloading curves do not coincide, Fig. 5.3.2a, but form a **hysteresis loop**

(ii) there is a dependence on the rate of straining $d\varepsilon / dt$, Fig. 5.3.2b; the faster the stretching, the larger the stress required

(iii) there may or may not be some permanent deformation upon complete unloading, Fig. 5.3.2a



(a) (b)

Figure 5.3.2: Response of a Viscoelastic material in the Tension test; (a) loading and unloading with possible permanent deformation (non-zero strain at zero stress), (b) different rates of stretching

The effect of *rate* of stretching shows that the viscoelastic material *depends on time*. This contrasts with the elastic material; it makes no difference whether an elastic material is loaded to some given stress level for one second or one day, or quickly or slowly, the resulting strain will be the same. This rate effect can be seen when you push your hand through water – it is easier to do so when you push slowly than when you push fast.

Depending on how “fluid-like” or “solid-like” a material is, it can be considered to be a **viscoelastic fluid**, for example blood or toothpaste, or a **viscoelastic solid**, for example Silly Putty™ or foam. That said, the model for both and the theory behind each will be similar.

Viscoelastic materials will be discussed in detail in Chapter 10.

5.3.3 Plasticity

Plasticity has the following characteristics:

(i) The loading is elastic up to some threshold limit, beyond which permanent deformations occur

(ii) The permanent deformation, i.e. the **plasticity**, is time independent

This plasticity can be seen in Figs. 5.2.2-4. The threshold limit – the elastic limit – can be quite high but it can also be extremely small, so small that significant permanent deformations occur at almost any level of loading. The plasticity model is particularly useful in describing the permanent deformations which occur in metals, soils and other engineering materials. It will be discussed in further detail in Chapter 11.

5.3.4 Viscoplasticity

Finally, the viscoplastic model is a combination of the viscoelastic and plastic models. In this model, the plasticity is rate-dependent. One of the main applications of the model is in the study of metals at high temperatures, but it is used also in the modeling of a huge range of materials and other applications, for example asphalt, concrete, clay, paper pulp, biological cells growth, etc. This model will be discussed in Chapter 12.