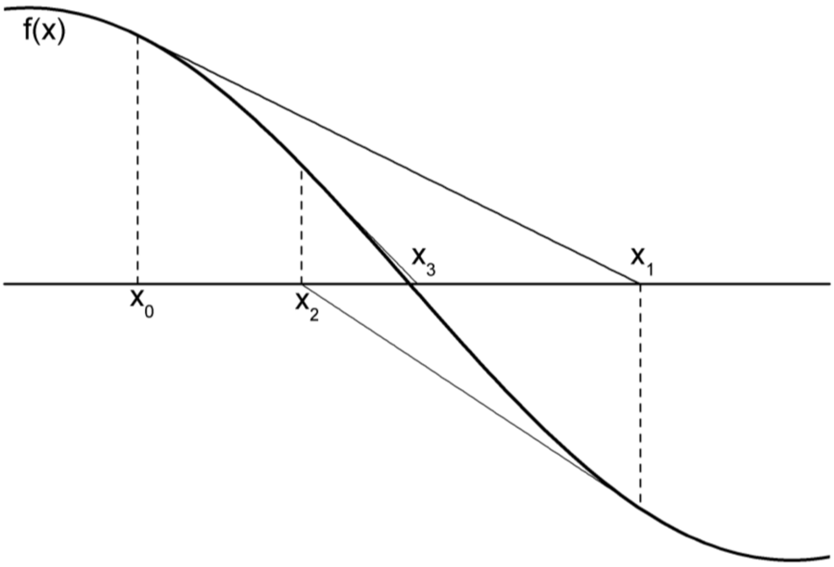
Newton-Raphson method:

The goal of the exercise is to apply the Newton-Raphson method to find the root of a function $f(x)$, that is to say find the x_{Root} value for which $f(x_{Root}) = 0$.

It is assumed that we have an initial value (x_0) close enough to x_{Root}. Using the values of the function $f(x_0)$ and of its derivative $f'(x_0)$, one can calculate the tangent line crossing zero at the point x_1. Then, the value (x_1) provides a new approximation to the root, which can be employed to calculate a new tangent line crossing zero at the point x_2. Thus, the process can be iterated according to the formula:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$



The algorithm is as follows:

1. Specify the values for x_0 and $Tolerance$.
2. Calculate $f(x_i)$ and $f'(x_i)$.
3. Calculate the new approximation to the root (x_{i+1}).
4. Iterate the steps 2 and 3 until the $Error$ is below a given $Tolerance$:

$$Error \equiv \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \leq Tolerance$$

5. Print the root value (x_{Root}).

Exercise:

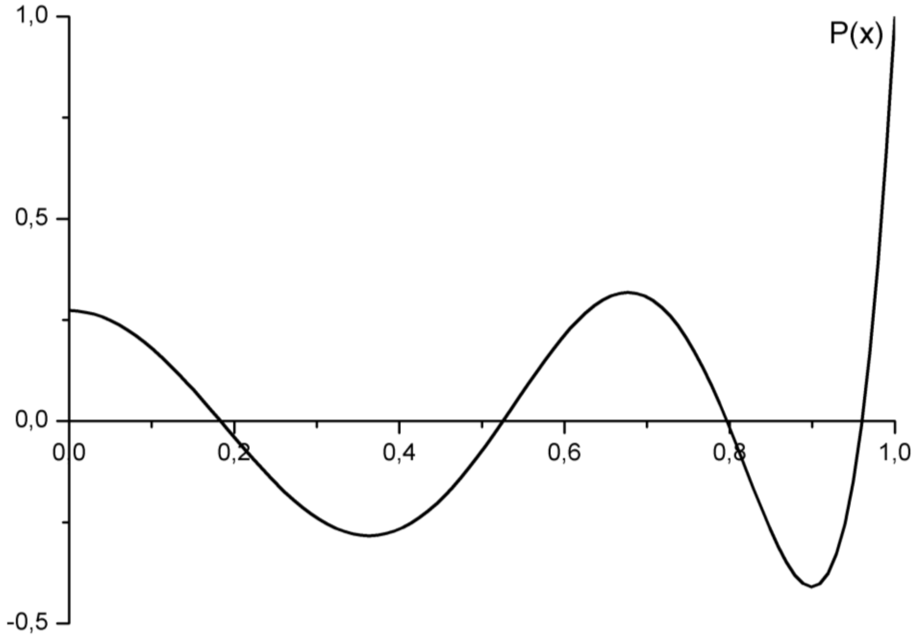
1) Find the root of the function $f(x) = \cos x - x$ starting with the initial value $x_0 = 0$.

- Derive the analytical expression for $f'(x)$.
- Calculate the root (x_{Root}) with 8 digits of accuracy and compare the number of necessary iterations with the bisection method.

2) Find the 4 roots of the polynomial $P(x)$ in the interval [0,1].

$$P(x) = \frac{6435x^8 - 12012x^6 + 6930x^4 - 1260x^2 + 35}{128}$$

- Indicate the values of the 4 roots on the graph:



- What happens if one starts with the initial value $x_0 = 0.4$?

3) Use the Newton-Raphson method to calculate the value of x such that: $x = 13^{2/3}$