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Hybrid method:

The goal of the exercise is to combine the bisection and Newton-Raphson methods to find the root of a function $f(x)$, that is to say find the x_{Root} value for which $f(x_{Root}) = 0$.
It is assumed that x_{Root} is in the interval $[x_L, x_R]$ (for the bisection method) and the initial value for the Newton-Raphson method is taken as $x_0 = x_L$.
Then, the program should decide if it should do a bisection step or a Newton-Raphson step. For that, it must check if a Newton-Raphson step stays in the interval $[x_L, x_R]$, i.e. the inequality,

$$x_L \le x_0 - \frac{f(x_0)}{f'(x_0)} \le x_R$$

must be verified and can be written as

$$\begin{aligned} &\text{If } f'(x_0) > 0: \quad (x_0 - x_L) f'(x_0) - f(x_0) \ge 0 \ge (x_0 - x_R) f'(x_0) - f(x_0) \\ &\text{If } f'(x_0) < 0: \quad (x_0 - x_L) f'(x_0) - f(x_0) \le 0 \le (x_0 - x_R) f'(x_0) - f(x_0) \end{aligned} \tag{1}$$

Thus, if the inequality (1) is verified the program should do a Newton-Raphson step, otherwise it should do a bisection step.
Additionally, the interval $[x_L, x_R]$ should be adjusted after each bisection and Newton-Raphson steps.
The process should be iterated until the *Error* is below a given *Tolerance*:

$$Error \equiv \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \le Tolerance$$

Exercise:

1) Test the program on the polynomial $P(x)$ with $x_L = x_0 = 0.4$ and $x_R = 0.7$

$$P(x) = \frac{6435x^8 - 12012x^6 + 6930x^4 - 1260x^2 + 35}{128}$$

Give the number of bisection and Newton-Raphson steps.

2) Find the root of the function $f(x) = x^2 - 2x - 2$ in the interval $[0,3]$.