Lagrange and Hermite interpolations:

The goal of the exercise is to implement the *Lagrange* and *Hermite* interpolation methods to approximate a function $f(x)$ by a polynomial $p(x)$.

- In the *Lagrange* interpolation method it is assumed that we know the function $f(x)$ at n points $[x_j, f(x_j)]$ with $j=1,\ldots,n$. The Lagrange interpolating polynomial is given by:

$$p(x)=\sum_{j=1}^{n} l_{j,n}(x)f(x_j)$$

with

$$l_{j,n}(x)=\frac{(x-x_1)(x-x_2)\ldots(x-x_{j-1})(x-x_{j+1})\ldots(x-x_n)}{(x_j-x_1)(x_j-x_2)\ldots(x_j-x_{j-1})(x_j-x_{j+1})\ldots(x_j-x_n)}=\prod_{i\neq j}^{n}\frac{x-x_i}{x_j-x_i}$$

- In the *Hermite* interpolation method it is assumed that we know the function $f(x)$ and its derivatives $f'(x)$ at n points $[x_j, f(x_j), f'(x_j)]$ with $j=1,\ldots,n$. The Hermite interpolating polynomial is given by:

$$p(x)=\sum_{j=1}^{n} h_{j,n}(x)f(x_j)+\sum_{j=1}^{n}\bar{h}_{j,n}(x)f'(x_j)$$

with

$$h_{j,n}(x)=\left[1-2(x-x_j)l'_{j,n}(x_j)\right]l_{j,n}^2(x)$$
$$\bar{h}_{j,n}(x)=(x-x_j)l_{j,n}^2(x)$$

and

$$l'_{j,n}(x_j)=\frac{1}{x_j-x_1}+\frac{1}{x_j-x_2}+\ldots+\frac{1}{x_j-x_{j-1}}+\frac{1}{x_j-x_{j+1}}+\ldots+\frac{1}{x_j-x_n}=\sum_{i\neq j}^{n}\frac{1}{x_j-x_i}$$

Exercise:

- Interpolate the function $f(x)=e^x$ with the points x_j = -1.0 ; 0.5 ; 1.5 ; 2.0
- Interpolate the function $f(x)=\dfrac{1}{1+x^2}$ with the points x_j = -5.0 ; -4.0 ; -3.0 ;…; 4.0 ; 5.0
- For both functions calculate the *Lagrange* and *Hermite* polynomial $p(x)$ in the interval $-5\leq x\leq 5$ with a step of 0.1 and plot the results.