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Numerical Differentiation:

The goal of the exercise is to calculate the first and second derivatives of a function $f(x)$.
The *Forward Difference Approximations* are given by

$$f'(x) = \frac{1}{h}\left[f(x+h) - f(x)\right] + E(h) \qquad f''(x) = \frac{1}{h^2}\left[f(x) - 2f(x+h) + f(x+2h)\right] + E(h)$$

in which $E(h)$ is the error.

The *Central Difference Approximations* are given by

$$f'(x) = \frac{1}{2h}\left[f(x+h) - f(x-h)\right] + E(h) \qquad f''(x) = \frac{1}{h^2}\left[f(x+h) - 2f(x) + f(x-h)\right] + E(h)$$

Exercise:

- Calculate $f'(2)$ and $f''(2)$ for the function $f(x) = xe^x$ with *h*=0.05 ; 0.10 ;…; 0.45 ; 0.50 using the *Forward* and *Central Difference Approximations*. Compare to the exact results.

- For small values of *h* the error is given by $|E(h)| \approx h^n$, in which *n* is the order of the leading term in the error. This equation can be written as $\ln|E(h)| \approx n \ln h$, which takes the form of a linear function ($Y = AX$) with $Y \equiv \ln|E(h)|$, $X \equiv \ln h$ and *A*=*n*.

Calculate $|E(h)| \equiv |Numerical - Exact|$ for $f'(2)$ with *h*=0.05 ; 0.10 ;…; 0.45 ; 0.50 using the *Forward* and *Central Difference Approximations*.

Plot $\ln|E(h)|$ with respect to $\ln h$ on a graph and deduce the order of the error (*n*), which is given by the slope (*A*) of the line. The slope of the line can be determined with the *Least Squares* method and is given by

$$A = \frac{N\sum_{i=1}^{N} X_i Y_i - \sum_{i=1}^{N}\sum_{j=1}^{N} X_i Y_j}{N\sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2}$$

in which *N* is the number of points.