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Numerical Integration:

The goal of the exercise is to calculate the integral of a function $f(x)$:

$$I \equiv \int_{x_0}^{x_N} f(x)dx$$

where *N* is the number of segments of width *h*.

The *Trapezoid rule* is given by ($f_i \equiv f(x_i)$)

$$\int_{x_0}^{x_N} f(x)dx = h\left(\frac{f_0}{2} + f_1 + f_2 + \ldots + f_{N-1} + \frac{f_N}{2}\right) \text{ with } N=1,2,3,4,\ldots$$

The *Simpson rule* is given by

$$\int_{x_0}^{x_N} f(x)dx = \frac{h}{3}\left(f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \ldots + 2f_{N-2} + 4f_{N-1} + f_N\right) \text{ with } N=2,4,6,8,\ldots$$

The *Boole rule* is given by

$$\int_{x_0}^{x_N} f(x)dx = \frac{2h}{45}\left(7f_0 + 32f_1 + 12f_2 + 32f_3 + 14f_4 + \ldots + 14f_{N-4} + 32f_{N-3} + 12f_{N-2} + 32f_{N-1} + 7f_N\right)$$

with N=4,8,12,16,…

The *Euler-Maclaurin rule* including corrections to the *Trapezoid rule* up to the fourth order is given by

$$\int_{x_0}^{x_N} f(x)dx = h\left(\frac{f_0}{2} + f_1 + f_2 + \ldots + f_{N-1} + \frac{f_N}{2}\right) + \frac{h^2}{12}(f_0^{(1)} - f_N^{(1)}) - \frac{h^4}{720}(f_0^{(3)} - f_N^{(3)})$$

with N=1,2,3,4,…

Exercise:

- Calculate $I = \int_0^{\pi} \sin x dx$ with the *Trapezoid*, *Simpson*, *Boole* and *Euler-Maclaurin rules* for N=4,8,16,…,1024. Compare to the exact result.