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Richardson/Romberg schemes to calculate Derivatives/Integrals:

The goal of the exercise is to calculate the first and second derivatives as well as the integral of a function $f(x)$.

$$f'(x) \quad ; \quad f''(x) \quad ; \quad I \equiv \int_{x_0}^{x_N} f(x)dx$$

The *Richardson Extrapolation Scheme* for first and second derivatives starts with the *Central Difference Approximation* given by

$$D_1(h) \equiv \frac{1}{2h}\left[f(x+h) - f(x-h)\right] \qquad \text{or} \qquad D_1(h) \equiv \frac{1}{h^2}\left[f(x+h) - 2f(x) + f(x-h)\right]$$

Then, the next approximations to the derivatives are obtained with the recursive relation:

$$D_{i+1}(h) = \frac{4^i D_i(h) - D_i(2h)}{4^i - 1}$$

The *Romberg Integration Scheme* starts with the *Trapezoid rule* given by

$$I_{m,0} \equiv h\left(\frac{f_0}{2} + f_1 + f_2 + \ldots + f_{N-1} + \frac{f_N}{2}\right) \text{ with } N = 2^m$$

Then, the next approximations to the integral are obtained with the recursive relation:

$$I_{m,k} = \frac{4^k I_{m,k-1} - I_{m-1,k-1}}{4^k - 1}$$

Exercise:

- Considering the function $f(x) = xe^x$, generate the *Richardson Tables* for $f'(2)$ and $f''(2)$ going from *h*=0.4 to *h*=0.05. Print the values with 12 digits and compare to the exact results.

h	D_1	D_2	D_3	D_4
0.4	$D_1(0.4)$			
0.2	$D_1(0.2)$	$D_2(0.2)$		
0.1	$D_1(0.1)$	$D_2(0.1)$	$D_3(0.1)$	
0.05	$D_1(0.05)$	$D_2(0.05)$	$D_3(0.05)$	$D_4(0.05)$

- Generate the *Romberg Table* for $I = \int_0^{\pi} \sin x dx$ going from *m*=2 to *m*=5. Print the values with 12 digits and compare to the exact result.

m	$I_{m,0}$	$I_{m,1}$	$I_{m,2}$	$I_{m,3}$
2	$I_{2,0}$			
3	$I_{3,0}$	$I_{3,1}$		
4	$I_{4,0}$	$I_{4,1}$	$I_{4,2}$	
5	$I_{5,0}$	$I_{5,1}$	$I_{5,2}$	$I_{5,3}$