Introduction to Laboratory 1

Laboratory 1 concerns the implementation of the **Bisection method**, which is an iterative method to find the root of a function (i.e. to find the x_0 value for which $f(x_0) = 0$). In the Bisection method we start with an initial interval $[x_L, x_R]$ comprising the root x_0 (see the Figure 1 for an example). Then, we calculate the middle value of the interval $x_M = (x_L + x_R)/2$ (see Figure 1). Next, the size of the interval $[x_L, x_R]$ is decreased by half. In the case of Figure 1, we see that x_0 is in the interval $[x_M, x_R]$. Therefore, in this case the new interval for the next iteration should be $[x_M, x_R]$ (i.e. we replace x_L by x_M). Starting with this new interval we iterate the process until convergence is obtained, i.e. until the interval or the relative error (see exercise 1) becomes smaller than a given tolerance.

To be able to decrease the interval size, we need to find at each iteration if x_0 is comprised in $[x_L, x_M]$ or in $[x_M, x_R]$. This can be easily found by checking if the function $f(x)$ has similar or different signs at the interval boundaries. The algorithm works as follows:

If $f(x_L) * f(x_M) > 0$ then the function $f(x)$ does not change sign in the interval $[x_L, x_M]$ (i.e. $f(x)$ is positive like on Figure 1 or it is negative). That means that x_0 is not in this interval but is in the other interval $[x_M, x_R]$. Else if $f(x_L) * f(x_M) < 0$ then it means that x_0 is in the interval $[x_L, x_M]$ (i.e. the function $f(x)$ crosses the x axis somewhere between x_L and x_M).

If x_0 is in the interval $[x_L, x_M]$ then this interval should be use as a initial interval for the next iteration, otherwise we should use $[x_M, x_R]$.

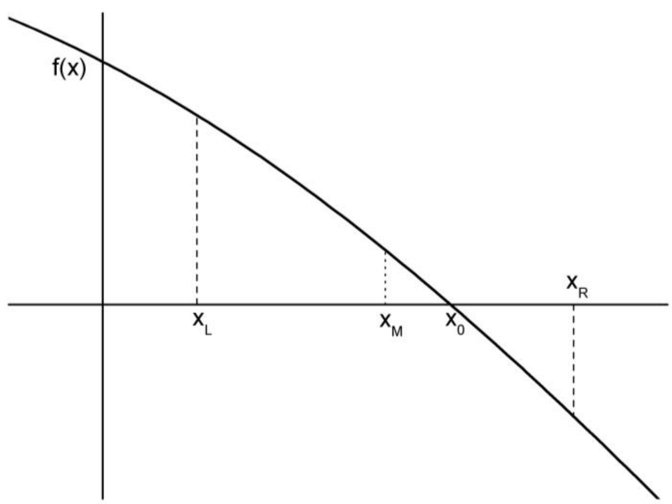

Figure 1: Example illustrating the first step of the Bisection method.

Solution:

1) For the function $f(x) = \cos x - x$, starting with $x_L = 0$, $x_R = 1$ and *Tolerance*=10^{-8}, the program does 28 iterations. The root x_0 is equal to 0.739 085 13.