Bernoulli equations

M.W.

Mathematics Teaching and Distance Learning Centre
Gdańsk University of Technology

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1. The Bernoulli equation

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Definition

The Bernoulli equation is an equation of the form

$$\frac{dy}{dx} + p(x)y = q(x)y^n \qquad (1)$$

where $p(x)$ and $q(x)$ are given functions of x continuous in the range in which it is required to integrate equation (1) $n \neq 0, 1$ (for $n = 0, 1$ the equation is linear).

By the substitution $z = \frac{1}{y^{n-1}}$ the Bernoulli equation can be reduced to a linear form and integrated as a linear equation.

Example

Solve the equation

$$y' - xy = -xy^3$$

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Solution

$$y^2(1 + Ce^{-x^2}) = 1$$

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$$xy' + y = y^2 \ln x$$

by the methods of variation of a constant.

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Solution

$$y = \frac{1}{1 + Cx + \ln x}$$

Substitution of variables may reduce some nonlinear equations of the first order to linear or Bernoulli equations.

Example

Solve the equation

$$y' + \sin y + x \cos y + x = 0$$

Hint: use identities $\sin 2y = 2 \sin y \cos y$, $\cos 2y = 2 \cos^2 y - 1$ and then substitution $\tan \frac{y}{2} = z$.

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Solution

$$\tan\frac{y}{2} = 1 - x + Ce^{-x}$$

Exercises

1. $y' + 2xy = 2xy^2$,
2. $3xy^2y' - 2y^3 = x^3$,
3. $(x^3 + e^y)y' = 3x^2$,
4. $2y' \ln x + \frac{y}{x} = y^{-1} \cos x$.

Exercises

By substituting the variable reduce the following nonlinear equation to linear or Bernoulli equations and solve them

1. $y' - \tan y = e^x \frac{1}{\cos y}$,
2. $y' = y(e^x + \ln y)$,
3. $y' \cos y + \sin y = x + 1$,
4. $yy' + x \sin 2y = 2xe^{-x^2} \cos^2 y$.