POCHODNE

Na podstawie definicji, znaleźć pochodną funkcji w zadanym punkcie:

1. $y = x^2 + 1$ w pkt. x_0
2. $y = \sqrt{x}$ w pkt. $x_0 > 0$
3. $y = \sin 2x$ w pkt. x_0
4. $y = \ln x$ w pkt. $x_0 > 0$

Obliczyć:

5. $y'(1)$ dla $y = \frac{1}{3}x^3 + 2x^2 + 1$
6. $y'(-2)$ dla $y = \frac{1}{x} + x$
7. $y'(-2)$ dla $y = 4\text{arctg}x$
8. $y'(\frac{1}{4})$ dla $y = \arcsin\sqrt{x}$

Obliczyć y' niżej podanych funkcji:

9. $y = 3x^5 - \frac{1}{2}x^2 + x + \frac{1}{x}$
10. $y = 10 + 2\sqrt[3]{x} + \frac{1}{\sqrt{x}}$
11. $y = 2\text{arctg}x + \pi$
12. $y = \frac{2x}{x-1}$
13. $y = x^4(x^2 + 1)$
14. $y = (ax + b)^6$
15. $y = (x^3 + 2x + 1)^4$
16. $y = \frac{1}{(2x+1)^3}$
17. $y = \sqrt{x^2 + 4}$
18. $y = \sqrt[4]{3x + 1}$
19. $y = (3x + 2)\sqrt{1 - x}$
20. $y = x^2 - \ln(2 - x^2)$
21. $y = e^{3x} + 5e^{-x}$
22. $y = x\ln x - x$
23. $y = (x^2 + 1)e^{-2x}$
24. $y = \ln(x + \sqrt{1 + x^2})$
25. $y = x\ln^2 x - 2x\ln x + 2x$
26. $y = \text{arctg}x - \text{arcctg}\frac{1}{x}$
27. $y = x\text{arctg}x - \frac{1}{2}\ln(1 + x^2)$
28. $y = \frac{1}{2}x\sqrt{1 - x^2} + \frac{1}{2}\arcsin x$
29. $y = \text{arctg}\frac{1+x}{1-x}$
30. $y = \cos(4x) - 2\sin\frac{x}{2} + 3\text{tg}x$
31. $y = x^3\sin x$
32. $y = \frac{\cos x}{1 - \sin x}$
33. $y = \cos^2 x$
34. $y = \cos(2x) + 2\sin^2 x$
35. $y = 3\sin^2 x - \sin^3 x$
36. $y = \frac{x}{\cos^2 x} - \text{tg}x$
37. $y = \sin^2 x \sin x^2$
38. $y = x\arcsin x$
39. $y = (\arcsin x)^2$
40. $y = x\sin x\,\text{arctg}x$
41. $y = \frac{x}{1 + x^2} - \text{arctg}x$
42. $y = (\arccos x + \arcsin x)^{100}$
43. $y = \ln^3 x$
44. $y = x\log x$
45. $y = x\sin x\ln x$
46. $y = x^{100}\ln x$
47. $y = \ln\left(\text{arctg}\frac{1}{1+x}\right)$
48. $y = \frac{1 - \ln x}{1 + \ln x}$
49. $y = x10^x$
50. $y = e^x\cos x$
51. $y = xe^x(\cos x + \sin x)$
52. $y = \sin 2^x$
53. $y = \arcsin(\sin x)$
54. $y = \sqrt[3]{x^4\sqrt{x}}$
55. $y = \sin(e^{x^2+3x-2})$
56. $y = \sin^2(\cos 3x)$
57. $y = \left(\frac{x+1}{x}\right)^x$
58. $y = x^{x^2}$
59. $y = x^{\sin x}$
60. $y = (\ln x)^x$
61. $y = (\text{arctg}x)^x$
62. $y = x^{x^x}$

Odpowiedzi

1. $2x_0$
2. $\frac{1}{2\sqrt{x_0}}$
3. $2\cos 2x_0$
4. $\frac{1}{x_0}$
5. $y'(1) = 5$
6. $y'(-2) = \frac{3}{4}$
7. $y'(-2) = \frac{4}{5}$
8. $y'(\frac{1}{4}) = \frac{2}{\sqrt{3}}$
9. $y' = 15x^4 - x + 1 - \frac{1}{x^2}$
10. $y' = \frac{2}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-\frac{3}{2}}$
11. $y' = \frac{2}{1+x^2}$
12. $y' = -\frac{2}{(x-1)^2}$
13. $y' = 6x^5 + 4x^3$
14. $y' = 6a(ax+b)^5$
15. $y' = 4(x^3 + 2x + 1)^3(3x^2 + 2)$
16. $y' = -\frac{6}{(2x+1)^4}$
17. $y' = \frac{x}{\sqrt{x^2+4}}$
18. $y' = \frac{3}{4\sqrt[4]{(3x+1)^3}}$
19. $y' = \frac{-9x+4}{2\sqrt{1-x}}$
20. $y' = \frac{2x(x^2-3)}{x^2-2}$
21. $y' = 3e^{3x} - 5e^{-x}$
22. $y' = \ln x$
23. $y' = 2(-x^2 + x - 1)e^{-2x}$
24. $y' = \frac{1}{\sqrt{x^2+1}}$
25. $y' = \ln^2 x$
26. $y' = 0$
27. $y' = \operatorname{arctg} x$
28. $y' = \sqrt{1 - x^2}$
29. $y' = \frac{1}{1+x^2}$
30. $y' = -4\sin(4x) - \cos\frac{x}{2} + 3\frac{1}{\cos^2 x}$
31. $y' = 3x^2 \sin x + x^3 \cos x$
32. $y' = \frac{1}{1-\sin x}$
33. $y' = -\sin(2x)$
34. $y' = 0$
35. $y' = \frac{3}{2}\sin(2x)(2 - \sin x)$
36. $y' = 2x\frac{\sin x}{\cos^3 x}$
37. $y' = 2\sin x(x\sin x\cos x^2 + \cos x \sin x^2)$
38. $y' = \arcsin x + \frac{x}{\sqrt{1-x^2}}$
39. $y' = \frac{2\arcsin x}{\sqrt{1-x^2}}$
40. $y' = \sin x \operatorname{arctg} x + x\cos x \operatorname{arctg} x + \frac{x\sin x}{1+x^2}$
41. $y' = \frac{-2x^2}{(1+x^2)^2}$
42. $y' = 0$
43. $y' = \frac{3\ln^2 x}{x}$
44. $y' = \frac{\ln x + 1}{\ln 10}$
45. $y' = (\sin x + x\cos x)\ln x + \sin x$
46. $y' = x^{99}(100\ln x + 1)$
47. $y' = -\frac{1}{(x^2+2x+2)\operatorname{arctg}\frac{1}{1+x}}$
48. $y' = -\frac{2}{x(1+\ln x)^2}$
49. $y' = 10^x(1 + x\ln 10)$
50. $y' = e^x(\cos x - \sin x)$
51. $y' = e^x(\cos x + \sin x + 2x\cos x)$
52. $y' = 2^x \ln 2 \cos 2^x$
53. $y' = \frac{\cos x}{|\cos x|}$
54. $y' = \frac{3}{2}\sqrt{x}$
55. $y' = (2x + 3)e^{x^2+3x-2}\cos(e^{x^2+3x-2})$
56. $y' = -3\sin(3x)\sin(2\cos 3x)$
57. $y' = \left(\frac{x+1}{x}\right)^x \left(\ln\frac{x+1}{x} - \frac{1}{x+1}\right)$
58. $y' = x^{x^2+1}(2\ln x + 1)$
59. $y' = x^{\sin x}\left(\cos x\ \ln x + \frac{\sin x}{x}\right)$
60. $y' = (\ln x)^x\left(\frac{1}{\ln x} + \ln\ln x\right)$
61. $y' = (\operatorname{arctg} x)^x\left[\ln(\operatorname{arctg} x) + \frac{x}{(1+x^2)\operatorname{arctg} x}\right]$
62. $y' = x^{x^x}x^x\left(\ln^2 x + \ln x + \frac{1}{x}\right)$