Ex. 1. Probability

Karolina Tura-Gawron, Ph.D.

Elementary probability

For equally likely outcomes,

Probability= Number of successful outcomes/Number of possible outomes

Task 1. A group of 20 university students contains eight who are in their first year of study. A student is picked up at random to represent the group at a meeting. Find the probability that the student is not in the first year of study.

Task 2. Two fair coins are tossed. Show the possible outcomes. Find the probability that two heads are obtained.

Task 3. The probability of an event occuring is 0.27. What is the probability that it will not occur?

Task 4. The pupils in the junior class were asked how many brothers and sisters they had. Their answers are shown in the table. Find the probability that a child chosen at random from the class comes from a family with three children.

Number of brothers and sisters	0	1	2	3	4	5
Number of pupils	4	12	8	3	2	1

Task 5. The table summarises the results of all the driving tests taken at a Test Center during the first week of September.

	Male	Female
Pass	32	43
Fail	8	15

A person is chosen at random from those who took their test that week.

a) Find the probability that the person passed the driving test.

b) Find the probability that the person was a female who failed the driving test.

c) A male is chosen. Find the probability that he did not pass the test.

Task 6. Two ordinary unbiased dice are thrown. Find the probability that:

a) the sum on the two dice is 3,

b) the sum on the two dice exceeds 9,

c) the two dice show the same numer,

d) the numbers on the two dice differ by more than 2.

Probability- combined and exclusive events

Events A and B are said to be combined if they can occur at the same time.

Probability rule for combined events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Task 7. In a class of 20 children, 4 of the 9 boys and 3 of the 11 girls are in the athletic team. A person from the class is chosen to be in the 'egg and spoon' race on Sports Day. Find the probability that the person chosen is:

a) in the athletics team,

b) female,

c) a female member of the athletics team,

d) a female or in the athletics team.

Task 8. Events C and D are such that P(C) = 19/30, P(D)= 2/5, $P(C \cup D) = 4/5$. Find $P(C \cap D)$.

Task 9. In a survey, 15% of the participants said that they had never bought lottery tickets or a premium bonds, 73% had bought lottery tickets and 49% had bought a premium bonds. Find the probability that a person chosen at random from those taking part in the survey:

    a.   had bought lottery tickets or premium bonds,

    b.   had bought lottery tickets and premium bonds,

    c.   had bought lottery tickets only.

Events A and B are said to be exclusive if they can not occur at the same time.

Probability rule for exclusive events: $P(A \cup B) = P(A) + P(B)$.

Task 10. In race in which there are no dead heats, the probability that John wins is 0.3, the probability that Paul wins is 0.2 and the probability that Mark wins is 0.4. Find the probability that:

    a.   John or Mark wins,

    b.   John or Paul or Mark wins,

    c.   someone else wins.

Conditional probability

If A and B are two elements, not necessarily from the same experiment, than the conditional probability that A occurs, given that B has already occured, is written:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Task 11. In a certain college 65% of the students are full-time students, 55% of the students are female, 35% of the students are male full-time students. Find the probability that:

    a.   a student chosen at random from all the students in the college is the part time student,

    b.   a student chosen at random from all the students in the college is female and a part time student,

    c.   a student chosen at random from all the female students in the college is a part time student.

Task 12. A group of girls at a school is entered for Advanced Level Mathematics modules. Each girl takes only module M1 or only module M2 or both M1 and M2. The probability that a girl is taking M2 given that she is taking M1 is 1/5. The probability that a girl is taking M1 given that she is taking M2 is 1/3. Find the probability that:

    a.   a girl selected at random is taking both M1 and M2,

    b.   a girl selected at random is taking only M1.

Independent events

If either of the events A and B can occur without being affected by the other, then the two events are independent: $P(A \cap B) = P(A) * P(B)$.

Task 13. In a group of 60 students, 20 study History, 24 study French, 8 study both History and French. Are the events 'a student studies History' and 'a student studies French' independent?

Task 14. Events A and B are independent and P(A)=1/3 and $P(A \cap B) = 1/12$. Find P(B) and $P(A \cup B)$.