

Lab. 3. Special discrete probability distributions

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	Bernoulli distribution	Discrete uniform distribution	Binomial distribution	Discrete Poisson distribution
Features	„p”-success „q”- failure	Finite number of values are equally likely to be observed	„n” trials. Trials are independent. „p”-success „q”- failure The binomial random variable „k” is the number of „p” in „n” trials	Probability of success „p” in one trial is small. Number of trials „n” is large
Probability function	$P(X = 1) = p$ $P(X = 0) = 1 - p = q$	$P(X = x_i) = \frac{1}{n}$	$P(X = k) = C_n^k p^k q^{n-k}$ $C_n^k = \frac{n!}{k!(n-k)!}$	$np = \lambda$ $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$
Expected value	p	$\frac{1}{n} \sum_{i=1}^n x_i$	np	$np = \lambda$
Standard deviation	\sqrt{pq}	$\sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - EX)^2}$	\sqrt{npq}	$\sqrt{np} = \sqrt{\lambda}$

Task 1. A game consists of tossing three times symmetrical coin. The winner is the one who gets the largest number of heads. What is the probability of tossing the three tails, one head, three heads? The number of heads is value of a binomial random variable. Find:

- a) the probability distribution,
- b) the mean and standard deviation of the distribution,
- c) the probability of tossing at least one head,
- d) the probability of tossing at least two heads.

Task 2. It is known that the average number of bacteria in the view of the microscope is fixed at 0.7. Assume that a random variable X (which values are equal to the number of bacteria in the view of the microscope) is 0,1,2, ...

This variable has a Poisson distribution. Find:

- a) the probability that in the view will be more than two bacteria,
- b) the probability that in the view will be a less or three bacteria.

Task 3. An investor (who holds on the stock exchange shares of 6 companies) participates in the trading game. The probability of an increase in the share prices (bull) on the next trading session is equal to the probability of a decrease in the share prices (bear). The number of increases is a random variable.

- a) Find the probability distribution and cumulative distribution function.
- b) Find the expected value and standard deviation.
- c) Find the probability of increase in prices of:
 - i. one company,
 - ii. the two companies,
 - iii. most of the two companies,
 - iv. three companies
 - v. at least three companies,

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vi. all companies.

Task.4. We examine the reliability of a bank's computer system. It is assumed that the mistake in the conducting of bank's customers accounts should not occur more than $1/1000$. Find the probability that in a randomly selected sample of 500 bank's customers accounts, there will be:

- a) one error,
- b) at most two errors.