Ex. 2. Probability: Probability Trees and Bayes Theorem

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Probability tree

How to use the tree:

1.Multiply the probabilities along the branches to get the end results, so for the first outcome, use the fact that *P(T and G)=P(T) x P(G given T).*

2.On any set of branches that meet at a point, the probabilities must add up to 1.

3.Check that all the end results add up to 1.

4.To answer any questions find the relevant end results. If more than one satisfy the requirements, add these end results together.

Task 1. In a certain selection of flower seeds 2/3 have been treated to improve germination and 1/3 have been left untreated. The seeds which have been treated have a probability of germination of 0.8, wheras the untreated seeds have a probability of germination of 0.5.

   a)   Find the probability that a seed , selected at random, will germinate.

The seeds were sown and given time to germinate.

   b)   Find the probability that a seed selected at random had been treated, given that it had germinated.

Task 2. A manufacturer makes writing pens. The manufacturer employs an inspector to check the quality of his product. The inspector tested a random sample of the pens from a large batch and calculated the probability of any pen being defective as 0.025. Carmel buys two of the pens made by manufacturer.

   a)   Calculate the probability that both pens are defective.

   b)   Calculate the probability that exactly one of the pens is defective.

Task 3. When a person needs a minicab, it hired from one of three firms, X, Y and Z. Of the hirings 40% are from X, 50% are from Y and 10% are from Z. For cabs hired from X, 9% arrive late, the corresponding percentages for cabs hired from firms Y and Z being 6% and 20% respectively. Calculate the probability that the next cab hired:

   a)   will be from X and will not arrive late,

   b)   will arrive late.

   c)   Given that a call is made for a minicab and that it arrives late, find, to three decimal places, the probability that it came from Y.

Bayes' Theorem

For *i=1,2,3,…,n*

$$P(A_i|B) = \frac{P(B|A_i) \times P(A_i)}{P(B|A_1) \times P(A_1) + P(B|A_2) \times P(A_2) + \cdots + P(B|A_n) \times P(A_n)}$$

Or shorter

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

Task 4. A computer program generates random questions in arithmetic that children have to answer within a fixed time. The probability of the first question being answered correctly is 0.8. Whenever a question is answered correctly, the next question generated is more difficult, and the probability of a correct answer being given is reduced by 0.1. Whenever a question is answered wrongly, the next question is of the same standard, and the probability of a correct answer being given remains unchanged.

a) Find the probability that the second question is answered correctly.

b) By extending the tree, or otherwise, find the probability that the second question is answered correctly given that third question is answered correctly.