Ex. 3. Probability distribution for continuous random variables

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Probability density function *f(x)* (p.d.f)

$$f(x) \geq 0 \ \wedge \ \int_{-\infty}^{+\infty} f(x)dx = 1$$

$$P(a < X \leq b) = P(a \leq X \leq b) = P(a \leq X < b) = P(a < X < b) = \int_{a}^{b} f(x)dx$$

Expected value, $EX = \int_{-\infty}^{+\infty} xf(x)dx = m$

Variance, $D^2(X) = \int_{-\infty}^{+\infty} (x-m)^2 f(x)dx$, $D^2(X)=E(X-EX)^2=E(X^2)-(EX)^2$

Cumulative distribution function *F(x)* (c.d.f.)

F(x)=P(X<x)=P(X≤x)

$$P(a < X \leq b) = P(a \leq X \leq b) = P(a \leq X < b) = P(a < X < b) = F(b) - F(a)$$

$$0 \leq F(x) \leq 1$$

$$\lim_{x \to -\infty} F(x) = 0 \ \vee \ \lim_{x \to +\infty} F(x) = 1$$

$$F(x) = \int_{-\infty}^{x} f(x)dx$$

Task 1. The continuous random variable X has p.d.f. *f(x)* where

$$f(x) = \begin{cases} 0 & for \ x \leq 5 \\ cx & for \ 5 < x \leq 15 \\ 0 & for \ x > 15 \end{cases}$$

a) Find the value of the constant *c* and write the p.d.f.

b) Find c.d.f.

c) Find $P(X > 7); P(X \leq 10); P(8 \leq X < 15)$.

d) Find the EX and the Variance.

Task 2. The continuous random variable X has c.d.f. *F(x)* where

$$F(x) = \begin{cases} 0 & dla \ x \leq 2 \\ 4(1-\frac{5}{x}) & dla \ 2 < x \leq k \\ 1 & dla \ x > k \end{cases}$$

a) Find p.d.f.

b) Find the value of the constant *k* and rewrite the c.d.f.

c) Find $P(X > 3); P(1 \leq X \leq 4)$.

Task 3. The continuous random variable X has p.d.f. *f(x)* where

$$f(x) = \begin{cases} xe^{-x} & dla \ x \geq 0 \\ 0 & dla \ x < 0 \end{cases}$$

a) Find c.d.f.

b) Find $P(X > 3); P(1 \leq X \leq 4)$. .