

## Lab. 4. Special continuous probability distributions

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	Uniform probability distribution	Normal distribution
<b>Probability density function</b>	$f(x) = \begin{cases} 0 & \text{dla } x < a \\ \frac{1}{b-a} & \text{dla } a \leq x \leq b \\ 0 & \text{dla } x > b \end{cases}$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
<b>Cumulative distribution function</b>	$F(x) = \begin{cases} 0 & \text{dla } x \leq a \\ \frac{x-a}{b-a} & \text{dla } a < x \leq b \\ 1 & \text{dla } x > b \end{cases}$	$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{(v-\mu)^2}{2\sigma^2}} dv$
<b>Expected value</b>	$\frac{a+b}{2}$	$\mu$
<b>Standard deviation</b>	$\frac{b-a}{2\sqrt{3}}$	$\sigma$
<b>Notes</b>	Probability is spread evenly over the range of $\langle a, b \rangle$ .	Standard Normal Variable $Z \sim N(0,1)$ . $Z = \frac{X - \mu}{\sigma}$

Task 1. We throw a random point on the range  $[0, 9]$ . Let  $X$  be the random variable, describing the distance of this point from the end of the range.  $X$  has the uniform distribution.

- a) Find the probability density function,
- b) Find cumulative distribution function
- c) Find  $P(X > 7)$ .
- d) Find the mean and standard deviation.

Task 2. The average of the yield of wheat "Grana" is 55 q/ha, and the standard deviation 5q/ha. The distribution of this yield is normal.

- a) Find the probability that the yield of wheat is greater than 65q / ha,
- b) Find the probability that the yield of wheat is in the range  $(\mu - 3\sigma; \mu + 3\sigma)$

Task 3. The profit of the company has a normal distribution with the average 100 (thousand zł), and the standard deviation of 10 (thousand zł). Find the probability that the company makes a profit of over 120 (thousand zł).

Task 4. Weight of the men is a random variable with a normal distribution, average of 70 kg and a standard deviation of 10 kg. Find the probability that from the population of men we choose a person weighing:

- a) more than 80 kg,
- b) below 60 kg,
- c) in the range  $\langle 60-80 \rangle$  kg,
- d) in the range  $\langle 80-90 \rangle$  kg.

$$F(z) = P(Z \leq z)$$

z	-0,09	-0,08	-0,07	-0,06	-0,05	-0,04	-0,03	-0,02	-0,01	-0,00	z
-3,80	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	-3,80
-3,70	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	-3,70
-3,60	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	,0001	-3,60
-3,50	,0002	,0002	,0002	,0002	,0002	,0002	,0002	,0002	,0002	,0002	-3,50
-3,40	,0002	,0003	,0003	,0003	,0003	,0003	,0003	,0003	,0003	,0003	-3,40
-3,30	,0003	,0004	,0004	,0004	,0004	,0004	,0004	,0005	,0005	,0005	-3,30
-3,20	,0005	,0005	,0005	,0006	,0006	,0006	,0006	,0006	,0007	,0007	-3,20
-3,10	,0007	,0007	,0008	,0008	,0008	,0008	,0009	,0009	,0009	,0010	-3,10
-3,00	,0010	,0010	,0011	,0011	,0011	,0012	,0012	,0013	,0013	,0013	-3,00
-2,90	,0014	,0014	,0015	,0015	,0016	,0016	,0017	,0018	,0018	,0019	-2,90
-2,80	,0019	,0020	,0021	,0021	,0022	,0023	,0023	,0024	,0025	,0026	-2,80
-2,70	,0026	,0027	,0028	,0029	,0030	,0031	,0032	,0033	,0034	,0035	-2,70
-2,60	,0036	,0037	,0038	,0039	,0040	,0041	,0043	,0044	,0045	,0047	-2,60
-2,50	,0048	,0049	,0051	,0052	,0054	,0055	,0057	,0059	,0060	,0062	-2,50
-2,40	,0064	,0066	,0068	,0069	,0071	,0073	,0075	,0078	,0080	,0082	-2,40
-2,30	,0084	,0087	,0089	,0091	,0094	,0096	,0099	,0102	,0104	,0107	-2,30
-2,20	,0110	,0113	,0116	,0119	,0122	,0125	,0129	,0132	,0136	,0139	-2,20
-2,10	,0143	,0146	,0150	,0154	,0158	,0162	,0166	,0170	,0174	,0179	-2,10
-2,00	,0183	,0188	,0192	,0197	,0202	,0207	,0212	,0217	,0222	,0228	-2,00
-1,90	,0233	,0239	,0244	,0250	,0256	,0262	,0268	,0274	,0281	,0287	-1,90
-1,80	,0294	,0301	,0307	,0314	,0322	,0329	,0336	,0344	,0351	,0359	-1,80
-1,70	,0367	,0375	,0384	,0392	,0401	,0409	,0418	,0427	0,436	,0446	-1,70
-1,60	,0455	,0465	,0475	,0485	,0495	,0505	,0516	,0526	,0537	,0548	-1,60
-1,50	,0559	,0571	,0582	,0594	,0606	,0618	,0630	,0643	,0655	,0668	-1,50
-1,40	,0681	,0694	,0708	,0721	,0735	,0749	,0764	,0778	,0793	,0808	-1,40
-1,30	,0823	,0838	,0853	,0869	,0885	,0901	,0918	,0934	,0951	,0968	-1,30
-1,20	,0985	,1003	,1020	,1038	,1056	,1075	,1093	,1112	,1131	,1151	-1,20
-1,10	,1170	,1190	,1210	,1230	,1251	,1271	,1292	,1314	,1335	,1357	-1,10
-1,00	,1379	,1401	,1423	,1446	,1469	,1492	,1515	,1539	,1562	,1587	-1,00
-0,90	,1611	,1635	,1660	,1685	,1711	,1736	,1762	,1788	,1814	,1841	-0,90
-0,80	,1867	,1894	,1922	,1949	,1977	,2005	,2033	,2061	,2090	,2119	-0,80
-0,70	,2148	,2177	,2206	,2236	,2266	,2296	,2327	,2358	,2389	,2420	-0,70
-0,60	,2451	,2483	,2514	,2546	,2578	,2611	,2643	,2676	,2709	,2743	-0,60
-0,50	,2776	,2810	,2843	,2877	,2912	,2946	,2981	,3015	,3050	,3085	-0,50
-0,40	,3121	,3156	,3192	,3228	,3264	,3300	,3336	,3372	,3409	,3446	-0,40
-0,30	,3483	,3520	,3557	,3594	,3632	,3669	,3707	,3745	,3783	,3821	-0,30
-0,20	,3859	,3897	,3936	,3974	,4013	,4052	,4090	,4129	,4168	,4207	-0,20
-0,10	,4247	,4286	,4325	,4364	,4404	,4443	,4483	,4522	,4562	,4602	-0,10
0,00	,4641	,4681	,4721	,4761	,4801	,4840	,4880	,4920	,4960	,5000	0,00

z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09	z
0,00	,5000	,5040	,5080	,5120	,5160	,5199	,5239	,5279	,5319	,5359	0,00
0,10	,5398	,5438	,5478	,5517	,5557	,5596	,5636	,5675	,5714	,5753	0,10
0,20	,5793	,5832	,5871	,5910	,5948	,5987	,6026	,6064	,6103	,6141	0,20
0,30	,6179	,6217	,6255	,6293	,6331	,6368	,6406	,6443	,6480	,6517	0,30
0,40	,6554	,6591	,6628	,6664	,6700	,6736	,6772	,6808	,6844	,6879	0,40
0,50	,6915	,6950	,6985	,7019	,7054	,7088	,7123	,7157	,7190	,7224	0,50
0,60	,7257	,7291	,7324	,7357	,7389	,7422	,7454	,7486	,7517	,7549	0,60
0,70	,7580	,7611	,7642	,7673	,7704	,7734	,7764	,7794	,7823	,7852	0,70
0,80	,7881	,7910	,7939	,7967	,7995	,8023	,8051	,8078	,8106	,8133	0,80
0,90	,8159	,8186	,8212	,8238	,8264	,8289	,8315	,8340	,8365	,8389	0,90
1,00	,8413	,8438	,8461	,8485	,8508	,8531	,8554	,8577	,8599	,8621	1,00
1,10	,8643	,8665	,8686	,8708	,8729	,8749	,8770	,8790	,8810	,8830	1,10
1,20	,8849	,8869	,8888	,8907	,8925	,8944	,8962	,8980	,8997	,9015	1,20
1,30	,9032	,9049	,9066	,9082	,9099	,9115	,9131	,9147	,9162	,9177	1,30
1,40	,9192	,9207	,9222	,9236	,9251	,9265	,9279	,9292	,9306	,9319	1,40
1,50	,9332	,9345	,9357	,9370	,9382	,9394	,9406	,9418	,9429	,9441	1,50
1,60	,9452	,9463	,9474	,9484	,9495	,9505	,9515	,9525	,9535	,9545	1,60
1,70	,9554	,9564	,9573	,9582	,9591	,9599	,9608	,9616	,9625	,9633	1,70
1,80	,9641	,9649	,9656	,9664	,9671	,9678	,9686	,9693	,9699	,9706	1,80
1,90	,9713	,9719	,9726	,9732	,9738	,9744	,9750	,9756	,9761	,9767	1,90
2,00	,9772	,9778	,9783	,9788	,9793	,9798	,9803	,9808	,9812	,9817	2,00
2,10	,9821	,9826	,9830	,9834	,9838	,9842	,9846	,9850	,9854	,9857	2,10
2,20	,9861	,9864	,9868	,9871	,9875	,9878	,9881	,9884	,9887	,9890	2,20
2,30	,9893	,9896	,9898	,9901	,9904	,9906	,9909	,9911	,9913	,9916	2,30
2,40	,9918	,9920	,9922	,9925	,9927	,9929	,9931	,9932	,9934	,9936	2,40
2,50	,9938	,9940	,9941	,9943	,9945	,9946	,9948	,9949	,9951	,9952	2,50
2,60	,9953	,9955	,9956	,9957	,9959	,9960	,9961	,9962	,9963	,9964	2,60
2,70	,9965	,9966	,9967	,9968	,9969	,9970	,9971	,9972	,9973	,9974	2,70
2,80	,9974	,9975	,9976	,9977	,9977	,9978	,9979	,9979	,9980	,9981	2,80
2,90	,9981	,9982	,9982	,9983	,9984	,9984	,9985	,9985	,9986	,9986	2,90
3,00	,9987	,9987	,9987	,9988	,9988	,9989	,9989	,9989	,9990	,9990	3,00
3,10	,9990	,9991	,9991	,9991	,9992	,9992	,9992	,9992	,9993	,9993	3,10
3,20	,9993	,9993	,9994	,9994	,9994	,9994	,9994	,9995	,9995	,9995	3,20
3,30	,9995	,9995	,9995	,9996	,9996	,9996	,9996	,9996	,9996	,9997	3,30
3,40	,9997	,9997	,9997	,9997	,9997	,9997	,9997	,9997	,9997	,9998	3,40
3,50	,9998	,9998	,9998	,9998	,9998	,9998	,9998	,9998	,9998	,9998	3,50
3,60	,9998	,9998	,9999	,9999	,9999	,9999	,9999	,9999	,9999	,9999	3,60
3,70	,9999	,9999	,9999	,9999	,9999	,9999	,9999	,9999	,9999	,9999	3,70
3,80	,9999	,9999	,9999	,9999	,9999	,9999	,9999	,9999	,9999	,9999	3,80