Ex. 5. Central Limit Theorem

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Central Limit Theorem (CLT)

Consider a random sample of *n* observations selected from a population (any probability distribution) with the mean μ and standard deviation σ. Then, when *n* is sufficiently large, the sampling distribution of $\overline{X}$ will be approximately a normal distribution with mean $\mu_{\overline{X}} = \mu$ and standard deviation $\sigma_{\overline{X}} = \sigma / \sqrt{n}$. The larger the sample size, the better will be the normal approximation to the sampling distribution of $\overline{X}$,

Task. 1. Thirty random observations are taken from each of the following distributions and the sample mean calculated. Find, in each case, the probability that the sample mean exceeds 5.

a) X is the number of the telephone calls made in the evening to a counselling service, where $X \sim Po(4.5)$.

b) X is the number of heads obtained when unbiased coin is tossed nine times.

c) X is distributed uniformly throughout the range $2 \leq x \leq 7$.

Task.2. The foreman of a bottling plant has observed that the amount of soda pop in each '32-ounce' bottle is actually a normally distributed random variable, with a mean of 32.2 ounces and a standard deviation of 0.3 ounces.

a) Find the probability that, if a customer buys one bottle, it will contain at least 32 ounces.

b) Find the probability that, if a customer buys a carton of four bottles, the mean will be at least 32 ounces.

Task 3. The dean of the business school claims that the average weekly income of graduates of his school one year after graduation is 400$. If the dean's claim is correct and if weekly incomes are normally distributed with a standard deviation of 60$, what is the probability that the 36 randomly selected graduates have a mean income of less than 375$?