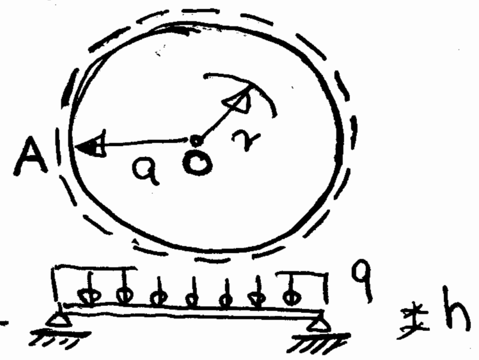
TSiP – ĆWICZENIE 12/1

1. Mając dane: $a, h, E, \nu, q=\text{const}$ podać funkcje i wykresy ugięć W, momentów płytowych M_{rr} i $M_{\varphi\varphi}$



Rozwiązanie ogólne: $W(r) = \frac{qr^4}{64D} + C_1 r^2 \ln r + C_2 r^2 + C_3 \ln r + C_4$

warunki ograniczające w środku płyty $\rightarrow$ $C_1 = C_3 = 0$

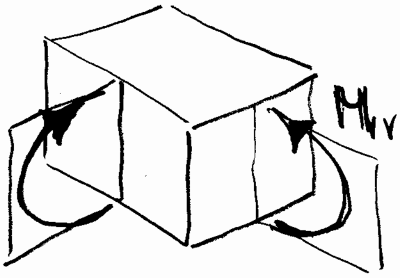
Warunki brzegowe: 1) $W(a) = 0$

2) $M_{rr}(a) = 0$

$$M_{rr} = -D\left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r}\frac{\partial w}{\partial r}\right)$$

$$M_{\varphi\varphi} = -D\left(\frac{1}{r}\frac{\partial w}{\partial r} + \nu\frac{\partial^2 w}{\partial r^2}\right)$$

$M_{\varphi\varphi}$ – moment zginający obwodowy



M_{rr} – moment zginający radialny

wyprowadzenie w ramach wykładu: $C_2 = -\frac{qa^2}{32D}\cdot\frac{3+\nu}{1+\nu}$; $C_4 = \frac{qa^4}{64D}\,\frac{5+\nu}{1+\nu}$

$$W(r) = \frac{qa^4}{64D}\left[\left(\frac{r}{a}\right)^4 - 2\,\frac{3+\nu}{1+\nu}\left(\frac{r}{a}\right)^2 + \frac{5+\nu}{1+\nu}\right] = \frac{q}{64D}\left(a^2-r^2\right)\left[\frac{5+\nu}{1+\nu}a^2 - r^2\right]$$

$$M_{rr}(r) = \frac{qa^2}{16}(3+\nu)\left[1-\left(\frac{r}{a}\right)^2\right]$$

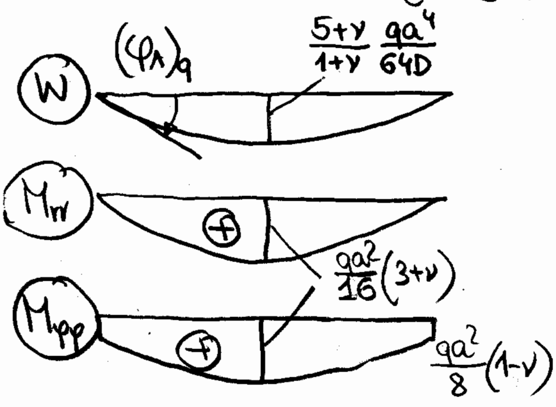
$$M_{\varphi\varphi}(r) = \frac{qa^2}{16}(3+\nu)\left[1-\frac{1+3\nu}{3+\nu}\left(\frac{r}{a}\right)^2\right]$$

$r=0$	$r=a$
$\frac{qa^2}{16}(3+\nu)$	0
$\frac{qa^2}{16}(3+\nu)$	$\frac{qa^2}{8}(1-\nu)$

$W_{max} = W(0) = \frac{5+\nu}{1+\nu}\,\frac{qa^4}{64D}$

kąt obrotu promienia przy brzegu (p.A)

$(\varphi_A)_q = \left.\frac{\partial w}{\partial r}\right|_{r=a} = -\frac{qa^3}{8D(1+\nu)}$



Dane liczbowe: $E = 200\,GPa$, $\nu = 0{,}2$
$h = 0{,}06\,m$, $a = 4\,m$
$q = 10\,kPa$

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{200\cdot10^6\cdot0{,}06^3}{12\cdot0{,}96} = 3750\,kNm$$

$$W_{max} = \frac{5{,}2}{1{,}2}\cdot\frac{10\cdot4^4}{64\cdot3750} = 4{,}62\,cm$$

Momenty płytowe:

– w środku płyty (p.O) $M_{rr} = M_{\varphi\varphi} = \frac{10\cdot16}{16}\cdot3{,}2 = 32\,\frac{kNm}{m}$

– na brzegu płyty (p.A) $M_{rr} = 0$; $M_{\varphi} = \frac{10\cdot16}{8}\cdot0{,}8 = 16\,\frac{kNm}{m}$

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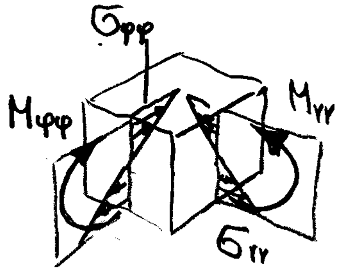


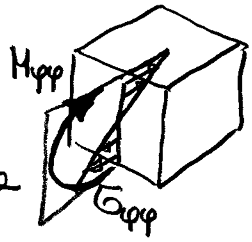
1 c.d. Naprężenia normalne:

p. O – kierunki (r) i (φ) dowolne prostopadłe

$$\sigma_{rr} = \frac{12 M_{rr}}{h^3} x_3 = \frac{12 \cdot 32}{0{,}06^3} x_3 = 1777{,}8 x_3 \; [\mathrm{MPa}], \quad (\sigma_{rr})_{max} = 53{,}3\ \mathrm{MPa}$$

$$\sigma_{\varphi\varphi} = \frac{12 M_{\varphi\varphi}}{h^3} x_3 = \frac{12 \cdot 32}{0{,}06^3} x_3 = 1777{,}8 x_3 \; [\mathrm{MPa}], \quad (\sigma_{\varphi\varphi})_{max} = 53{,}3\ \mathrm{MPa}$$

stąd $\sigma_{zast}(0) = 53{,}3\ \mathrm{MPa}$



p. A – brzeg płyty, $M_{rr} = 0 \Rightarrow \sigma_{rr} = 0$

$$\sigma_{\varphi\varphi} = \frac{12 M_{\varphi\varphi}}{h^3} x_3 = \frac{12 \cdot 16}{0{,}06^3} x_3 = 888{,}9 x_3 \; [\mathrm{MPa}], \quad (\sigma_{\varphi\varphi})_{max} = 26{,}7\ \mathrm{MPa}$$

stąd $\sigma_{zast}(A) = 26{,}7\ \mathrm{MPa}$

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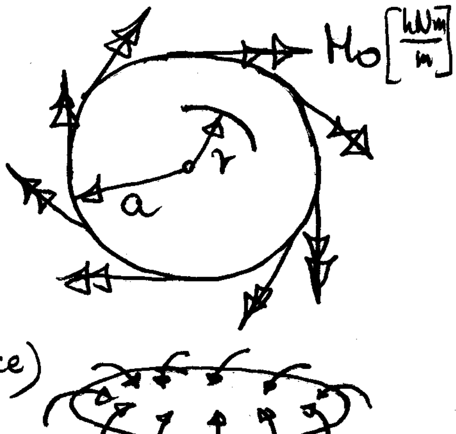
2 Mając dane a, h, E, ν, $M_0 = \mathrm{const}$ → obciążenie ciągłym momentem zginającym brzegowym podać funkcje i wykresy ugięć w, momentów M_{rr} i $M_{\varphi\varphi}$



$w(r) = C_1 r^2 + C_2$ (brak obciążenia ciągłego, redukcja wyrazów logarytmicznych – te same warunki ograniczające)

$$\left.\begin{aligned} w_{,r} &= 2C_1 r \\ w_{,rr} &= 2C_1 \end{aligned}\right\} \quad M_{rr} = -D\left(w_{,rr} + \frac{\nu}{r} w_{,r}\right) = -2DC_1(1+\nu)$$

Warunki brzegowe:

1) $w(a) = 0 \Rightarrow C_1 a^2 + C_2 = 0$ 2) $M_{rr}(a) = M_0 \Rightarrow -2DC_1(1+\nu) = M_0$

stąd $C_1 = -\dfrac{M_0}{2D(1+\nu)}$, $C_2 = \dfrac{M_0 a^2}{2D(1+\nu)}$

$$w(r) = -\frac{M_0 r^2}{2D(1+\nu)} + \frac{M_0 a^2}{2D(1+\nu)} = \frac{M_0}{2D(1+\nu)}\left(a^2 - r^2\right)$$

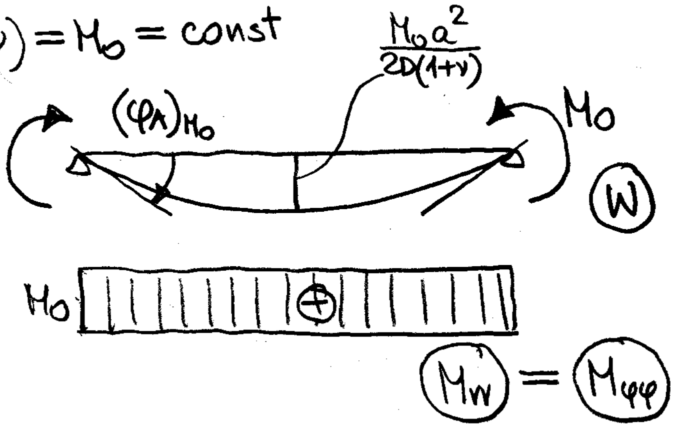
$$w_{max} = w(0) = \frac{M_0 a^2}{2D(1+\nu)}$$

$$M_{rr} = -D\left(w_{,rr} + \frac{\nu}{r} w_{,r}\right) = -2DC_1(1+\nu) = M_0 = \mathrm{const}$$

$$M_{\varphi\varphi} = -D\left(\frac{1}{r} w_{,r} + \nu w_{,rr}\right) = -2DC_1(1+\nu) = M_0 = \mathrm{const}$$

$$w_{,r} = -\frac{M_0 r}{D(1+\nu)}$$

$$(\varphi_A)_{M_0} = w_{,r}(a) = -\frac{M_0 a}{D(1+\nu)}$$



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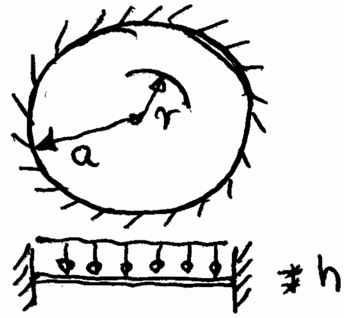
3▪ Mając dane $a, h, E, \nu, q = const$
podać funkcje i wykresy ugięć w, momentów M_{rr} i $M_{\varphi\varphi}$



$$w(r) = \frac{qr^4}{64D} + C_1 r^2 + C_2 \; ; \quad w_{,r} = \frac{qr^3}{16D} + 2C_1 r$$

warunki brzegowe: 1) $w_{,r}(a) = 0 \Rightarrow C_1 = -\frac{qa^2}{32D}$

2) $w(a) = 0 \Rightarrow \frac{qa^4}{64D} - \frac{qa^4}{32D} + C_2 = 0 \Rightarrow C_2 = -\frac{qa^4}{64D}$

$$w(r) = \frac{qr^4}{64D} - \frac{qa^2}{32D} r^2 + \frac{qa^4}{64D} = \frac{q}{64D}\left(r^2 - a^2\right)^2 = \frac{qa^4}{64D}\left[1 - \left(\frac{r}{a}\right)^2\right]^2 \; ; \; w_{max} = w(0) = \frac{qa^4}{64D}$$

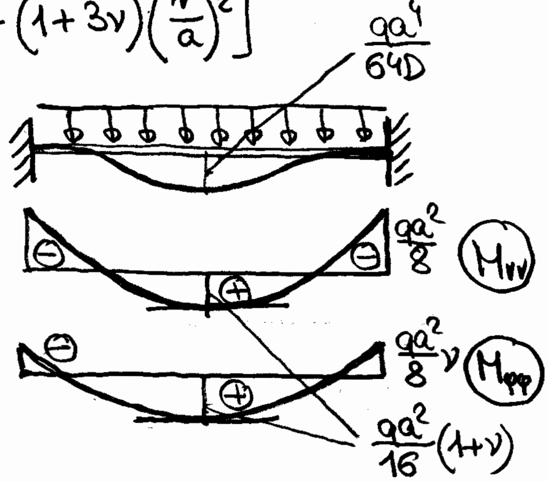
momenty płytowe:

$$M_{rr} = -D\left(w_{,rr} + \frac{\nu}{r} w_{,r}\right) = \frac{qa^2}{16}\left[(1+\nu) - (3+\nu)\left(\frac{r}{a}\right)^2\right]$$

$$M_{\varphi\varphi} = -D\left(\frac{1}{r} w_{,r} + \nu w_{,rr}\right) = \frac{qa^2}{16}\left[(1+\nu) - (1+3\nu)\left(\frac{r}{a}\right)^2\right]$$

środek (p.0) $M_{rr} = M_{\varphi\varphi} = \frac{qa^2}{16}(1+\nu)$

brzeg (p.A) $M_{rr} = -\frac{qa^2}{8}$, $M_{\varphi\varphi} = -\frac{qa^2}{8}\nu$



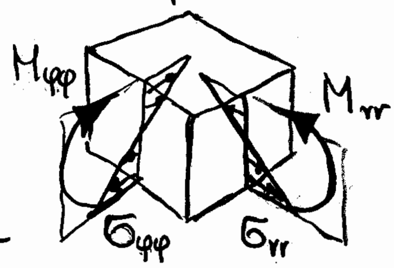
Wartości liczbowe (dane z poprzedniego zadania):

$$w_{max} = \frac{10 \cdot 4^4}{64 \cdot 3750} = 1{,}067\,cm \qquad q = 10\,MPa$$

	$r=0$ (środek)	$r=a$ (brzeg)
M_{rr}	$\frac{10\cdot4^2}{16}\cdot1{,}2 = 12$	$-\frac{10\cdot4^2}{8} = -20$
$M_{\varphi\varphi}$	$\frac{10\cdot4^2}{16}\cdot1{,}2 = 12$	$-\frac{10\cdot4^2}{8}\cdot0{,}2 = -4$

$\left[\frac{kNm}{m}\right]$

Naprężenia:



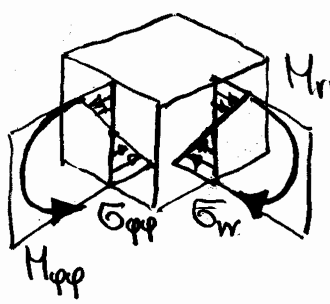
p.0 – kierunki (r) i (φ) dowolne prostopadłe

$$\sigma_{rr} = \frac{12 \cdot 12}{0{,}06^3} x_3 = 666{,}7 x_3 \,[MPa], \; (\sigma_{rr})_{max} = 20\,MPa$$

$$\sigma_{\varphi\varphi} = \frac{12 \cdot 12}{0{,}06^3} x_3 = 666{,}7 x_3 \,[MPa], \; (\sigma_{\varphi\varphi})_{max} = 20\,MPa$$

$$\sigma_{zast}(0) = 20\,MPa$$

p.1



$$\sigma_{rr} = -\frac{12 \cdot 20}{0{,}06^3} x_3 = -1111{,}1 x_3 \,[MPa], \; (\sigma_{rr})_{max} = 33{,}3\,MPa$$

$$\sigma_{\varphi\varphi} = -\frac{12 \cdot 4}{0{,}06^3} x_3 = -222{,}2 x_3 \,[MPa], \; (\sigma_{\varphi\varphi})_{max} = 6{,}67\,MPa$$

$$\sigma_{zast}(A) = \sqrt{33{,}3^2 + 6{,}67^2 - 33{,}3 \times 6{,}67} = 30{,}55\,MPa$$

(mniejsza od $\sigma_{rr} = 33{,}3\,MPa$)