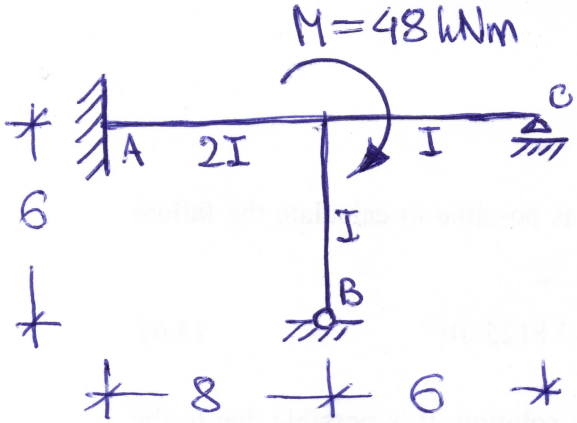
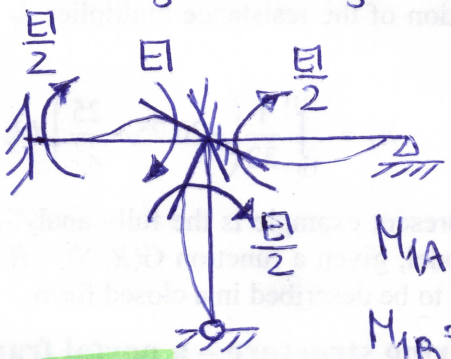
ĆW. 9/1



$n_g = 1\ (\varphi)$

brak momentów wyjściowych – obciążenie węzłowe – moment skupiony



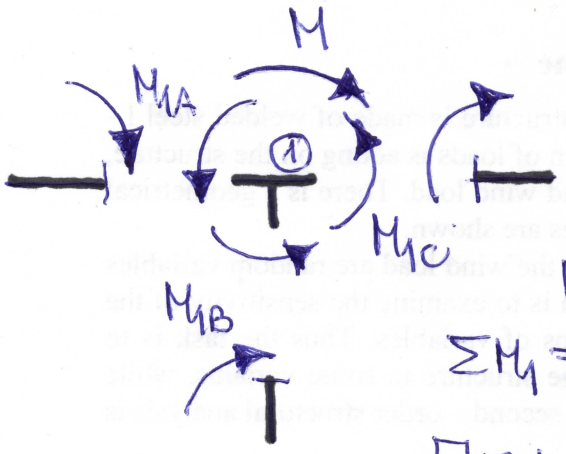
Sumaryczne momenty przywęzłowe:

$$M_{1A} = \frac{4E\cdot 2I}{8}\varphi = EI\varphi$$

$$M_{1B} = M_{1C} = \frac{3EI}{6}\varphi = \frac{EI}{2}\varphi$$

$$M_{A1} = \frac{2E\cdot 2I}{8}\varphi = \frac{EI}{2}\varphi$$

$\varphi = 1$



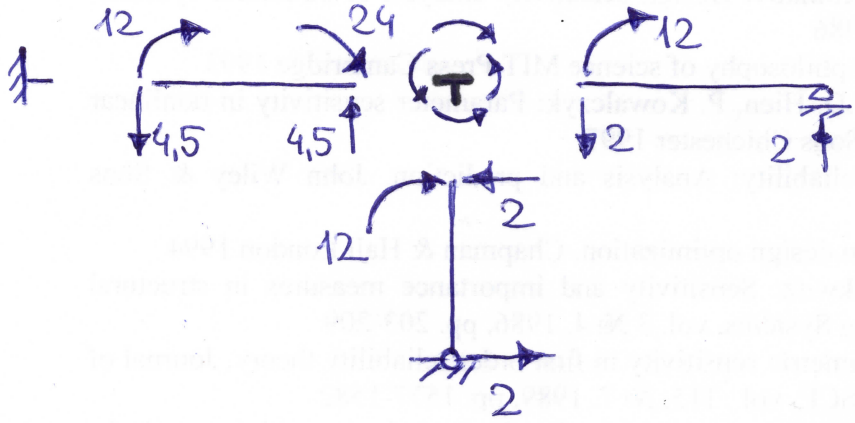
Równowaga węzła ①:

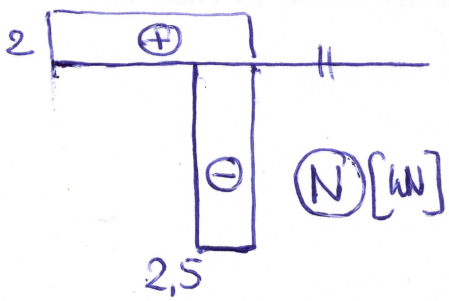
$$\Sigma M_1 = M_{1A} + M_{1B} + M_{1C} - M = 0 \quad \text{lub} \quad M_{1A} + M_{1B} + M_{1C} = M$$

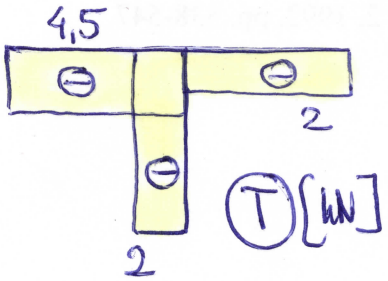
$$EI\varphi + \frac{EI}{2}\varphi + \frac{EI}{2}\varphi = 48 \Rightarrow \varphi = \frac{24}{EI}$$

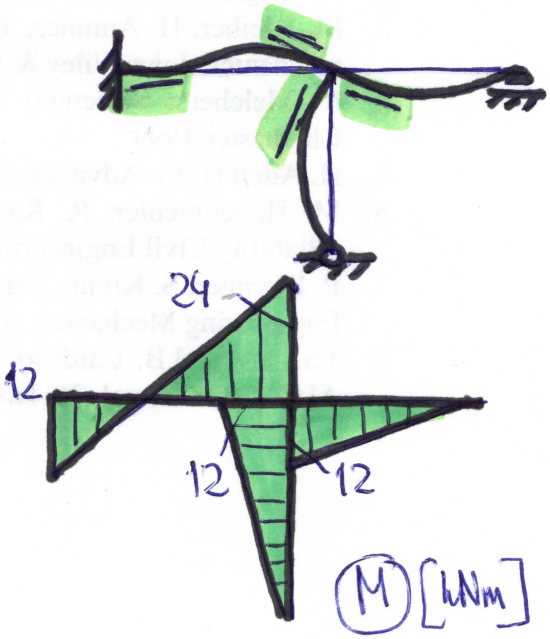
Wartości momentów przywęzłowych:

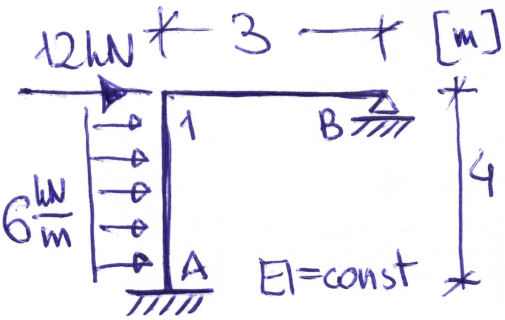
$$M_{1A} = 24\ kNm,\quad M_{1B} = M_{1C} = 12\ kNm,\quad M_{A1} = 12\ kNm$$



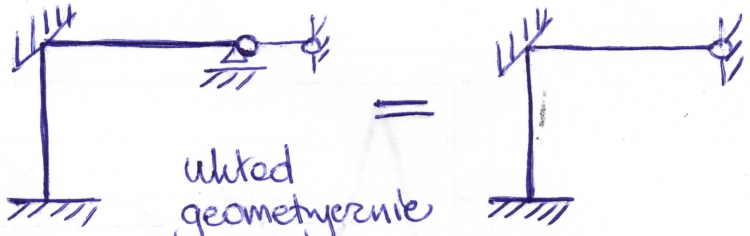




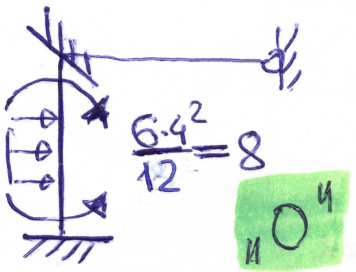


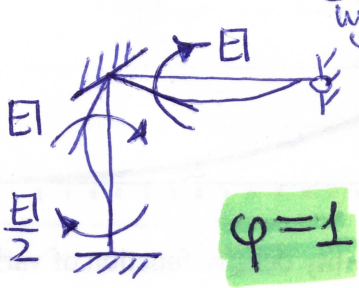


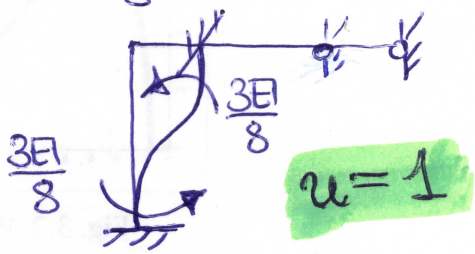
$n_g = 2(\varphi, u)$, układ przesuwny ĆW. 9/2



układ geometrycznie wyznaczalny







Sumaryczne momenty przywęzłowe:

$M_{1A} = 8 + EI\varphi - \frac{3}{8}EIu$

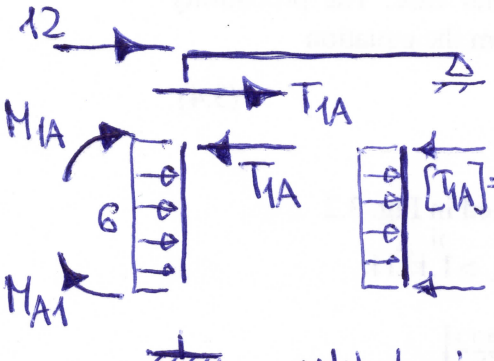
$M_{1B} = EI\varphi$

$M_{A1} = -8 + \frac{EI}{2}\varphi - \frac{3}{8}EIu$

Równanie równowagi momentów w p. 1:

$\Sigma M_1 = 8 + 2EI\varphi - \frac{3}{8}EIu = 0$

$16\varphi - 3u = -\frac{64}{EI}$



Równowaga wyciętego elementu 1-B:

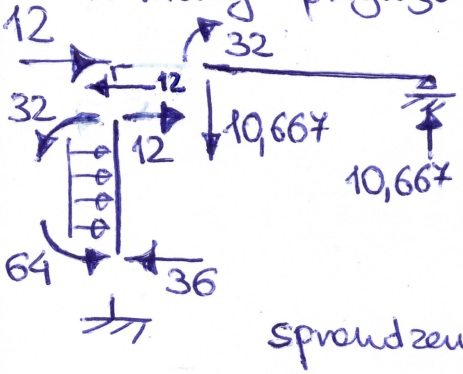
$T_{1A} = \frac{M_{A1} + M_{1A}}{4} + [T_{1A}] = \frac{3EI}{8}\varphi - \frac{3}{16}EIu + 12$

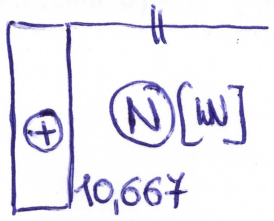
$\Sigma P_x = 0 \Rightarrow T_{1A} + 12 = \frac{3EI}{8}\varphi - \frac{3EI}{16}u + 24 = 0$

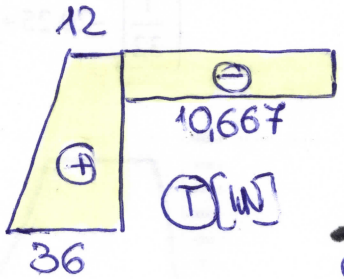
$2\varphi - u = -\frac{128}{EI}$

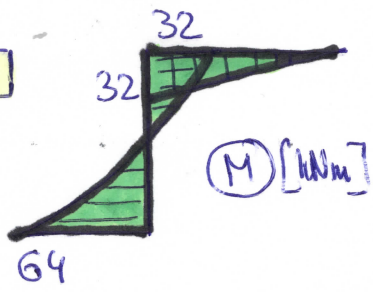
układ równań: $\varphi = \frac{32}{EI}$, $u = \frac{192}{EI}$

Momenty przywęzłowe: $M_{1A} = -32\,kNm$, $M_{1B} = 32\,kNm$, $M_{A1} = -64\,kNm$

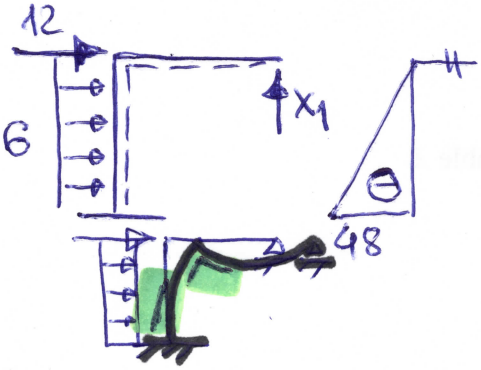


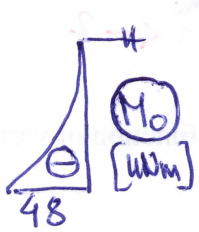


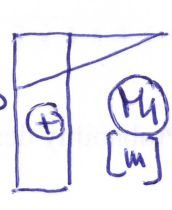




Sprawdzenie metodą sił:





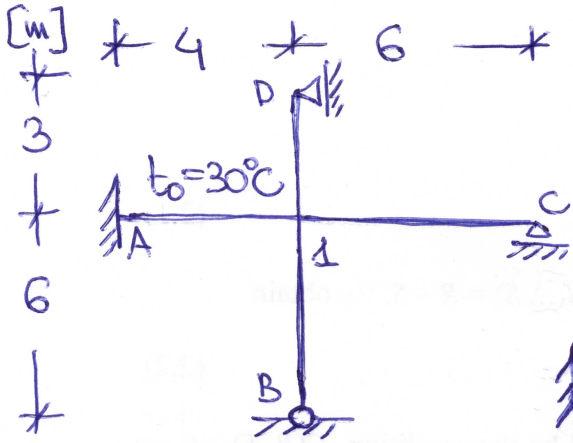


$\delta_{10} = -\frac{3}{EI}\left(\frac{1}{2} \cdot 48 \cdot 4 + \frac{1}{3} \cdot 48 \cdot 4\right) = \frac{-480}{EI}$

$\delta_{11} = \frac{1}{EI}\left(3 \cdot 3 \cdot 4 + \frac{1}{3} \cdot 3 \cdot 3 \cdot 3\right) = \frac{45}{EI}$

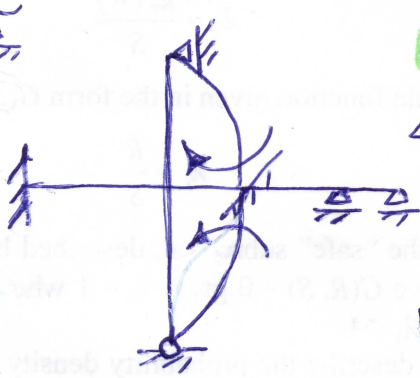
$X_1 = 10{,}667\,kN$

itd.



$\alpha_t = 10^{-5}\frac{1}{°C}$ $EI = 10^5\ kNm^2$ Ćw. 9/3

$n_g = 1\ (\varphi)$



$_4O^4$ – momenty wyjściowe

$\Delta t = 10^{-5}\cdot 4\cdot 30 = 1{,}2\cdot 10^{-3}\ m$

$M^0_{1B} = -\frac{3\cdot 10^5}{6^2}\cdot 1{,}2\cdot 10^{-3} = -10\ kNm$

$M^0_{1D} = \frac{3\cdot 10^5}{3^2}\cdot 1{,}2\cdot 10^{-3} = 40\ kNm$

sumaryczne momenty przywęzłowe

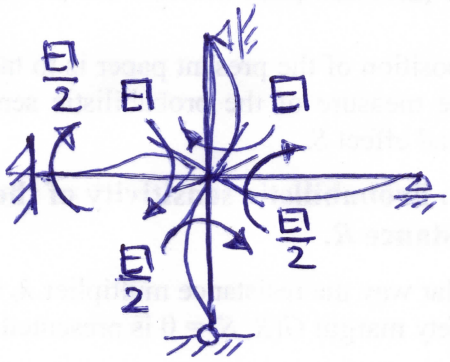
$M_{A1} = \frac{EI}{2}\varphi$

$M_{1A} = -10 + EI\varphi$

$M_{1B} = -10 + \frac{EI}{2}\varphi$

$M_{1C} = \frac{EI}{2}\varphi$

$M_{1D} = 40 + EI\varphi$



$\varphi = 1$

równanie równowagi: $\Sigma M_1 = 0 \Rightarrow 3EI\varphi + 30 = 0 \Rightarrow \varphi = -\frac{10}{EI}$

Wartości momentów przywęzłowych:

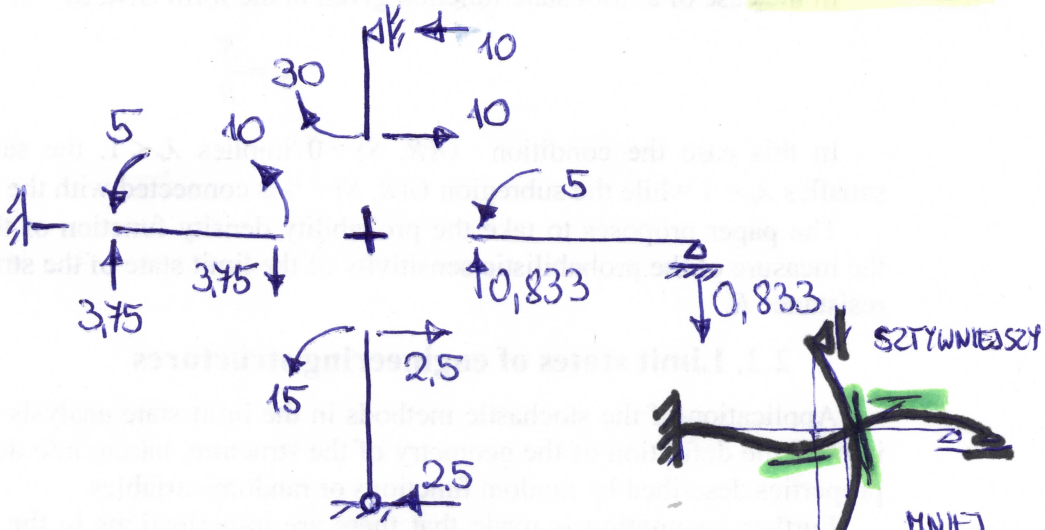
$M_{A1} = -5\ kNm$

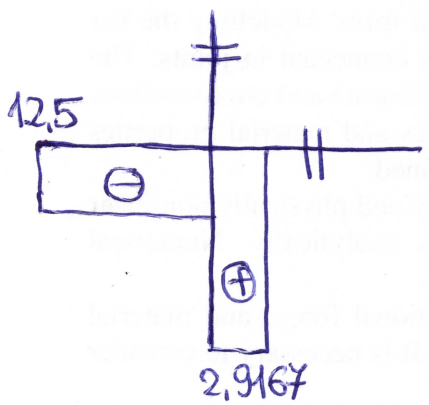
$M_{1A} = -10\ kNm$

$M_{1B} = -15\ kNm$

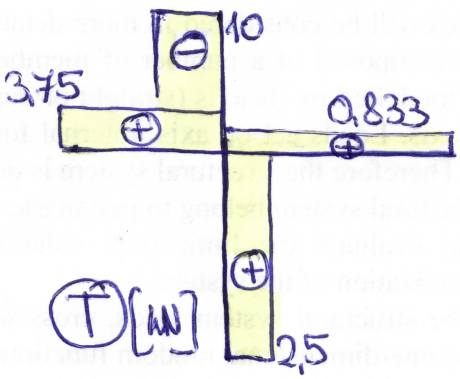
$M_{1C} = -5\ kNm$

$M_{1D} = 30\ kNm$

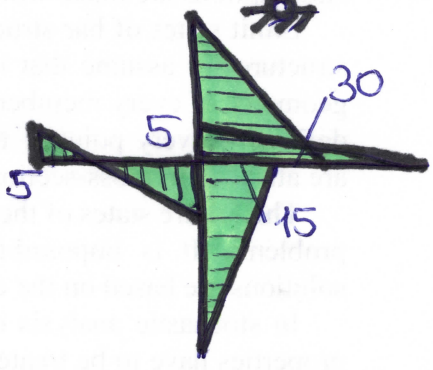




(N)[kN]

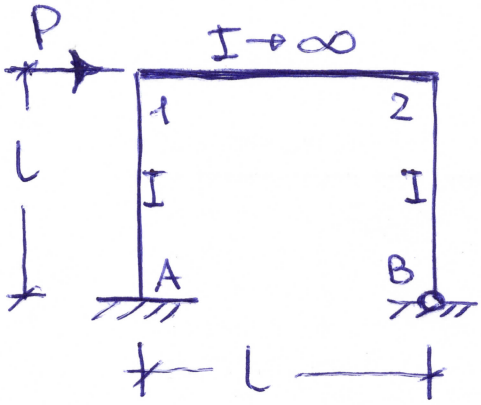


(T)[kN]



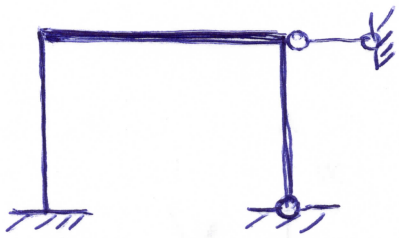
(M)[kNm]

$N_{1-B} = \frac{15}{4} - \frac{5}{6} = \frac{35}{12} = 2{,}9167\ kN$



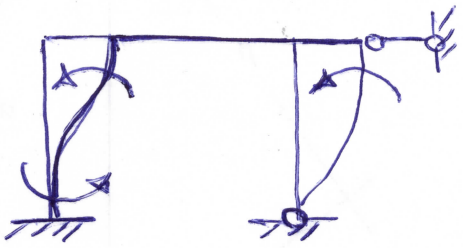
$n_g = 1$ (u)

układ geometrycznie wyznaczalny



brak momentów wyjściowych – obciążenie węzłowe

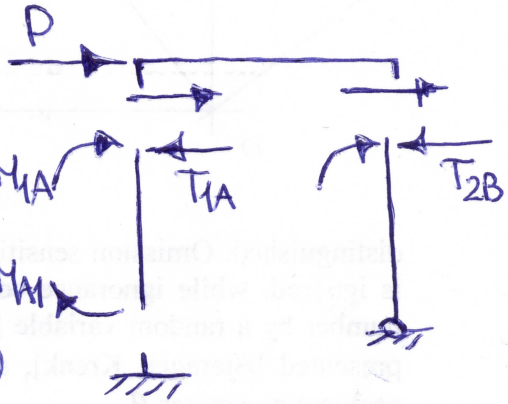
$u = 1$



Momenty przywęzłowe:

$$M_{A1} = M_{1A} = -\frac{6EI}{L^2}u, \quad M_{2B} = -\frac{3EI}{L^2}u$$

Równowaga wyciętego elementu 1-2:



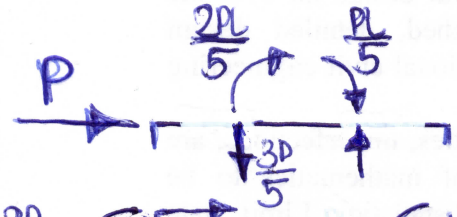
$$T_{1A} = \frac{M_{1A}+M_{A1}}{L} = -\frac{12EI}{L^3}u$$

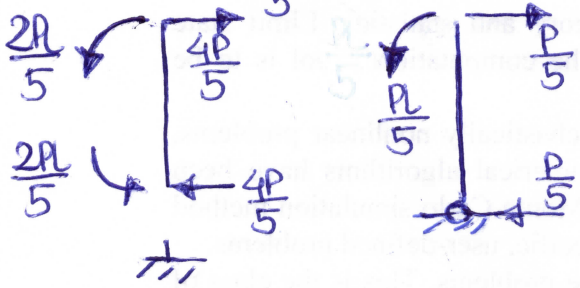
$$T_{2B} = \frac{M_{2B}}{L} = -\frac{3EI}{L^3}u$$

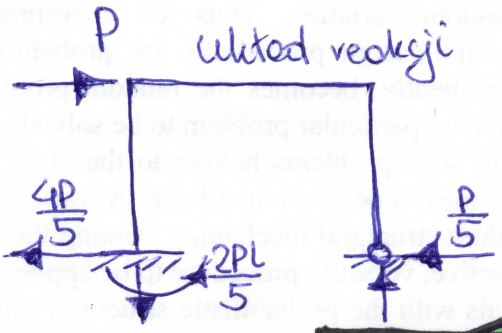
$$\Sigma P_x = P + T_{1A} + T_{2B} = P - \frac{15EI}{L^3}u = 0$$

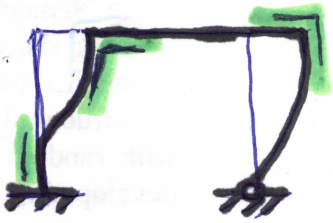
$$u = \frac{PL^3}{15EI} \Rightarrow M_{A1} = M_{1A} = -\frac{2PL}{5}$$

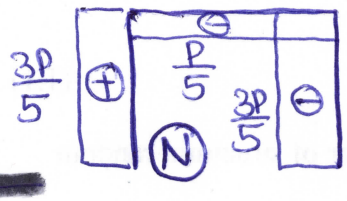
$$M_{2B} = -\frac{PL}{5}$$

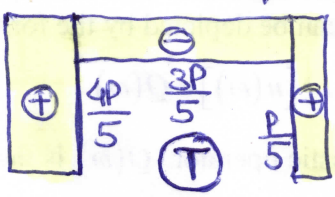


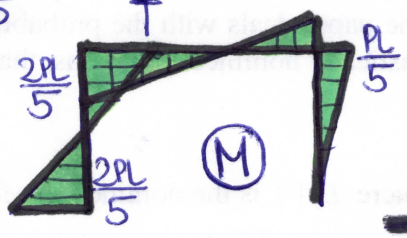


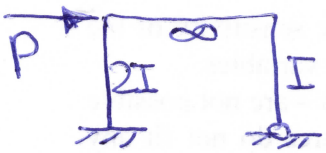


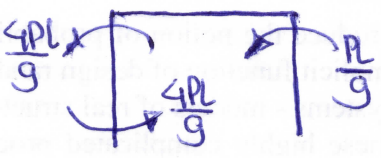


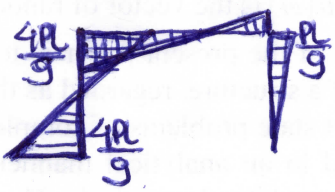


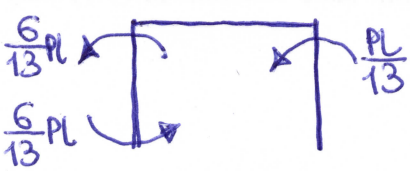


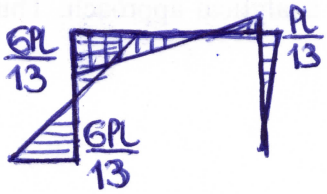




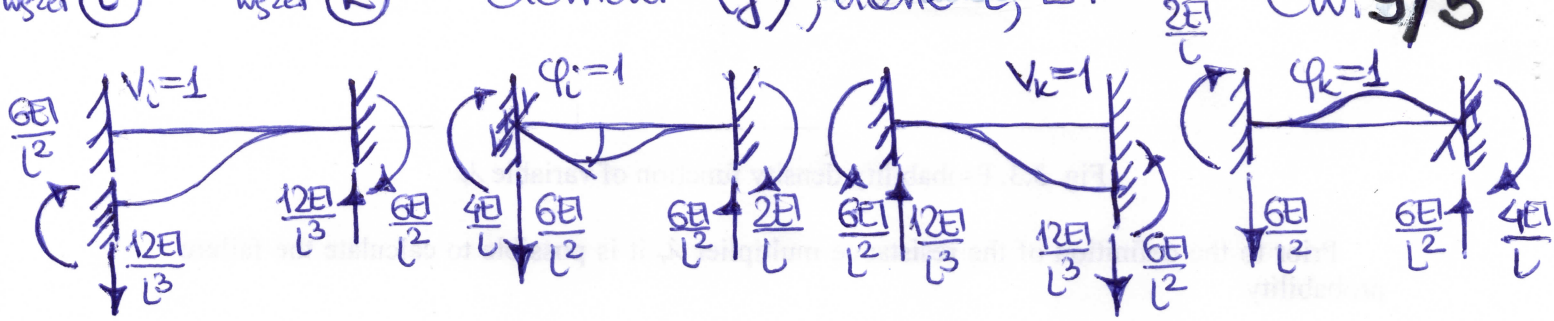


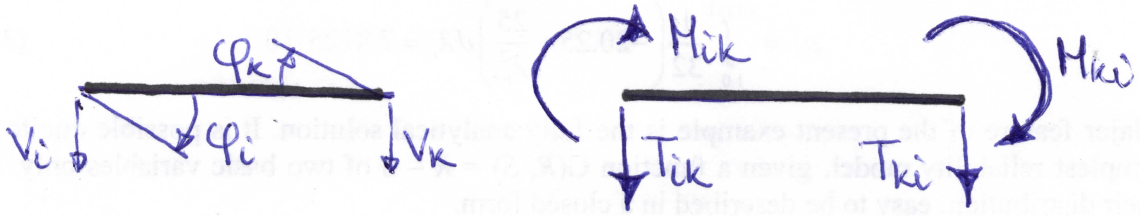






węzeł (i) węzeł (k) element (j), dane: L, EI ĆW. 9/5





$$T_{ik} = \frac{12EI}{L^3} V_i + \frac{6EI}{L^2} \varphi_i - \frac{12EI}{L^3} V_k + \frac{6EI}{L^2} \varphi_k$$

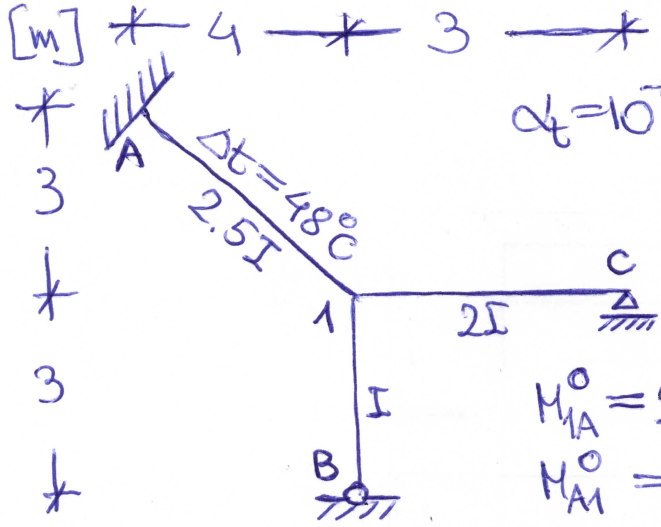
$$M_{ik} = \frac{6EI}{L^2} V_i + \frac{4EI}{L} \varphi_i - \frac{6EI}{L^2} V_k + \frac{2EI}{L} \varphi_k$$

$$T_{ki} = -\frac{12EI}{L^3} V_i - \frac{6EI}{L^2} \varphi_i + \frac{12EI}{L^3} V_k - \frac{6EI}{L^2} \varphi_k$$

$$M_{ki} = \frac{6EI}{L^2} V_i + \frac{2EI}{L} \varphi_i - \frac{6EI}{L^2} V_k + \frac{4EI}{L} \varphi_k$$

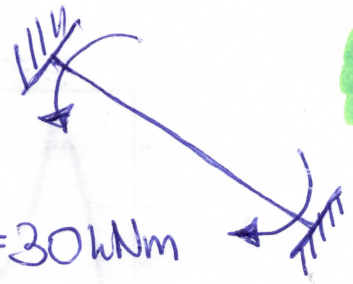
$$\begin{Bmatrix} T_{ik} \\ M_{ik} \\ T_{ki} \\ M_{ki} \end{Bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} V_i \\ \varphi_i \\ V_k \\ \varphi_k \end{Bmatrix}$$

$$S^{(j)} = K^{(j)} \cdot D^{(j)}$$



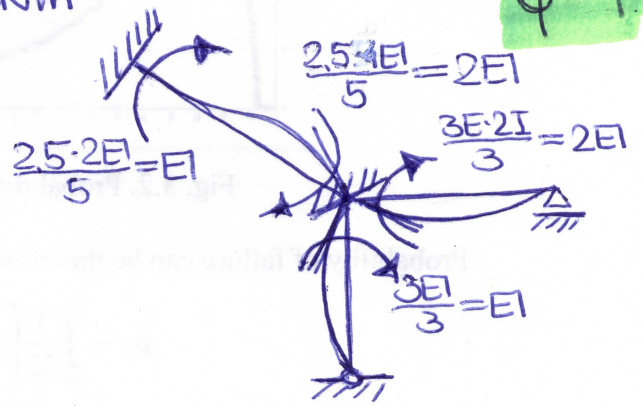
CW 9/6

$\alpha_t = 10^{-5}\frac{1}{°C}$ $h = 0{,}4\,m$ $EI = 10^4\,kNm^2$



$M_{1A}^0 = 2{,}5\cdot 10^4 \frac{10^{-5}\cdot 48}{0{,}4} = 30\,kNm$

$M_{A1}^0 = -30\,kNm$



Sumaryczne momenty przywęzłowe:

$M_{1A} = 30 + 2EI\varphi$

$M_{1B} = EI\varphi$

$M_{1C} = 2EI\varphi$

$M_{A1} = -30 + EI\varphi$

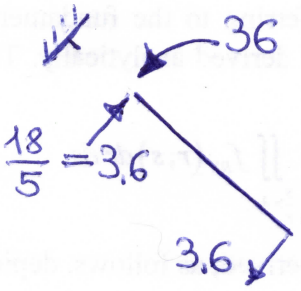
Równanie równowagi: $\Sigma M_1 = 0 \Rightarrow 30 + 5EI\varphi = 0 \Rightarrow \varphi = -\frac{6}{EI}$

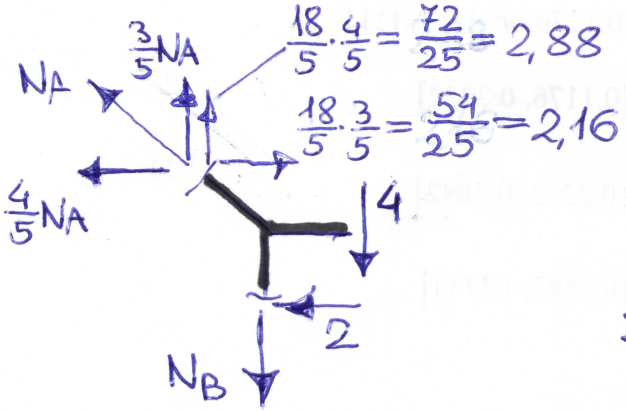
$M_{1A} = 18\,kNm$

$M_{1B} = -6\,kNm$

$M_{1C} = -12\,kNm$

$M_{A1} = -36\,kNm$





$\Sigma P_x = \frac{4}{5}N_A - \frac{54}{25} + 2 = 0$

$N_A = \frac{1}{5} = 0{,}2\,kN$

$\Sigma P_y = \frac{3}{5}N_A + \frac{72}{25} - N_B - 4 = 0$

$N_B = -1\,kN$