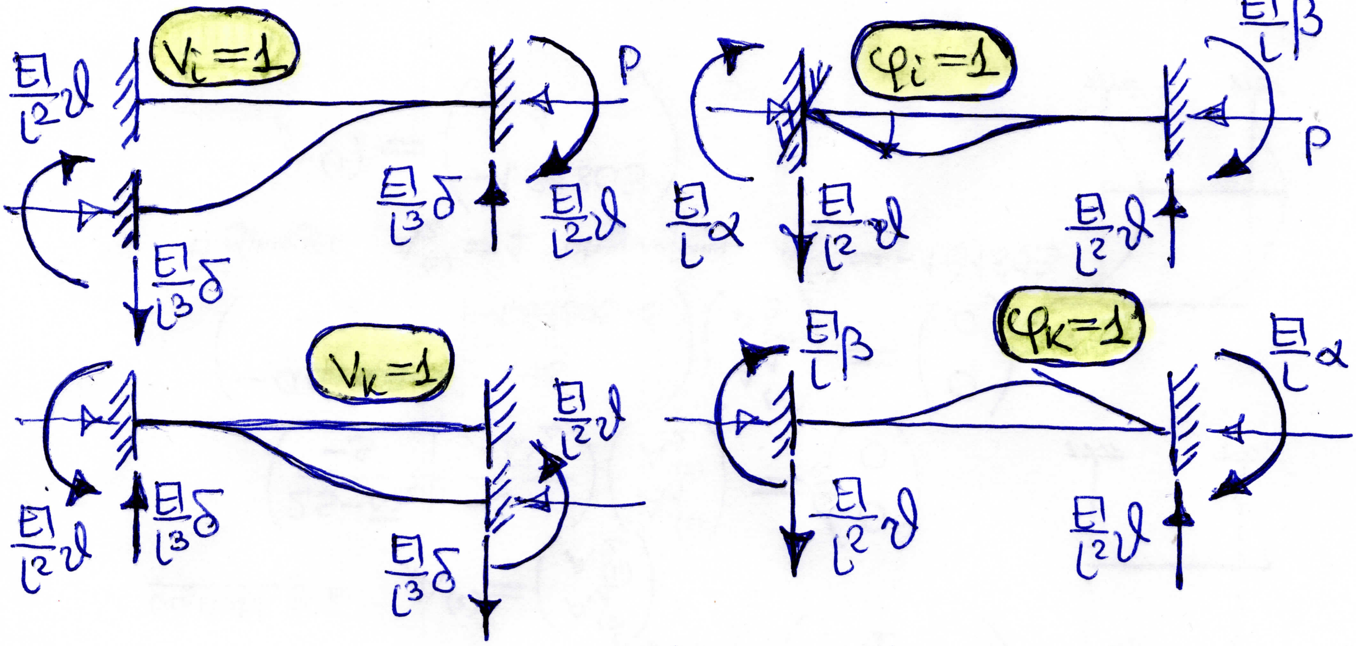
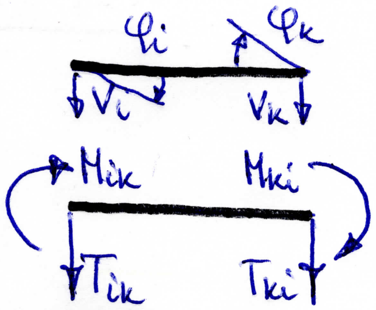
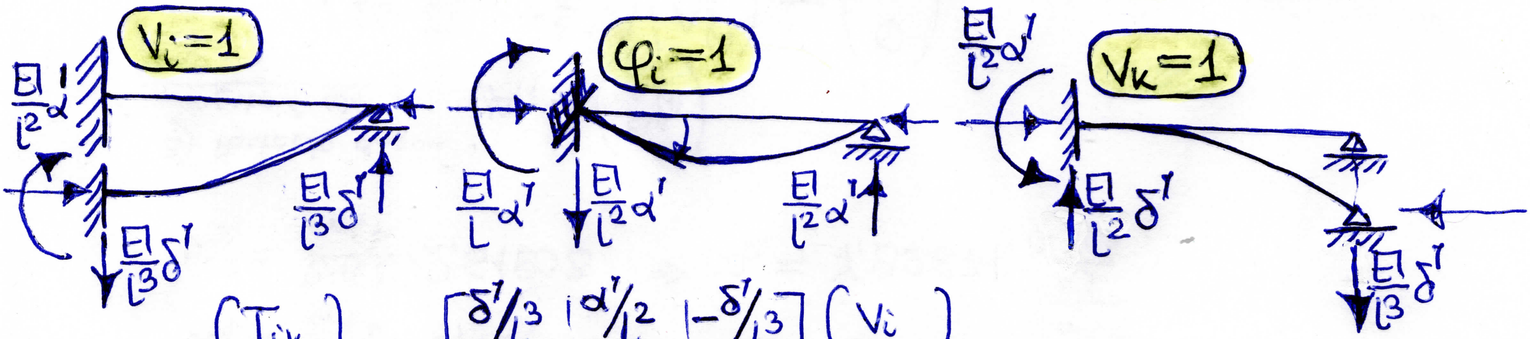
STATECZNOŚĆ UKŁADÓW RAMOWYCH

WYJŚCIOWE SIŁY PRZYWĘZŁOWE Z UWZGLĘDNIENIEM WPŁYWU SIŁY OSIOWEJ (tzw. teoria II rzędu), SIŁA P, PARAMETRY l, EI

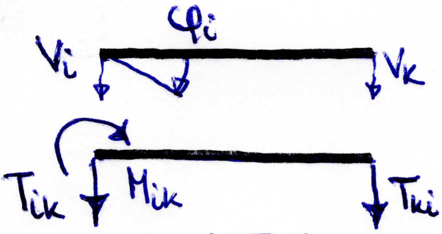


$$\begin{Bmatrix} T_{ik} \\ M_{ik} \\ T_{ki} \\ M_{ki} \end{Bmatrix} = EI \begin{bmatrix} \delta/l^3 & \vartheta/l^2 & -\delta/l^3 & \vartheta/l^2 \\ & \alpha/l & -\vartheta/l^2 & \beta/l \\ & & \delta/l^3 & -\vartheta/l^2 \\ & & & \alpha/l \end{bmatrix} \begin{Bmatrix} V_i \\ \varphi_i \\ V_k \\ \varphi_k \end{Bmatrix}$$





$$\begin{Bmatrix} T_{ik} \\ M_{ik} \\ T_{ki} \end{Bmatrix} = EI \begin{bmatrix} \delta'/l^3 & \alpha'/l^2 & -\delta'/l^3 \\ & \alpha'/l & -\alpha'/l^2 \\ & & \delta'/l^3 \end{bmatrix} \begin{Bmatrix} V_i \\ \varphi_i \\ V_k \end{Bmatrix}$$



$\alpha, \beta, \vartheta, \delta$ oraz α', δ' → funkcje parametru $\lambda = \sqrt{\frac{Pl^2}{EI}}$

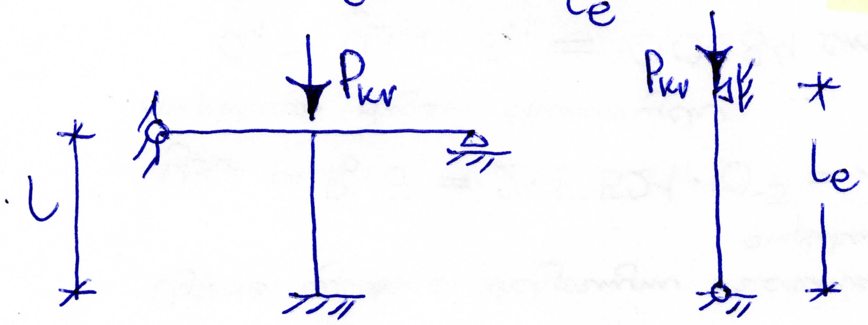
gdy $\lambda \to 0$ (przypadek $P \to 0$): $\alpha(\lambda) \to 4$, $\beta(\lambda) \to 2$, $\vartheta(\lambda) \to 6$, $\delta(\lambda) \to 12$

$\alpha'(\lambda) \to 3$, $\delta'(\lambda) \to 3$

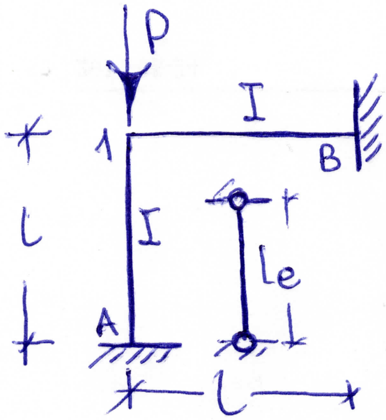
Siła krytyczna wyboczenia giętnego: $P_{kr} = \lambda^2 \frac{EI}{l^2}$

długość wyboczeniowa l_e (długość swobodna na wyboczenie)
– długość elementu (pręta) przegubowego, przy której siła krytyczna wg wzoru Eulera jest taka sama, jak siła w danym pręcie układu w chwili wyboczenia.

$$\lambda^2 \frac{EI}{l^2} = \pi^2 \frac{EI}{l_e^2} \Rightarrow \boxed{l_e = \frac{\pi l}{\lambda}}$$



$$\lambda = \sqrt{\frac{Pl^2}{EI}}$$



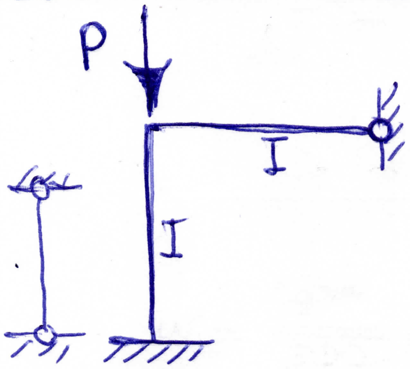
$$M_{1A} = \frac{EI}{l}\alpha(\lambda)\varphi$$

$$M_{1B} = \frac{4EI}{l}\varphi$$

$$\Sigma M_1 = \frac{EI}{l}\left[\alpha(\lambda)+4\right]\varphi = 0$$

$$\alpha(\lambda) = -4 \Rightarrow \lambda = 5{,}33 \quad , \quad P_{kr} = 28{,}41\frac{EI}{l^2}$$

$$l_e = \frac{\pi l}{\lambda} = 0{,}59l$$

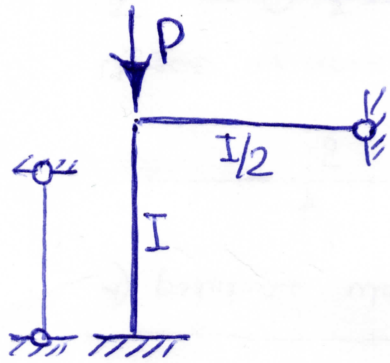


$$M_{1A} = \frac{EI}{l}\alpha(\lambda)\varphi \qquad M_{1B} = \frac{3EI}{l}\varphi$$

$$\Sigma M_1 = \frac{EI}{l}\left[\alpha(\lambda)+3\right]\varphi = 0$$

$$\alpha(\lambda) = -3 \Rightarrow \lambda = 5{,}19 \quad , \quad P_{kr} = 26{,}94\frac{EI}{l^2}$$

$$l_e = \frac{\pi l}{\lambda} = 0{,}61\,l$$

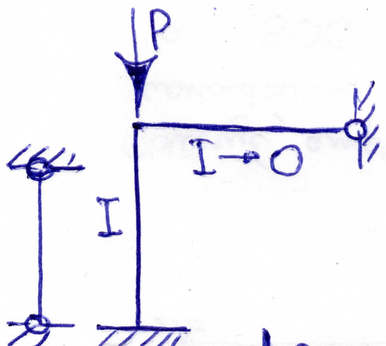


$$M_{1A} = \frac{EI}{l}\alpha(\lambda)\varphi \qquad M_{1B} = \frac{3EI}{2l}\varphi$$

$$\Sigma M_1 = \frac{EI}{l}\left[\alpha(\lambda)+\frac{3}{2}\right]\varphi = 0$$

$$\alpha(\lambda) = -1{,}5 \Rightarrow \lambda = 4{,}91, \quad P_{kr} = 24{,}11\frac{EI}{l^2}$$

$$l_e = \frac{\pi l}{\lambda} = 0{,}64l$$



$$M_{1A} = \frac{EI}{l}\alpha(\lambda)\varphi \qquad M_{1B} = 0$$

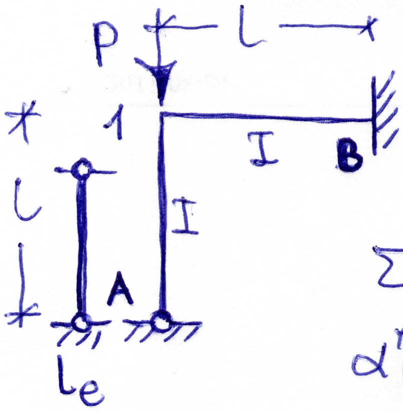
$$\Sigma M_1 = \frac{EI}{l}\alpha(\lambda)\varphi = 0$$

$$\alpha(\lambda) = 0 \Rightarrow \lambda = 4{,}49 \quad , \quad P_{kr} = 20{,}16\frac{EI}{l^2}$$

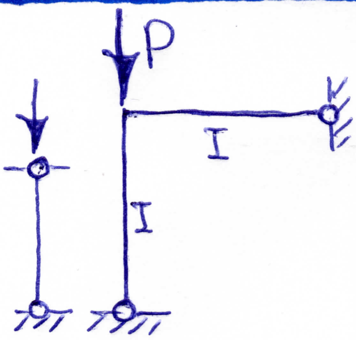
$$l_e = \frac{\pi l}{\lambda} = 0{,}7l$$

$$P_{kr} = \frac{\pi^2 EI}{(0{,}7l)^2} = 20{,}14\frac{EI}{l^2}$$

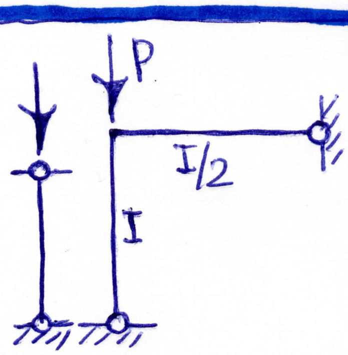
$$\lambda = \sqrt{\frac{Pl^2}{EI}}$$ 12/4



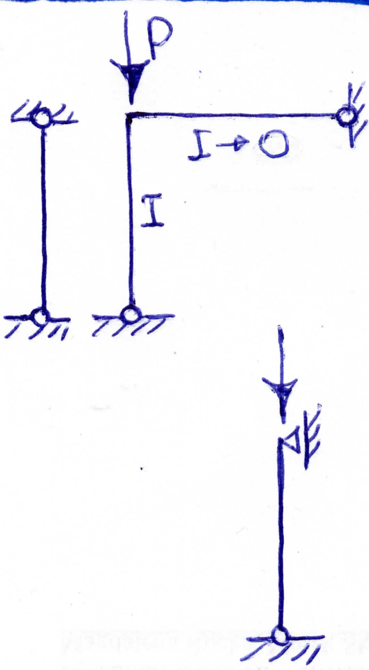
$$M_{1A} = \frac{EI}{L}\alpha'(\lambda)\varphi$$
$$M_{1B} = \frac{4EI}{L}\varphi$$
$$\Sigma M_1 = \frac{EI}{L}\left[\alpha'(\lambda)+4\right]\varphi = 0$$
$$\alpha'(\lambda) = -4 \Rightarrow \lambda = 3{,}83 \;,\; P_{kr} = 14{,}67\frac{EI}{L^2}$$
$$l_e = \frac{\pi L}{\lambda} = 0{,}82L$$



$$M_{1A} = \frac{EI}{L}\alpha'(\lambda)\varphi \qquad M_{1B} = \frac{3EI}{L}\varphi$$
$$\Sigma M_1 = \frac{EI}{L}\left[\alpha'(\lambda)+3\right]\varphi = 0$$
$$\alpha'(\lambda) = -3 \Rightarrow \lambda = 3{,}73 \;,\; P_{kr} = 13{,}91\frac{EI}{L^2}$$
$$l_e = \frac{\pi L}{\lambda} = 0{,}84L$$



$$M_{1A} = \frac{EI}{L}\alpha'(\lambda) \qquad M_{1B} = \frac{3EI}{2L}\varphi$$
$$\Sigma M_1 = \frac{EI}{L}\left[\alpha'(\lambda)+\frac{3}{2}\right]\varphi = 0$$
$$\alpha'(\lambda) = -1{,}5 \Rightarrow \lambda = 3{,}51 \qquad P_{kr} = 12{,}32\frac{EI}{L^2}$$
$$l_e = \frac{\pi L}{\lambda} = 0{,}90L$$



$$M_{1A} = \frac{EI}{L}\alpha'(\lambda) \qquad M_{1B} = 0$$
$$\Sigma M_1 = \frac{EI}{L}\cdot\alpha'(\lambda) = 0$$
$$\alpha'(\lambda) = 0 \Rightarrow \lambda = 3{,}14 \;,\; P_{kr} = 9{,}87\frac{EI}{L^2}$$
$$l_e = \frac{\pi L}{\lambda} = L$$