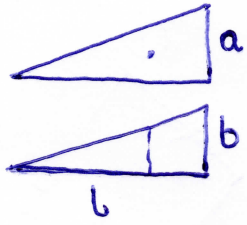
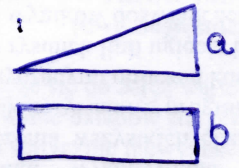


MECHANIKA BUDOWLI - wzupetnienie - **CATKOWANIE**

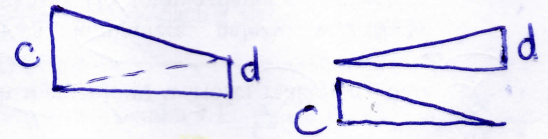
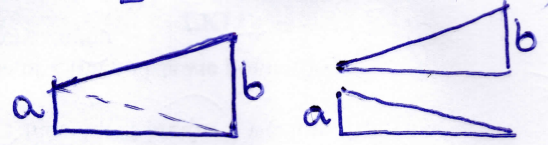
GRAFICZNE WYKRESOW ($I = \int_L F_1 F_2 dx$, podstawa L)



$$I = \frac{1}{2} a l \cdot \frac{2}{3} b = \frac{1}{3} a b l$$



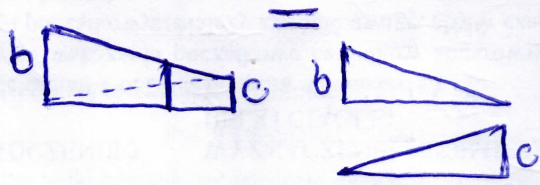
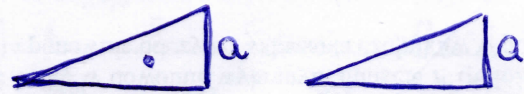
$$I = \frac{1}{2} a l \cdot b = \frac{1}{2} a b l$$



$$I = \frac{1}{3} a c l + \frac{1}{3} b d l + \frac{1}{6} a d l + \frac{1}{6} b c l$$



$$I = \frac{1}{2} a l \cdot \frac{1}{3} b = \frac{1}{6} a b l$$

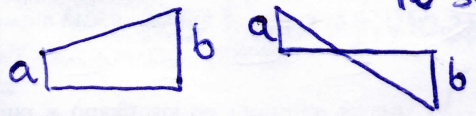


$$\begin{aligned} I &= \frac{1}{2} a l \cdot \frac{2}{3} c + \frac{1}{2} a l \cdot \frac{1}{3} b = \\ &= \frac{1}{3} a c l + \frac{1}{6} a b l \\ &= \frac{1}{2} a l \left(\frac{2}{3} c + \frac{1}{3} b \right) \end{aligned}$$

$$I = a l \frac{b+c}{2}$$

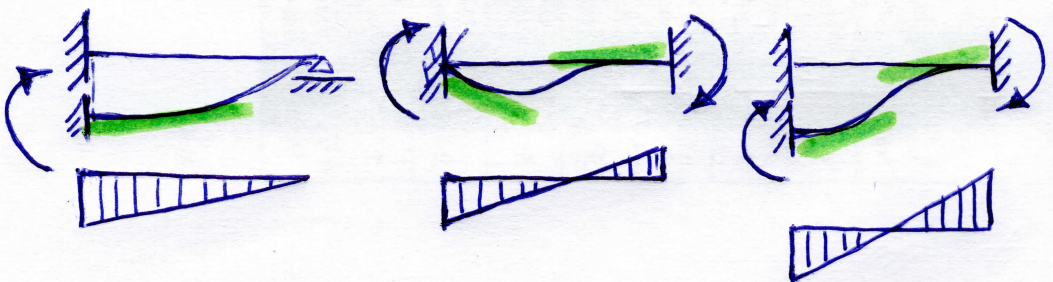
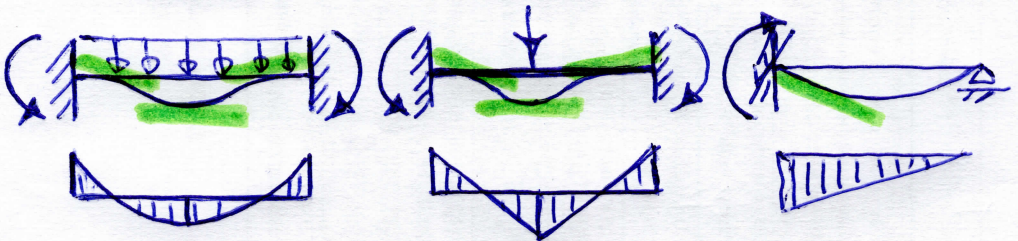
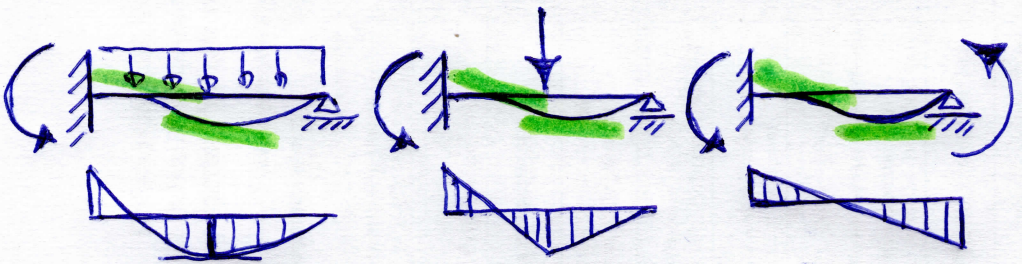
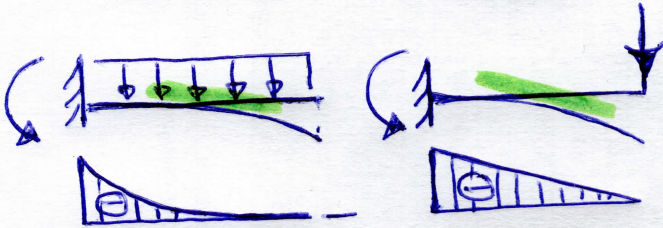
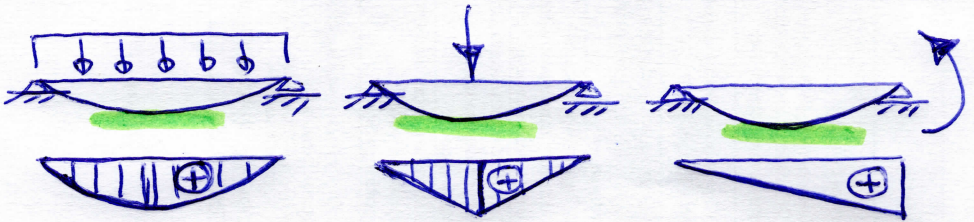
(ew. trapez na dwie trzjasty - to samo)

UWAGA:



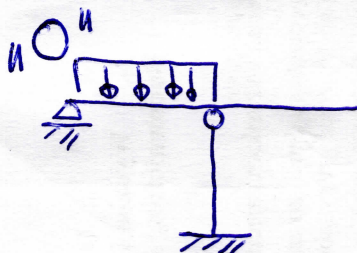
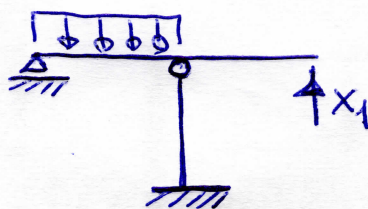
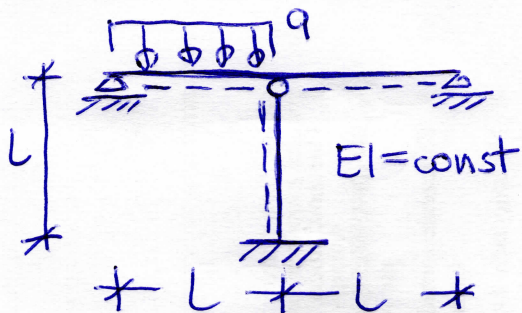
to samo

SZKICE LINII UGIĘCIA, ZGODNOŚĆ Z WYKRESAMI MOMENTÓW ZGINAJĄCYCH

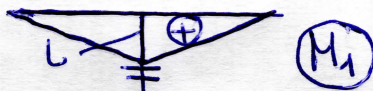
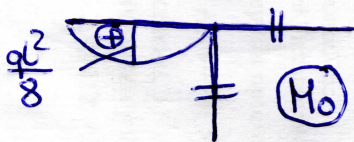
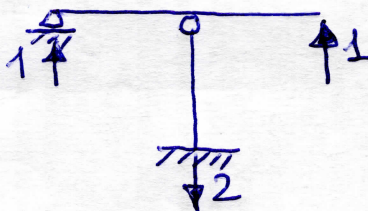


METODA SIŁ-ZADANIA WZUPERNIAJĄCE

$$t=2 \quad r=7 \quad n_s=7-6=1$$



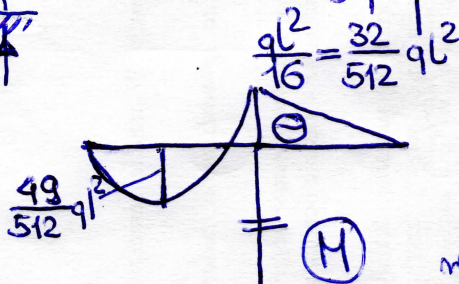
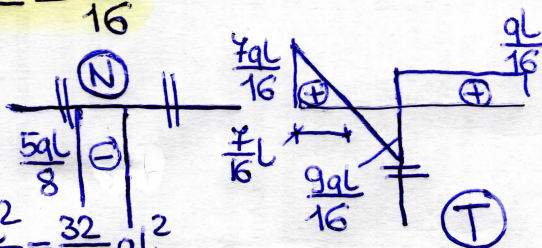
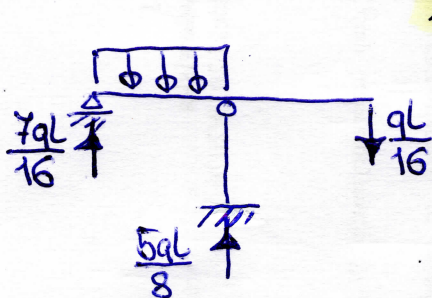
$$X_1 = 1$$



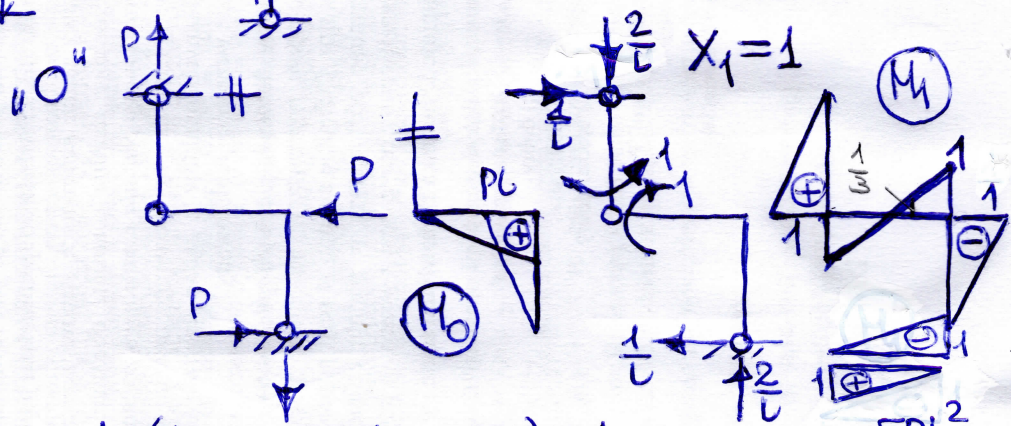
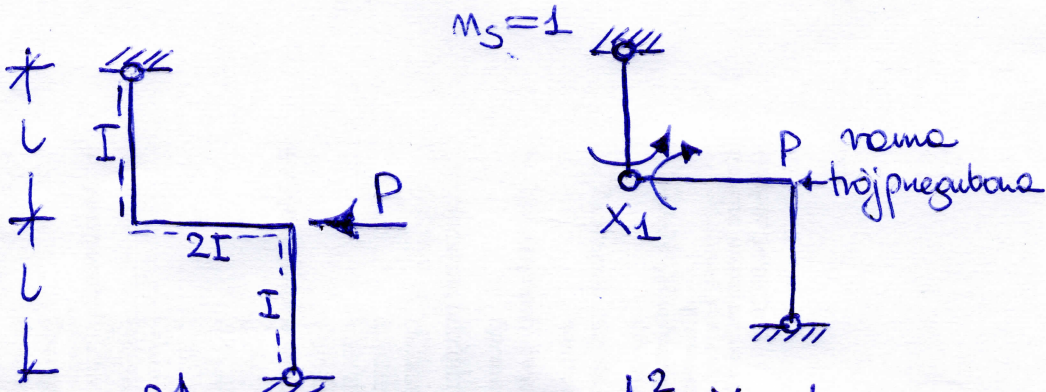
$$\delta_{10} = \frac{1}{EI} \cdot \frac{2}{3} \cdot L \cdot \frac{qL^2}{8} \cdot \frac{L}{2} = \frac{qL^4}{24EI}$$

$$\delta_{11} = \frac{1}{EI} \cdot 2 \cdot \frac{1}{3} \cdot L \cdot L \cdot L = \frac{2L^3}{3EI}$$

$$X_1 = -\frac{qL}{16}$$



Jest to praktycznie
rozciągające belkowe,
również - siła normalna
w elemencie pionowym

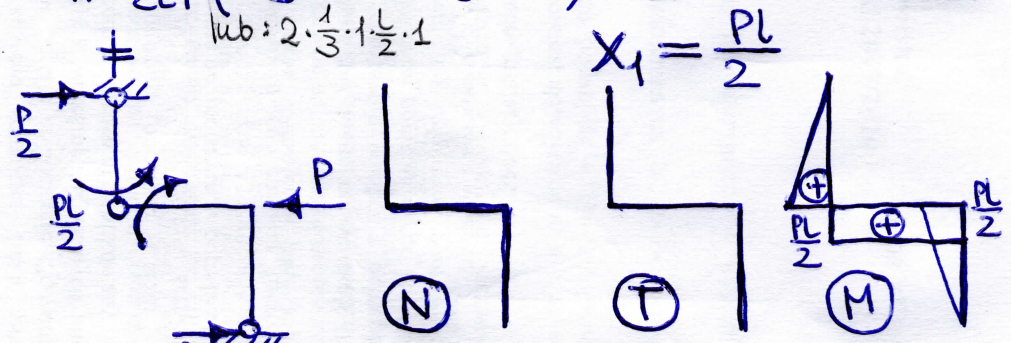


$$\delta_{10} = \frac{1}{2EI} \left(\frac{1}{6} \cdot L \cdot PL \cdot 1 - \frac{1}{3} \cdot L \cdot PL \cdot 1 \right) - \frac{1}{EI} \cdot \frac{1}{3} \cdot L \cdot PL \cdot 1 = -\frac{5PL^2}{12EI}$$

lub: $-\frac{1}{2} \cdot L \cdot PL \cdot \frac{1}{3}$

$$\delta_{11} = \frac{1}{2EI} \left(2 \cdot \frac{1}{3} \cdot L \cdot 1 \cdot 1 - 2 \cdot \frac{1}{6} \cdot L \cdot 1 \cdot 1 \right) + \frac{1}{EI} \cdot 2 \cdot \frac{1}{3} \cdot L \cdot 1 \cdot 1 = \frac{5L}{6EI}$$

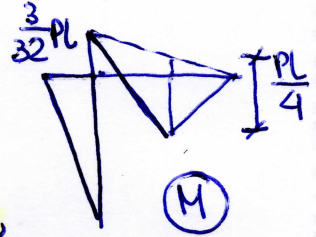
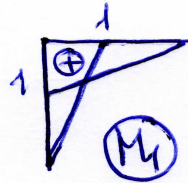
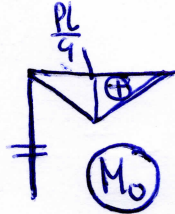
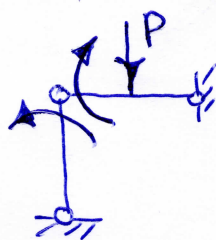
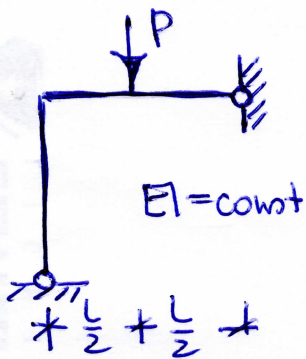
lub: $2 \cdot \frac{1}{3} \cdot 1 \cdot \frac{L}{2} \cdot 1$



gdy środkowy element o sztywności mEI :

$$\left. \begin{aligned} \delta_{10} &= \frac{-PL^2}{mEI} \cdot \frac{1}{6} - \frac{PL^2}{3EI} = \frac{-PL^2}{6EI} \left(2 + \frac{1}{m} \right) \\ \delta_{11} &= \frac{L}{mEI} \cdot \frac{1}{3} + \frac{2L}{3EI} = \frac{L}{3EI} \left(2 + \frac{1}{m} \right) \end{aligned} \right\} X_1 = \frac{PL}{2}$$

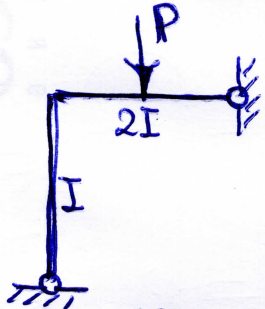
niezależnie od m



$$\delta_{10} = \frac{1}{EI} \cdot \frac{1}{2} \cdot L \cdot \frac{PL}{4} \cdot \frac{1}{2} = \frac{PL^2}{16EI}$$

$$\delta_{11} = \frac{1}{EI} \cdot 2 \cdot \frac{1}{3} \cdot L \cdot 1 \cdot 1 = \frac{2L}{3EI}$$

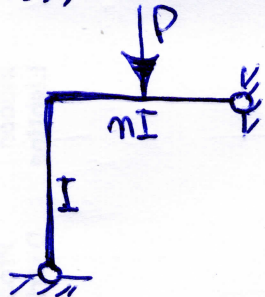
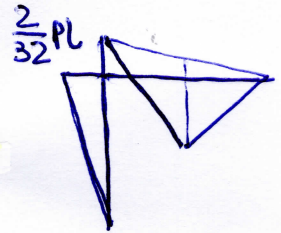
$$X_1 = -\frac{3}{32} PL$$



$$\delta_{10} = \frac{1}{2} \cdot \frac{PL^2}{16EI} = \frac{PL^2}{32EI}$$

$$\delta_{11} = \frac{L}{3EI} + \frac{1}{2} \cdot \frac{L}{3EI} = \frac{L}{2EI}$$

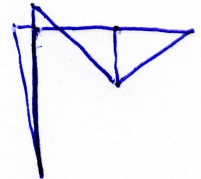
$$X_1 = -\frac{1}{16} PL$$



$$\delta_{10} = \frac{1}{n} \cdot \frac{PL^2}{16EI} = \frac{PL^2}{16nEI}$$

$$\delta_{11} = \frac{L}{3EI} + \frac{1}{n} \cdot \frac{L}{3EI} = \frac{n+1}{n} \cdot \frac{L}{3EI}$$

$$X_1 = -\frac{3}{16(n+1)} PL$$



$n \rightarrow \infty \Rightarrow$ sztywność słupa zdecydowanie mniejsza od sztywności belki, $X_1 \rightarrow 0$

