1. INTRODUCTION

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Structural reliability theory and its applications

Deterministic approach is still dominant in structural engineering.

Design calculations are characterized by the use of the following:

- specified minimum material properties,
- specified maximum load intensities,
- prescribed procedures for structural response computing.

Structural engineering design may be outlined by the following statements:

- the knowledge of the actual structural performance is insufficient,
- actual stresses are rarely known,
- deflections are rarely observed or monitored,
- **the real strength reserves are generally not known.**

Both the lack of information on the actual behaviour of structures and relatively high safety factors introduced in the codes may lead to **a view that absolute safety can be achieved.**

Of course, absolute safety is unobtainable and undesirable, since infinite resources are necessary to achieve it.

Some risk of unacceptable structural performance must be tolerated.

The main object of structural design is to ensure, at an acceptable probability level, that a given structure serves its intended purpose at any time during its specified design lifetime.

The problem is much more complex than to specify just a single probability.

As a means of assessing the relative importance of structural failures as a cause of death, some comparative statistics are given.

Activity/Cause	Number of deaths per hour per 10^8 persons
Mountaineering (International)	2700
Air travel (International)	120
Deep water trawling	59
Car travel	56
Coal mining	21
Construction sites	7.7
Manufacturing	2.0
Accidents at home (all)	2.1
Accidents at home (able-bodied persons)	0.7
Fire at home	0.1
STRUCTURAL FAILURES	0.002
Natural causes (average, all ages)	129
Males aged 30 (all causes)	15
Females aged 30 (all causes)	13
Males aged 50 (all causes)	84
Females aged 50 (all causes)	51

Table 1.1 Comparative death risk. [Average 1970-1973 in U.K. based on Central Statistical Office, Abstract 1974].

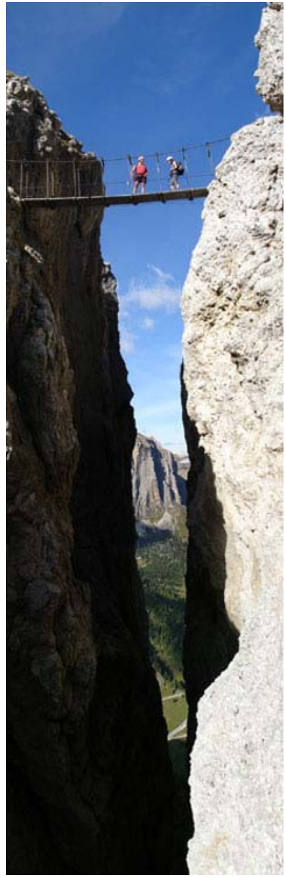
These figures show that the risk to life from structural failures may be considered negligible.

For the 3 year period reported, the average number of deaths per annum directly attached to structural failure was 14, with almost equal extent of failures during construction and failures of completed structures.

In order to assess the importance of structural failures, **economic consequences of collapse** and unserviceability **should be noted.**

In fact **economic aspects are dominant**
since the marginal returns in terms of lives saved for each additional amount of money invested in improving the safety of structures may be small in comparison with the benefits of investing the same sum in, say, road safety or health care.

Voluntary risks of higher grade are accepted worldwide, but they should not be taken into account while considering structural safety.



1.1 Structural Codes

The deterministic format of structural codes describes minimum standards for design, construction and workmanship for various types of structures.

The majority of codes evolve in time to allow for the following updates:

- analysis of new types of structures,
- the effects of improved understanding of structural behaviour,
- the effects of tolerance changes or quality control procedures,
- better knowledge and modelling of structural actions.

The recent generation of structural codes (Eurocodes) is more scientific in its format than the previous documents.

The benefits of these improved codes are as follows:

- increased overall safety for the same construction costs,
- the same or more consistent safety levels of safety with reduced construction costs,
- a combination of these two.

A further aim is the trend to create design procedures related, with a high confidence, to completely new forms of construction without the prior need for prototype testing.

These aims and benefits can only be achieved by a rational assessment of various uncertainties inherent to structures and a study of their interactions.

This is the essence of structural reliability analysis.

1.2 UNCERTAINTY

All quantities (except physical and mathematical constants) that currently enter into engineering calculations are in reality associated with some uncertainty.

This fact has been implicitly recognized in current and previous codes.

PN-ISO 2394.
General reliability rules for civil engineering structures:
“If the **uncertainty** of a given basic variable is considered significant … it should be represented by a **random variable**.”

The magnitudes of all variables are either bounded or restricted within specified limits by the appropriate control standards.

These bounding values should be the basis for design.

In structural engineering, however, such a statement is inappropriate for a number of reasons:

- **upper limits to individual loads and lower limits to material strength are not easily identified in practice** (e.g. building occupancy loads, wind loads, yield stress of steel, cube or cylinder strength of concrete),
- even if such natural limits exist, **their direct use in design is extremely uneconomic**,
- **limits imposed by quality control and testing are not completely effective**, particularly in the case of parameters assessed by destructive tests only or in the case of variations affected by sampling and use of material (e.g. concrete),
- even if recognizable limits exist, **their use may be irrational**.

Example

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Consider a column supporting *n* floors of a building,
the loads are varying independently in time.

The load on each floor is physically restricted by a hypothetical fail-safe device so that it cannot exceed a specified maximum value, **each load stays at this maximum value for 1% of its time**.

The rational design load for the column is generally less than the sum of the maximum values of component loads.

This design load will, of course, depend on the number of storeys supported and the design life of the structure.

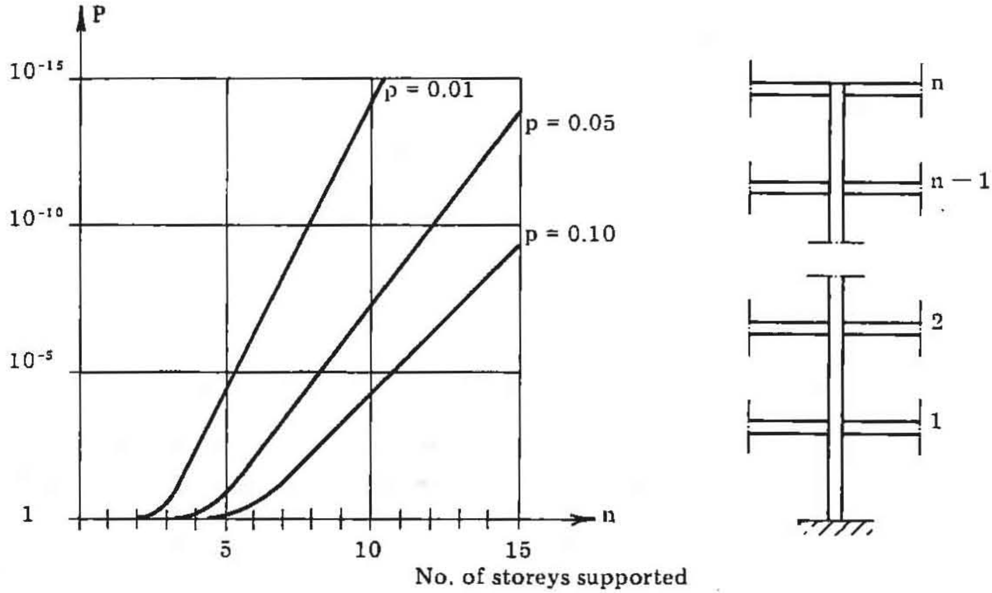

Figure shows the probabilities *P* that **the maximum column load at ground level will reach the sum of the maxima of the individual floor loads** (i.e. the maximum possible column load) at some time during a 50 year period.

Assumptions:

- floor loads are independent,
- the loads remain constant for an hour and then switch to a new random value,
- there is a 1% chance ($p = 0.01$) for each load to reach its maximum value after each renewal (i.e. each floor is loaded to its maximum value for approximately 1% of the time).

The figure shows that even taking six floors the probability of maximum column load occurrence in 50 years is equal to 10^{-5} .

Even in the case of each floor loaded to its maximum value for about 10% of the time ($p = 0.1$), the probability of maximum column load occurring is still very small, taking 10 or more supported floors.

In such cases it would be irrational and uneconomic to design for the worst possible combination.

Basic Variables

In order to quantify uncertainties in the field of structural engineering and for further reliability analysis it is necessary to define a set of **basic variables**.

They are basic parameters governing static or dynamic structural response.

Examples of structural basic variables:

- mechanical properties of materials,
- structural dimensions,
- unit weights of materials,
- intensities of environmental loads,
- live load values

The “basic” attribute means that they are the most fundamental parameters recognized and used by designers and analysts in structural analysis.

It is generally impracticable to collect sufficient statistical data to model the variations in the strength of entire structural components directly.

An idealized view regards all basic variables of the problem **statistically independent**.

This is not always available, e.g. if the strength of a structure is dependent on different mechanical properties known to be correlated, e.g. the tensile strength and the compressive strength of a batch of concrete.

Types of Uncertainty

Physical, statistical and model uncertainty are encountered in structural analysis.

Physical uncertainty

The reliability analyst has to be accustomed with the nature of variability of physical parameters, such as loads, material properties and dimensions.

This variability can be described in terms of probability distributions or stochastic processes.

Physical variability can be quantified only by examining sample data.

Since the sample sizes are limited by practical and economic reasons some uncertainty is still achieved.

This practical limit gives rise to the so-called ***statistical uncertainty***.

Statistical uncertainty

Statistics, compared to probability, concerns inferences drawn from sample observations.

Data may be collected for the purposes of building a probabilistic model of physical variability of a given parameter.

The model involves selection of a relevant probability distribution and determination of numerical values for its parameters.

Common probability distributions are described by a number (one to four) of parameters.

In practice sufficiently large samples are required to establish reliable estimates of the numerical values of these parameters.

The distribution parameters are therefore random variables, their uncertainty depends on the sample space volume and any prior knowledge.

This uncertainty is termed ***statistical uncertainty***, unlike physical variability it arises solely from the lack of information.

Model uncertainty

Both structural design and analysis make use of mathematical models relating the desired outcomes (e.g. the deflection at the mid-span of a reinforced concrete beam) to the values of input basic variables (e.g. load intensities, modulus of elasticity, duration of loading, etc.).

These models are generally deterministic in their form (e.g. linearly elastic structural analysis) although they may be probabilistic (e.g. peak response of an offshore structure to stochastic wave loading).

They may be based on an intimate understanding of mechanics of the problem (e.g. plastic collapse analysis of a steel portal frame) or they may be highly empirical (e.g. punching shear at tubular joint connections in offshore jacket structures).

It is rarely possible to make highly accurate predictions about the magnitude of the response of typical civil engineering structures to loading even when the governing input data are known.

Additional source of uncertainty is termed *model uncertainty* and occurs as a result of simplifying assumptions, unknown boundary conditions and as a result of the unknown effects of other variables and their interactions not included in the model.
For example, shear strength of nominally similar reinforced concrete beams exhibits considerable scatter even when due allowance has been made for the various known differences between test specimens.
The model uncertainty associated with a particular mathematical model may be expressed in terms of the probability distribution of a variable X_m defined as

$$X_m = \frac{\text{actual strength (response)}}{\text{predictal strength (response) using model}} \tag{0.1}$$

For a number of components and structures model uncertainties act strongly on structural reliability and should not be neglected.