Random imperfections of structural elements

Antoni Biegus "Probabilistyczna analiza konstrukcji"
(Probabilistic analysis of structures) PWN 1999

Deterministic approach to civil engineering design is based on the following assumptions:

- idealized structural geometry – cross-sectional shapes and the axes of elements,
- material homogeneity,
- initial state of cross-sections free of stresses (no initial stresses before loading),
- perfect manufacturing, workmanship and assemblage,
- proper operation, due to the design recommendations.

Ideal structures are usually taken for the load-carrying capacity assessment, but the **real structures may exhibit initial defects or discrepancies, called imperfections**.

In general, structural, technological and geometric imperfections may be encountered.

Their origin may be as follows:

– the process of element shaping (e.g. metallurgical process of element forming may produce initial stresses),
– the assemblage of elements into structures or substructures (e.g. assembly of component elements into a substructure may produce geometrical imperfections),
– transportation imperfections, assembly at the building site.

Examples of elements subjected to initial imperfections:

– components of bar elements (e.g. plate sheets – components of plate girders),
– assembly subsystems (e.g. columns, trusses),
– entire load-carrying systems (load-carrying frames).

Imperfections are random in nature, but many kinds of imperfections have been described thoroughly up till now.

The distinction is done for **computational** sake:

– random imperfections of cross-sectional resistance parameters, $w(\omega)$,

– random imperfections of cross-sectional geometry, $C(\omega)$,

– random coefficient $\alpha(\omega)$ for the element effort model.

The coefficient $\alpha(\omega)$ covers geometric imperfections of axes (planes) and technological stress imperfections of bars.

The standard steel design procedures usually link random geometry of a cross-section with a random strength of a material.

The **other initial imperfections** of geometry, construction, technology and assembly are covered by instability (buckling) coefficients or the structural analysis incorporating imperfections directly.

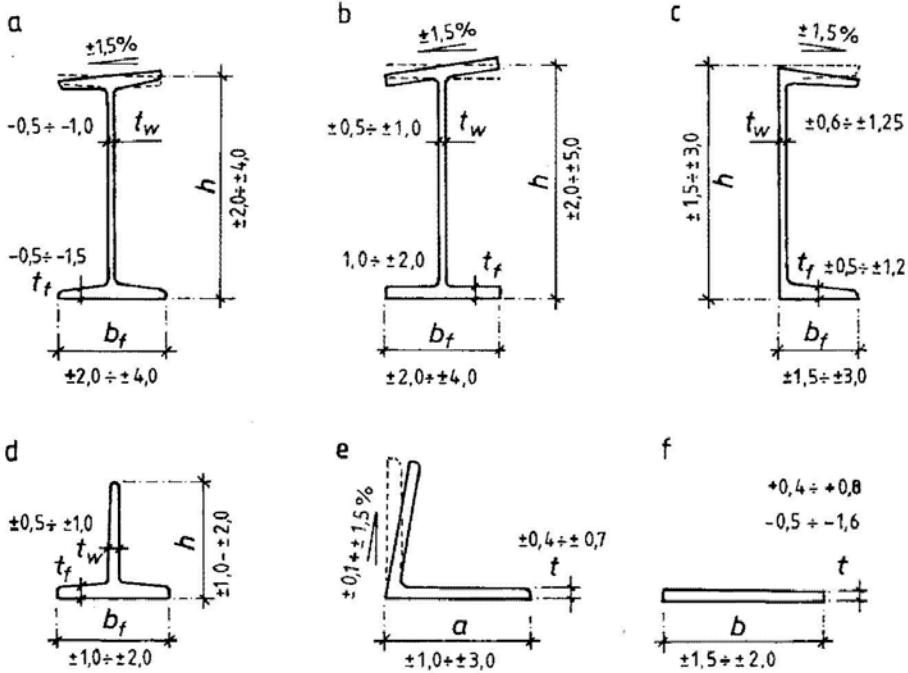
Random imperfections of cross-sectional dimensions of bars

The $C(\omega)$ coefficient captures **geometrical deviations of cross-sectional dimensions** of bars, from their nominal values.

Standards and catalogues of structural components (e.g. metallurgical products) are based on the so-called nominal values of dimensions, geometrical and stiffness parameters.

In fact the cross-sectional dimensions are randomly imperfect, usually bounded by the allowable values of **fabrication tolerances of element profiles**.
These allowable tolerances of dimensions of bars and two-dimensional elements, with respect to their nominal values, are specified in standards, not to be exceeded by the manufacturers. They make up the **quality check of metallurgic products.**

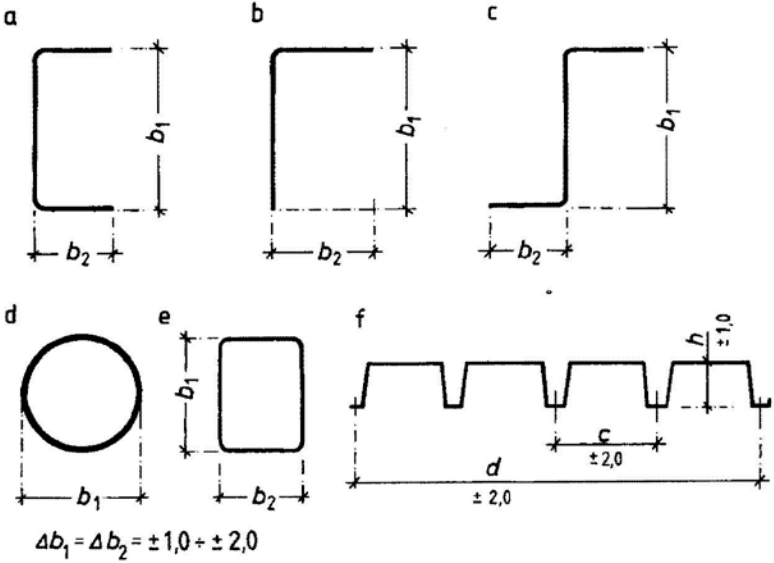
Fig.4.1 shows the cross-sectional geometry tolerances for hot-rolled sections, according to Polish metallurgic standards.



Rys. 4.1. Tolerancje wymiarów geometrycznych przekrojów poprzecznych wyrobów walcowanych na gorąco według polskich norm hutniczych

These standards allow for the following imperfections of hot-rolled profiles: dimensional imperfections of web and flanges, non-orthogonality of flange-web joints, torsion of the section components.

Fig. 4.2. shows the dimensional tolerances of cold formed profiles.



Rys. 4.2. Tolerancje wymiarów geometrycznych przekrojów poprzecznych wyrobów giętych na zimno

The Polish metallurgical standards allow for dimensional imperfections of cross-sections of ordinary and straight-flange I-sections, the allowable values are shown in Table 4.3

Tablica 4.3. Dopuszczalne tolerancje wykonania dwuteowników normalnych i równoległościennych według polskich norm hutniczych

<table>
<tr><th colspan="5">Dwuteowniki normalne
według PN-91/H-93407</th></tr>
<tr><th rowspan="2">Wyróżnik oznaczenia I</th><th colspan="4">Dopuszczalne odchyłki [mm]</th></tr>
<tr><th>h</th><th>b_f</th><th>t_w</th><th>t_f</th></tr>
<tr><td>80–140</td><td rowspan="3">±2,0</td><td rowspan="2">±1,5</td><td rowspan="5">±0,5</td><td>−0,5</td></tr>
<tr><td>160</td><td rowspan="5">−1,0</td></tr>
<tr><td>180–200</td><td rowspan="2">±2,0</td></tr>
<tr><td>220</td><td rowspan="6">±3,0</td></tr>
<tr><td>240–260</td><td rowspan="2">±2,5</td></tr>
<tr><td>280–300</td><td rowspan="2">±0,6</td></tr>
<tr><td>320</td><td rowspan="6">±3,0</td><td rowspan="6">−1,5</td></tr>
<tr><td>340–380</td><td>±0,7</td></tr>
<tr><td>400</td><td rowspan="2">±0,8</td></tr>
<tr><td>425</td><td rowspan="3">±4,0</td></tr>
<tr><td>450–500</td><td>±0,9</td></tr>
<tr><td>550–600</td><td>±1,0</td></tr>
</table>

<table>
<tr><th colspan="5">Dwuteowniki równoległościenne
według PN-91/H-93406</th></tr>
<tr><th rowspan="2">Wyróżnik oznaczenia IPE</th><th colspan="4">Dopuszczalne odchyłki [mm]</th></tr>
<tr><th>h</th><th>b_f</th><th>t_w</th><th>t_f</th></tr>
<tr><td>80–120</td><td>±2,0</td><td>±2,0</td><td>±0,5</td><td>±1,0</td></tr>
<tr><td>140–180</td><td>+3,0
−2,0</td><td>+3,0
−2,0</td><td rowspan="2">±0,75</td><td rowspan="2">±1,5</td></tr>
<tr><td>200–270</td><td rowspan="3">±3,0</td><td rowspan="2">+3,0</td></tr>
<tr><td>300–360</td><td rowspan="4">±1,0</td><td rowspan="4">±2,0</td></tr>
<tr><td>400</td><td rowspan="3">±4,0</td></tr>
<tr><td>450–500</td><td>±4,0</td></tr>
<tr><td>550–600</td><td>±5,0</td></tr>
</table>

Tablica 4.4. Tolerancje wymiarowe dwuteowników szerokostopowych według wymagań zachodnioeuropejskich [27]

<table>
<tr><th rowspan="2">Szerokość półki</th><th rowspan="2">Wysokość dwuteownika</th><th colspan="5">Odchyłki</th></tr>
<tr><th>Szerokość pasa</th><th>Wysokość profilu</th><th>Grubość półki</th><th>Grubość środnika</th><th>Powierzchnia przekroju</th></tr>
<tr><td>mm</td><td>mm</td><td>mm</td><td>mm</td><td>mm</td><td>nm</td><td>%</td></tr>
<tr><td rowspan="2">160</td><td>160</td><td rowspan="8">±3,0</td><td>+4,0
−2,0</td><td rowspan="2">±1,5</td><td rowspan="3">±1,0</td><td rowspan="8">±6,0</td></tr>
<tr><td>220</td><td rowspan="4">±3,0</td></tr>
<tr><td>220</td><td>260</td><td rowspan="6">±2,0</td></tr>
<tr><td>260</td><td>300</td><td rowspan="4">±1,5</td></tr>
<tr><td>300</td><td>400</td></tr>
<tr><td>400</td><td>500</td><td>±4,0</td></tr>
<tr><td>500</td><td>700</td><td rowspan="2">±5,0</td></tr>
<tr><td>700</td><td>1000</td><td>±2,0</td></tr>
</table>

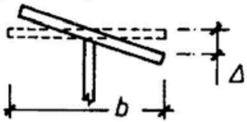
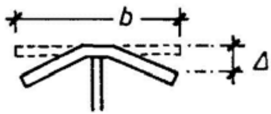
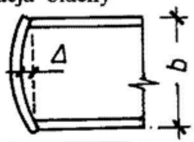
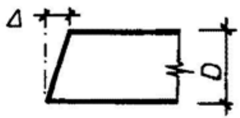
Table 4.4 shows dimensional tolerances of web-flange I-sections according to Western European requirements.

The allowable dimensional imperfections shown in tables 4.3 and 4.4 converge.

The Polish metallurgical standards allow for fabrication imperfections of an I-section depth (2,0-5,0 mm),
flange width (1,5-4,0 mm), web-flange non-orthogonality (up till 1,5%),
and thickness of plates (0,5-2,0 mm).
Metallurgical standards make these fabrication imperfections depend on the I-section depth.

The imperfections similar to those shown in Fig. 4.1 may occur in the load-carrying elements (beams, columns) of welded sections, made of sheets of random dimensions.

Tablica 4.5. Wybrane odchyłki dopuszczalne według PN-B-06200:1997 [104]

Nr	Rodzaj odchyłki	Parametr	Odchyłka dopuszczalna
1	Prostopadłość pasów 	Odchyłka od kąta prostego	$\lvert\Delta\rvert$ = większa z: (0,01b; 5 mm)
2	Płaskość pasów 	Odchyłka od płaskości	$\lvert\Delta\rvert$ = większa z: (b/150; 3 mm)
3	Deformacja blachy 	Deformacja Δ blachy	$\lvert\Delta\rvert$ = większa z: (b/150; 3 mm)
4	Prostokątność krawędzi 	Prostopadłość do osi podłużnej: – część nieobrobiona dla podparcia dociskowego, – część obrobiona dla podparcia dociskowego	$\Delta = \pm D/300$ $\Delta = \pm D/1000$

Standards allow for the following fabrication imperfections of welded cross-sections of girders:

- non-orthogonality of flange-web joints (table 4.5, 1st row),
- non-planar flanges (table 4.5, 2nd row),
- flange – web eccentricity $\Delta = \pm 5\ \text{mm}$,
- variation of depth h of a section

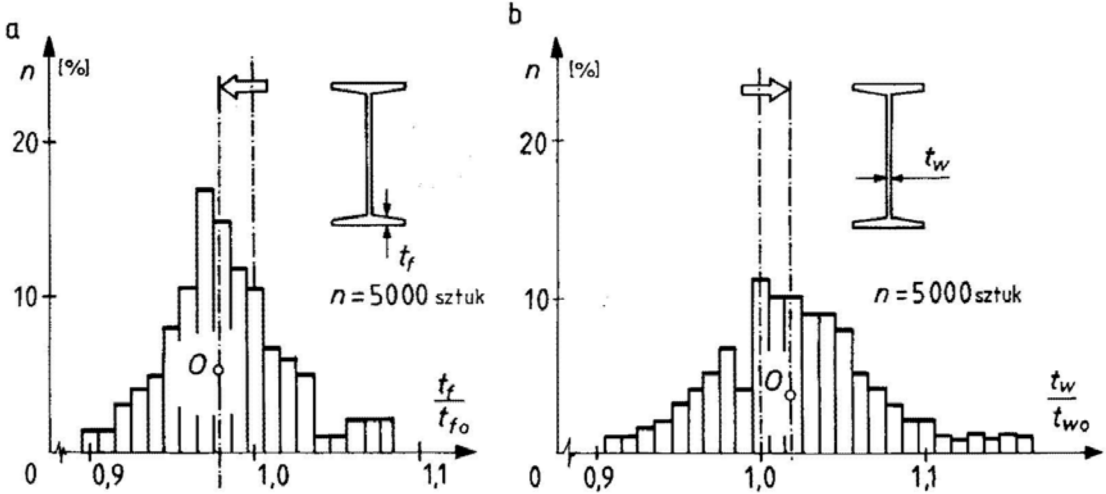
$$\Delta = \begin{cases} \pm 3\ \text{mm} \quad \text{dla } h \le 900\,\text{mm} \\ \pm 5\ \text{mm} \quad \text{dla } 900 \le h \le 1800\,\text{mm} \\ +8\ \text{mm};\ -5\ \text{mm} \quad \text{dla } h > 1800\,\text{mm} \end{cases}$$

The cross-sectional imperfections of hot-rolled metallurgic profiles.

Variation of flange width and depth of I-beams, angle and channel sections is relatively small while variation of thickness of their flanges and webs is relatively high.

The population experiments show that flange thickness of I-sections tends to decrease, while web thickness of I-sections tends to increase.

Figure 4.3. shows the histograms of thickness of flanges and webs of a population of 5000 hot-rolled I-beams.



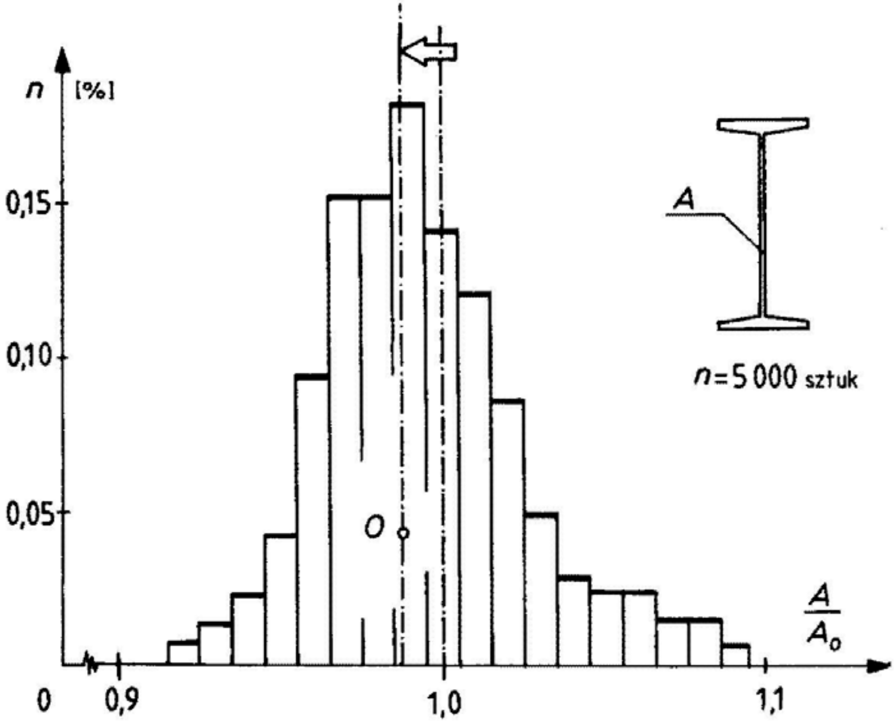
Rys. 4.3. Histogram grubości pasów (a) i grubości środników (b) dwuteowników walcowanych [1]

The coordinate of a centroid *O* of the **flange thickness histogram** (Fig. 4.3a) is less than unity, so the mean flange thickness is less than the nominal one.

The coordinate of a centroid *O*of the web thickness histogram (Fig. 4.3a) is greater than unity, so the mean web thickness is greater than the nominal one.

Reduction of a flange thickness produces decrement of moment of inertia and section modulus of a cross-section, consequently its bending load-carrying capacity, both elastic and plastic, related to the section of nominal dimensions.

Figure 4.4 shows the histogramof cross-sectional area, of a 5000 population of hot rolled I-beams,based on their web and flange variation studies, shown in Fig. 4.3.



Rys. 4.4. Histogram pola przekroju poprzecznego dwuteowników walcowanych [1]

The coordinate of a centroid O of the cross-sectional are a histogram (Fig. 4.4) is less than unity, so the mean cross-sectional area is less than the nominal one.

Reduction of a cross-sectional area produces decrement of axial load-carrying capacity of a cross-section, related to the case of nominal dimensions.

The histograms of other cross-sectional geometric parameters(e.g. $i(\omega)$, $W(\omega)$, $J(\omega)$, $W_{pl}(\omega)$ resemble those shown in Fig. 4.4.

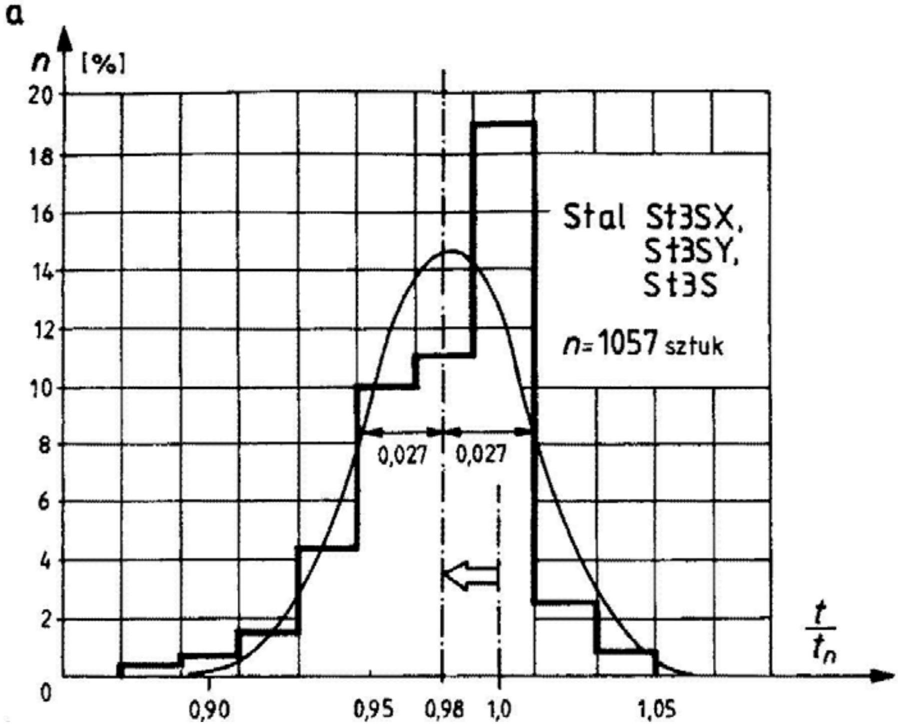
The magnitudes of their variations are experimentally proved to be high enough to incorporate them in the assessment of random load-carrying capacity of elements.

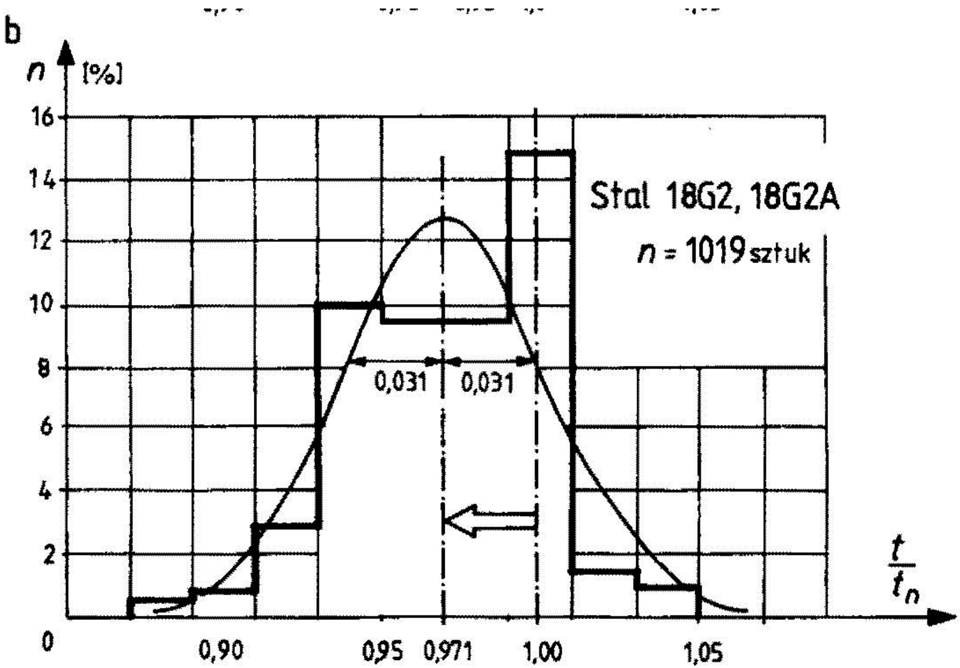
The dimensions of welded cross-sections are strongly affected by random variation of metal sheet thickness.

The case study of welded plate girders, of a thickness range 6–50 mm, results in the 0.2-0.4 mm thickness standard deviation and low-valued, 1-4%corresponding coefficients of variations.

The statistical variation of thickness of metallurgical steel products intended to civil engineering steel structures is estimated to approx. $v_t = 6{,}0\%$.

Variation of thickness *t* of a number of approx. 1000 steel sheets of grades St3 and 18G2 is shown in Fig. 4.5


a
n [%]
20
18
16
14
12
10
8
6
4
2
0
Stal St3SX,
St3SY,
St3S
n=1057 sztuk
0,027
0,027
t/tn
0,90
0,95
0,98
1,0
1,05

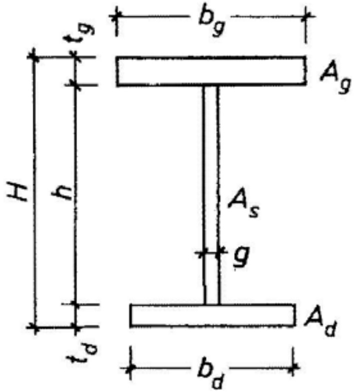


Rys. 4.5. Wyniki badań grubości *t* blach gatunku St3 i 18G2

The coordinate of a centroid *O* of the presented sheet thickness histograms (Fig. 4.5) is less than unity, so the mean thickness is less than the nominal one.

In the case of welded cross-sections, composed of sheets, relatively low coefficients of variation of flange and web thickness may act significantly on the random cross-sectional parameters.

In the following parts geometric parameters are analysed
of a welded I-section composed of sheets, as shown in Fig. 4.6



Rys. 4.6. Dwuteowy przekrój poprzeczny pręta spawanego z blach

Fig. 4.6 Welded I-section composed of plate sheets

The dimensions of sheets are assumed random, Gaussian distributed.

Random cross-sectional areas of plate sheets, $A_i(\omega)$ - components of the I-section (Fig. 4.6), are expressed by the formula

$$A_g(\omega) = b_g(\omega)t_g(\omega) \tag{1}$$

$$A_s(\omega) = h_s(\omega)g(\omega) \tag{2}$$

$$A_d(\omega) = b_d(\omega)t_d(\omega) \tag{3}$$

The mean values and standard deviations of cross-sectional areas of plates are equal to

$$\overline{A}_g = \overline{b}_g \overline{t}_g \tag{4}$$

$$\overline{A}_s = \overline{h}_s \overline{g} \tag{5}$$

$$\overline{A}_d = \overline{b}_d \overline{t}_d \tag{6}$$

$$s_{A_g} = \sqrt{s_{b_g}^2 + s_{t_g}^2} \tag{7}$$

$$s_{A_s} = \sqrt{s_h^2 + s_g^2} \tag{8}$$

$$s_{A_d} = \sqrt{s_{b_d}^2 + s_{t_d}^2} \tag{9}$$

where

$\overline{b}_g, \overline{h}, \overline{b}_d, \overline{t}_g, \overline{g}, \overline{t}_d$ – mean values of width and thickness of each plate,

$s_{b_g}, s_h, s_{b_d}, s_{t_g}, s_g, s_{t_d}$ – standard deviations of width and thickness of each plate.

Random cross-sectional area of an I-beam is a sum

$$A(\omega) = A_g(\omega) + A_s(\omega) + A_d(\omega) \tag{10}$$

Its mean value and standard deviation formulae (Fig. 4.6) hold the Gaussian random variable algebra

$$\overline{A} = \overline{A}_g + \overline{A}_s + \overline{A}_d \tag{11}$$

$$s_A = \sqrt{s_{A_g}^2 + s_{A_s}^2 + s_{A_d}^2} \tag{12}$$

Assuming both mean areas of flanges equal to $\bar{A}_p$, simplified form of the total area arises

$$\bar{A} = 2\bar{A}_p + \bar{A}_s \tag{13}$$

Standard deviation of the total area is obtained in two variants
– uncorrelated areas of both upper and lower flange
$(\rho_{A_g, A_p} = 0)$

$$s_A = \sqrt{2s_{A_g}^2 + s_{A_s}^2} \tag{14}$$

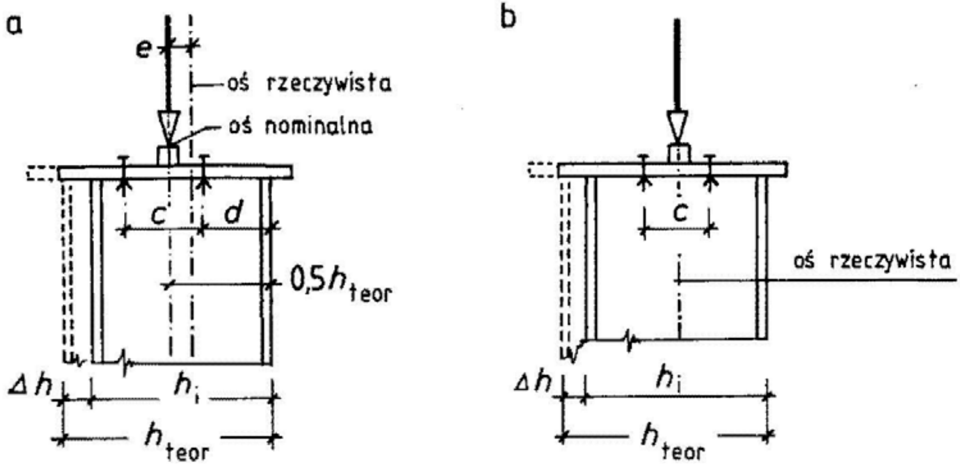
– correlated areas of both flanges
$(\rho_{A_g, A_p} = 1)$

$$s_A = \sqrt{4s_{A_g}^2 + s_{A_s}^2} \tag{15}$$

where S_{Ap} is a standard deviation of a single flange area.
The random cross-sectional geometrical parameters may be estimated with the use of computational tools.

Geometrical imperfections of metallurgic products yield the undesired strength and assembly effects in the case of neglecting them on **technical drawings**.
It is a common fault in engineering graphics not to mention geometric and operational imperfections, due to fabrication and fastening of steel elements.
This action usually leads to deterioration of structural elements – **decrement of both quality and bearing capacity**.
There are cases of structural failures caused by an improper graphical image of a structure.

The illustrative **example concerns the head of an I-section** of a column, transferring compressive load by means of a centering element (Fig. 4.9a)



Rys. 4.9. Błędne (a) i poprawne (b) wymiarowanie geometryczne głowicy słupa

Fig. 4.9 Failed (a) and proper (b) dimensioning of a column head

The demand is imposed on the contractor to locate the centering element at half the theoretical depth of a hot-rolled / welded I-section, the depth tolerance is equal to Δh.

This yields non-axial load transfer and, consequently, decrement of the limit load – in fact, both compression and bending occur.

Assembly problems also arise, while the bolt holes do not coincide with their theoretical position.

The static structural model is altering too, for the idealized state does not consider any assembly deviations.