2. 3. Simulation methods – the Monte Carlo method

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Reliability of engineering structures (in Polish)
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Structural reliability assessment by means of the Monte Carlo simulation method (MCS):

- generate a sequence of random numbers / random fields due to every random involved in the reliability analysis,
- state a reliability measure, being an outcome of physical experiments,
- classify the results to the zones of reliable or failed states,
- performing a sufficiently great population of realizations (N) compute the ratio of failed cases N_I to the total population N,
- the ratio $q = N_I / N$ is a structural unreliability measure, reliability $Q = 1 - q$).

Sufficient accuracy in reliability estimation requires a high population $N \geq (25 \div 100)q^{-1}$, e.g. 10000 ($q$ – anticipated failure probability)
Satisfactory results for practical problems may be achieved by means of a relatively low number of realizations, e.g. 10-30.

The Monte Carlo method is convenient for any structures, including those of nonlinear vector of structural performance, producing gross errors while linearized statistically.

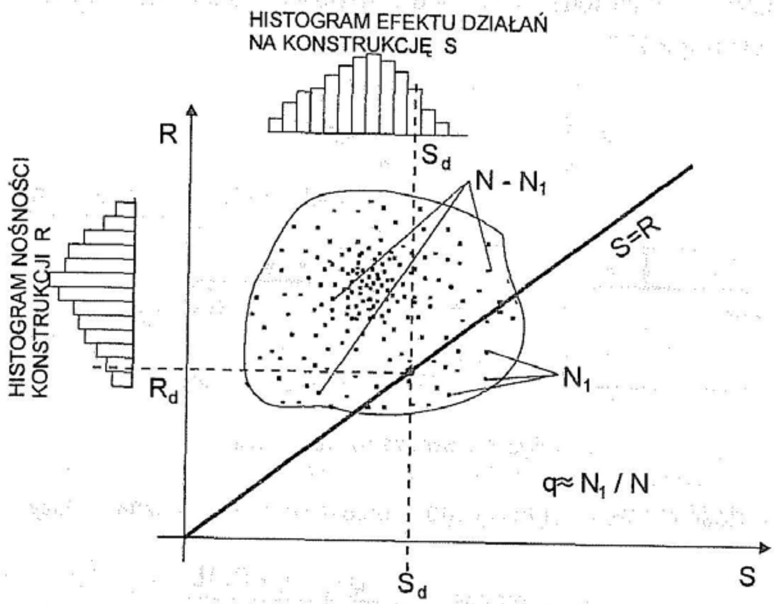
The advanced Monte Carlo techniques require a more comprehensive data on structural performance, including the failure regions. These methods improve significantly the result convergence. Examples are: importance sampling, directional, stratified, adaptive sampling, Latin Hypercube sampling.

The MCS was applied in a straightforward way to assess the structural reliability within the TERECO project (coordinator: prof. Pavel Marek, software: Milan Guštar - Prague, Polish participants: prof. Szczepan Woliński, prof. Ryszard Kowalczyk)

The assumptions prior to software making:

- all random variables: basic (material and geometric parameters, imperfections, structural actions), compound (multidimensional, correlated, action effects, limit parameters, etc.) and the results (reliability / safety, durability, serviceability measures, economic measures) are represented by bar histograms (Fig. 4.7),
- reliability check is the comparison of computed probability of failure / exceeding the limit values (ratio of the failed cases to the whole population) with the allowable probabilities.

The concept is illustrated in Fig. 4.7. (fundamental reliability case – two random variables: load effect *S* and resistance *R*).



Rys. 4.7. Symulacja Monte Carlo w dwuwymiarowej przestrzeni zmiennych losowych S i R

Fig. 4.7. Monte Carlo simulation in a two-dimensional space of S and R variables

The MCS package created by P. Marek and M. Guštar includes five procedures:

- LoadCom – load combination analysis, design loads due to various standards,

- M-Star – solver for a large family of algebraic, logarithmic, exponential and trigonometric equations composed of maximum 30 random variables expressed by bar histograms. It is a tool to analyze problems of load-carrying capacity assessment of elements and structures, load effect combinations (e.g. cross-sectional forces), failure probability, damage accumulation and serviceability criteria,

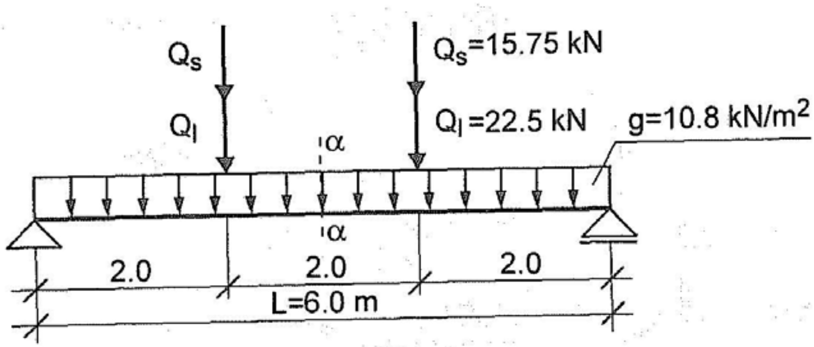
- AntHill – two- and multi-dimensional random variable analysis, e.g. reliability assessment, cross-sectional forces due to complex actions,

- DamAc – the impact of load duration to fatigue resistance of structures and reliability assessment incorporating rheological material phenomena,

- ResCom – structural load analysis.

Example

Apply the Monte Carlo method to compute the bending moment at a critical section $\alpha - \alpha$ of a beam, shown in Fig. 4.8 whose probability of exceedance is $q = 10^{-4}$.



Rys. 4.8. Schemat statyczny belki

Fig. 4.8. Static model of a beam

The maximum, midspan bending moment is given:

$$M_{sd} = \max M_{\alpha-\alpha} = \frac{ql^2}{8} + \frac{(Q_I + Q_S)L}{3} \qquad (1)$$

Loads and the beam length are random variables –products of their nominal (design) values and the random factor, derived experimentally and represented by bar histograms, shown in Fig. 4.9.

– dead load: g = 10.8*Gvar , Gvar = Dead1

– long-lasting (sustained) live load: $Q_I = 22.5*Q_{I,var}$

$Q_{I,var}$ = Long1

– short-lasting (transient) live load: $Q_S = 15.75*Q_{S,var}$

$Q_{S,var}$ = Short1

The span: $L = 6.0*L_{var}$ L_{var} = U1-05

The M-Star software was used for solution.

$M_{sd} = g*L\^2/8+(Q_I + Q_S) *L/3$

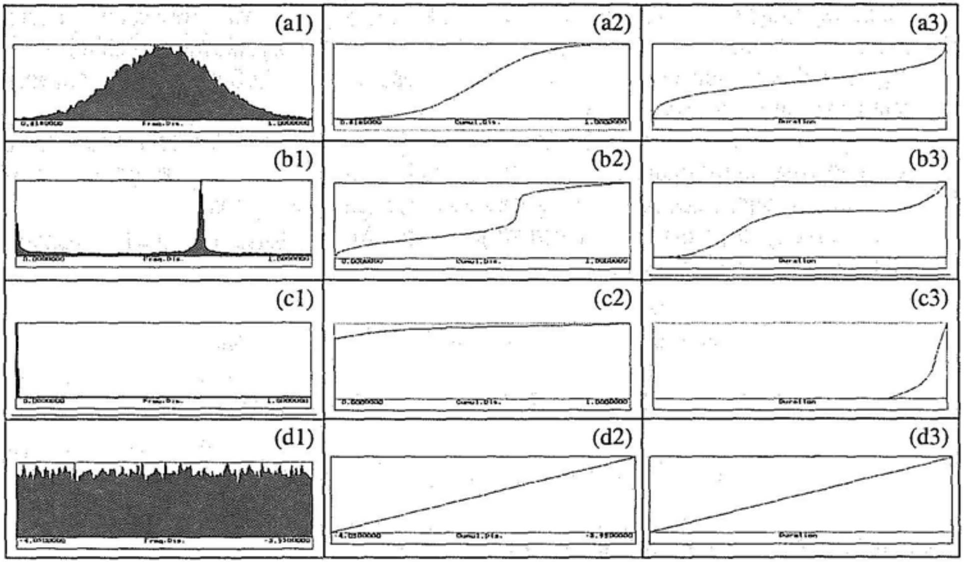
$g = 10.8*G_{var}$

$Q_I = 22.5 * Q_{I,var}$

$Q_{Sh} = 15.75*O_{s,var}$

$L = 6.0*L_{var}$

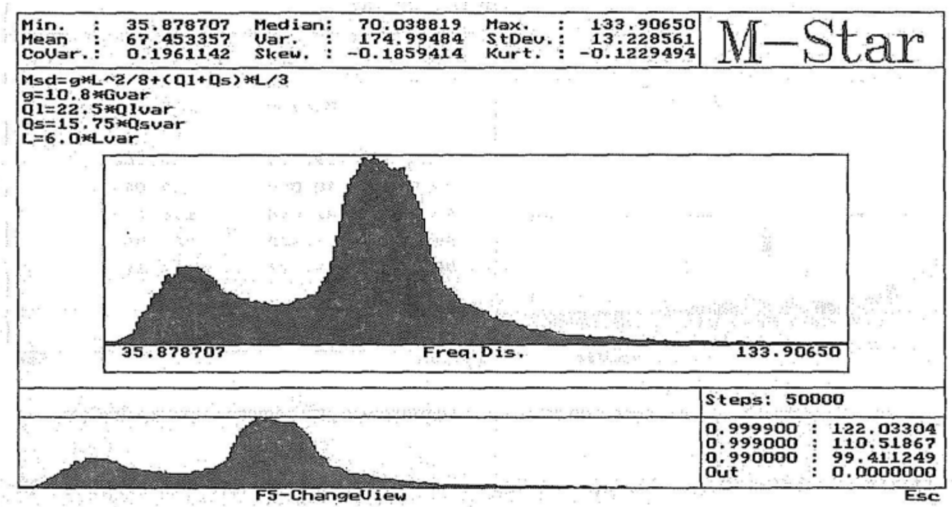
The basic variables were assumed according to Fig. 4.9.



Rys. 4.9. Histogramy słupkowe (a1, b1, c1, d1), dystrybuanty empiryczne (a2, b2, c2, d2) i krzywe rozkładu w czasie (a3, b3, c3, d3) bazowych zmiennych losowych Dead 1 (a), Long 1 (b), Short 1 (c), U1-05 (d)

Fig. 4.9. Bar histograms (a1, b1, c1, d1), empirical CDFs (a1, b1, c1, d1), time duration curves (a3, b3, c3, d3) of basic random variables: Dead 1(a). Long 1 (b), Short 1 (c), U1-05 (d).

The results of 50 000 realizations are shown in Fig. 4.10.



Rys. 4.10. Wydruk wyników obliczeń z przykładu 4.4 wykonanych za pomocą programu M-Star

Fig. 4.10. The M-Star output of an example

Histogram in Fig. 4.10 represents a random variable M_{sd} – bending moment at the critical section of a beam, due to loads g, Q_I i Q_s being uncorrelated random variables whose distributions are known, considering randomly variable beam length L. The result may be also a CDF or a time duration curve of M_{sd}.

The value M_{sd} = 122.03 kNm may be exceeded with the probability $q = 10^{-4}$.

The second solution variant uses the ResCom software. Nominal (design) load values were taken from the previous case.
The main difference is a deterministic beam length.

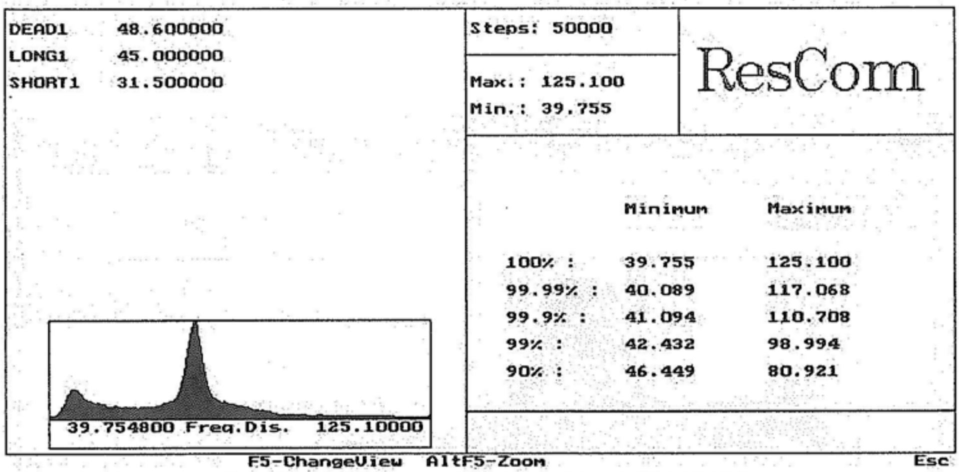
Critical bending moment is the sum
$M_{sd} = M_{sd}(g) + \mathrm{M}_{\mathrm{sd}}(O_I) + M_{sd}(O_s)$, the components are:

$$M_{sd}(g) = g \times L^2/8 = 10.8 * G_{var} * 6.0^2/8 = 48.60 * G_{var}$$

$$M_{sd}(Q_l) = Q_l \times L/3 = 22.5 * Q_{l,var} * 6.0/3 = 45.0 * Q_{l,var}$$

$$M_{sd}(Q_s) = Q_s \times L/3 = 15.75 * Q_{s,var} * 6.0/3 = 31.5 * Q_{s,var}$$

The result is shown in Fig. 4.11.



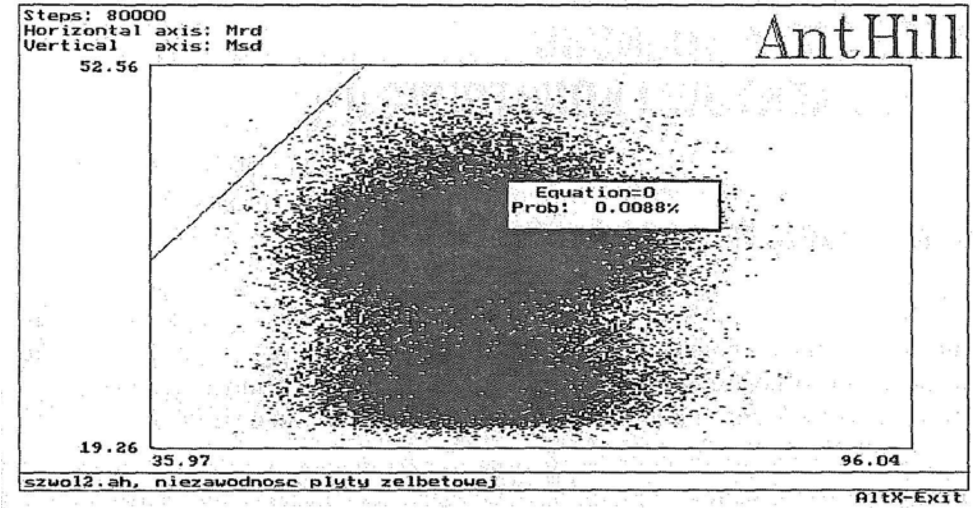
Rys. 4.11. Wydruk wyników obliczeń do przykładu 4.4 wykonanych przy pomocy programu ResCom

Fig. 4.11. The ResCom output of an example

The result is the value M_{sd} = 117.07 kNm to be exceeded with a probability $q = 10^{-4}$.

A lower M_{sd} value occurs, referring to the same $q = 10^{-4}$. It yields from a deterministic beam length assumption of the second variant.

The AntHill output



Rys. 4.13. Wydruk wyników obliczeń do przykładu 4.5, wykonanych za pomocą programu AntHill

Fig. 4.13. The AntHill output of an example