

5.6. Parallel connections of decisive elements

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Probabilistic analysis of structures (in Polish)

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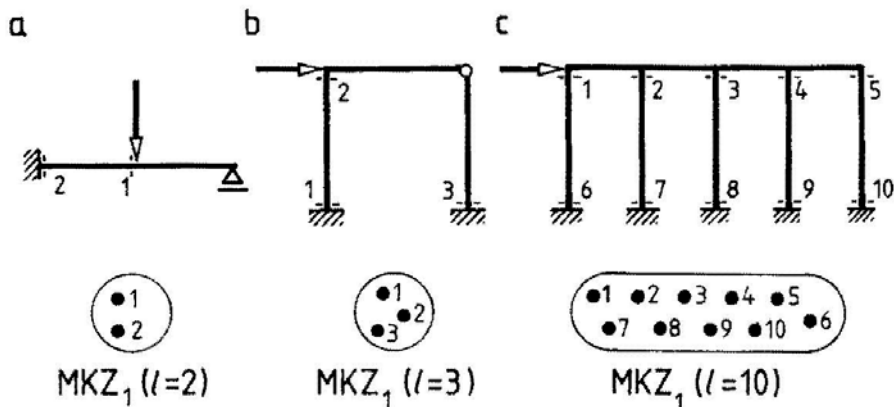
Parallel models match the structures which fail after all their decisive elements are disabled. A simple physical model is a bundle of parallel elements at tension.

After the resistance of the decisive elements is exhausted, they still work (e.g. plastic hinges in bar elements), re-distributing cross-sectional forces along the system members. Their equilibrium paths exhibit a horizontal, plastic phase. Examples are shown in Fig. 5.19.

Fig. **5.19a** presents a statically indeterminate beam. Failure means yield at two cross-sections, which are decisive elements, the only mechanism contains both elements, so $l = 2$.

Figs. **5.19 b** and **c** show systems of a greater number of decisive elements (cross-sectional yield) forming the only mechanism, they are $l = 3$ and $l = 10$, respectively.

A single mechanism of $l < 1$ decisive elements is a parallel reliability model – structural failure means failure of all components.



Rys. 5.19. Przykłady schematów konstrukcji o równoległym modelu niezawodnościowym

Fig. 5.19 denotes a single kinematic mechanism of a number of decisive elements greater than one. The problem is much more complex in the case of multiple, non-separable mechanisms.

The parallel models may be loaded incrementally, thus the weight resistances of elements are summing up. Random resistance $N(\omega)$ of a structure – parallel connection, is

$$N(\omega) = \sum_{i=1}^n a_i N_i(\omega) \quad (1)$$

$N(\omega)$ – resistance of the i -th decisive element,

a_i – weight of the i -th decisive element,

n – the number of decisive elements.

Random resistance of the i -th decisive element is described by its mean value \bar{N}_i and standard deviation s_{N_i} .

Mean value estimation of a parallel system resistance is

$$\bar{N} = \sum_{i=1}^n a_i \bar{N}_i \quad (2)$$

Standard deviation of a parallel system is estimated by

$$s_N = \sqrt{\sum_{i=1}^n a_i^2 s_{N_i}^2} \quad (3)$$

Design resistance of a parallel system on a given significance level

$$N_0 = \bar{N} - t_0 s_N \quad (4)$$

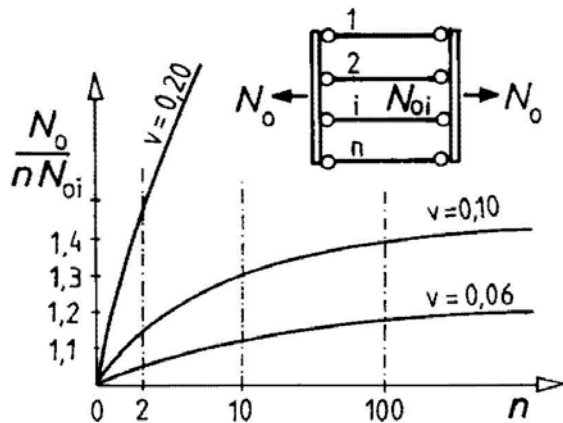
An important effect due to parallel systems is a statistical reliability improvement – a conjugate action of component resistances.

Thus the increment of decisive elements in mechanisms improves the system reliability.

Design resistance is greater for a parallel system N_0 is greater than for n separate decisive elements nN_{0i} , taking constant t_0 , as shown

$$\frac{\bar{N} - t_0 s_N}{n(\bar{N}_i - t_0 s_{N_i})} > 1 \quad (5)$$

Conjugate action of component resistances of parallel systems is shown in Fig. 5.20. Ordinates denote the ratio (5) while abscissas - the number of decisive elements n , in logarithmic scale. The curves correspond to the coefficients of variation $\nu = 0,06; 0,10; 0,20$.



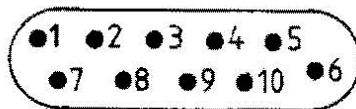
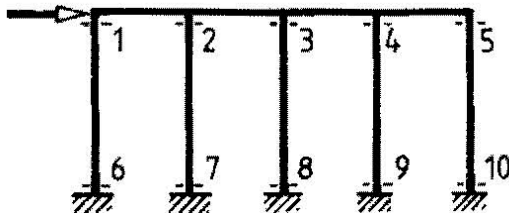
Rys. 5.20. Zmiana parametrów bezpieczeństwa konstrukcji, o równoległym modelu niezawodnościowym, w funkcji liczby jej elementów sprawczych

Problem.

A steel I200 section frame is shown in Fig. 5.19c. Moment resistance of a cross-section is denoted by a mean value

$\bar{N} = 81,0$ kNm and a standard deviation $s_N = 9,26$ kNm. Estimate:

- design resistance N_{0i} of a decisive element (single plastic hinge) due to a significance level $p(t) = p(3) = 0,99865$;



MKZ₁ ($l=10$)

- mean value and standard deviation of a random system resistance, and design resistance N_0 due to significance $p(t) = p(3) = 0,99865$,
- the ratio: system resistance N_0 / / resistance of n decisive elements (hinges), each one denoted by a N_{0i} value,
- system reliability under random load $P(\omega)h = X(\omega)$ of $\bar{X} = 600$ kNm and $S_x = 50$ kNm.

Design resistance of an element (single plastic hinge) for $p(3)$ is

$$N_{0i} = \bar{N} - t_0 s_N = 81,0 - 3 \cdot 9,26 = 53,22 \text{ kNm}$$

Mean resistance of a system of 10 parallel elements (plastic hinges)

$$\bar{N} = \sum_{i=1}^n a_i \bar{N}_i = 10 \cdot 1 \cdot 81,0 = 810 \text{ kNm}$$

its standard deviation equals

$$s_N = \sqrt{\sum_{i=1}^n a_i^2 s_{N_i}^2} = \sqrt{10 \cdot 1^2 \cdot 9,26^2} = 29,28 \text{ kNm}$$

Design resistance of a system for a significance level $p(t)=p(3)$ is

$$N_0 = \bar{N} - t_{0,s_N} = 810 - 3 \cdot 29,28 = 722,16 \text{ kNm}$$

The ratio N_0 / nN_{0i} is

$$\frac{N_0}{nN_{01}} = \frac{722,16}{10 \cdot 53,22} = 1,357$$

System reliability under random load of $\bar{X} = 600$ kNm and $s_x = 50$ kNm is estimated by a reliability index

$$t = \frac{\bar{N} - \bar{X}}{\sqrt{s_N^2 + s_X^2}} = \frac{810 - 600}{\sqrt{29,29^2 + 50^2}} = 3,62$$

Standard Gaussian tables show the investigated reliability $R(3,62) = 0,999853$, a value corresponding to $t = 3,62$.

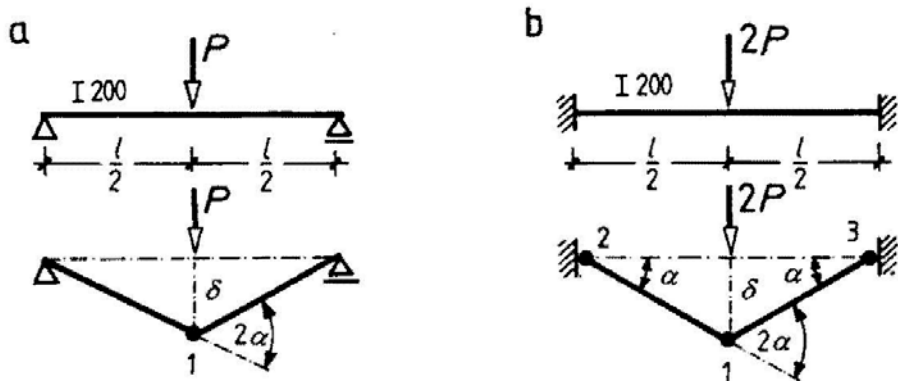
Problem

Two beams 4 m long, resistant to lateral torsional buckling, made of I200 of moment resistance $\bar{N} = 81,0$ kNm, $s_N = 9,26$ kNm. Task:

- design resistance of a simply supported beam, Fig 5.21a, in terms of a midspan force at a significance level $p(t) = p(3)$,

- reliability of a simply supported beam (Fig. 5.21a) due to a random midspan force of $\bar{X}_1 = 45,0$ kN, $s_{X1} = 4,5$ kN,

- resistance and reliability of a both sides clamped beam due to a random midspan force (Fig. 5.21 b) of $\bar{X}_2 = 90,0$ kN i $s_{X2} = 9,0$ kN.



Rys. 5.21. Schemat belki: a – swobodnie podpartej, b – obustronnie zamocowanej

Mean value and standard deviation of a simply supported beam resistance, in terms of its midspan load ($P=P_1=N_1$, $M_{pl} = N$) corresponds to the failure mode shown in Fig. 5.21a

$$P_1 \delta = 2\alpha M_{pl} \quad \alpha = \frac{\delta}{l/2} = \frac{2\delta}{l} \quad P_1 = N_1 = 2 \frac{2}{l} N = \frac{4}{l} N$$

$$\bar{N}_1 = \sum_{i=1}^n a_i \bar{N}_i = \frac{4}{l} \bar{N} = \frac{4}{4} \cdot 81,0 = 81 \text{ kN}$$

$$s_N = \sqrt{\sum_{i=1}^n a_i^2 s_{N_i}^2} = \sqrt{\left(\frac{4}{4}\right) \cdot 9,26^2} = 9,26 \text{ kNm}$$

Design resistance of a simply supported beam due to $p(3)$ equals

$$N_{01} = \bar{N}_1 - t s_{N_1} = 81,0 - 3 \cdot 9,26 = 53,22 \text{ kNm}$$

Reliability of a simply supported beam due to random load $X_1(\omega)$ is estimated by a reliability index of a safety margin

$$t = \frac{\bar{N} - \bar{X}}{\sqrt{s_N^2 + s_X^2}} = \frac{81 - 45}{\sqrt{9,26^2 + 4,5^2}} = 3,498$$

A table of standard Gaussian CDF shows $p(3,498) = 0,999760$. Mean value and standard deviation of a both sides clamped beam resistance, in terms of its midspan load ($2P = P_2 = N_2$, $M_{pb\ i} = N$) Fig. 5.21b, are based on a limit state equation (weights of N applied)

$$P_2\delta = \alpha M_{pl,2} + 2\alpha M_{pl,1} + \alpha M_{pl,3}$$

$$\bar{N}_2 = \sum_{i=1}^n a_i \bar{N}_i = \frac{4}{l} (0,5\bar{N} + \bar{N} + 0,5\bar{N}) = \frac{8}{l} \bar{N} = \frac{8}{4} \cdot 81,0 = 162 \text{ kN}$$

$$s_{N_2} = \sqrt{\sum_{i=1}^n a_i^2 s_{N_i}^2} = \frac{4}{l} \sqrt{(0,5 \cdot 9,26)^2 + 9,26^2 + (0,5 \cdot 9,26)^2} = 11,34 \text{ kNm}$$

Design resistance of a clamped beam due to $p(3)$ equals

$$N_{02} = \bar{N}_2 - t s_{N_2} = 162 - 3 \cdot 11,34 = 127,98 \text{ kNm}$$

Reliability of a clamped beam due to random load $X_2(\omega)$ is estimated by a reliability index t_2

$$t = \frac{\bar{N} - \bar{X}}{\sqrt{s_N^2 + s_X^2}} = \frac{162 - 90}{\sqrt{11,34^2 + 9,0^2}} = 4,972$$

A table of standard Gaussian CDF gives $p(4,972) = 0,9999996652$. Note, that loads in both cases result in equal deterministic safety, for

$$2\bar{X}_1 = \bar{X}_2, \quad \frac{\bar{X}_2 l}{8} = \frac{\bar{X}_1 l}{4}$$

Reliability of a simply supported beam of a series pattern is equal to 0,999760, less than the value for a clamped beam, of a parallel model, equal to 0,9999996652.

Plastic re-distribution of cross-sectional forces in a clamped beam results in its mean resistance twice the simply supported beam value

$$\bar{N}_2 / \bar{N}_1 = 2$$

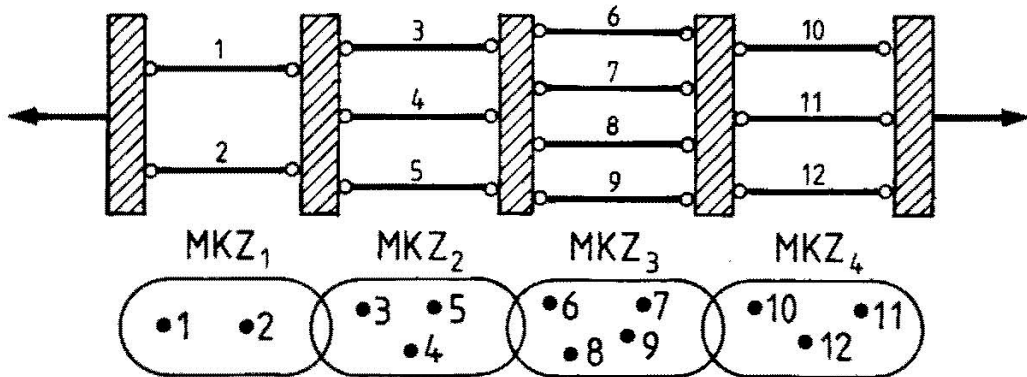
Design resistances of both beams due to significance level $p(3)$ give

$$N_{02} / N_{01} = 127,98 / 53,22 = 2,4047$$

This effect means statistical improvement of a statically indeterminate system.

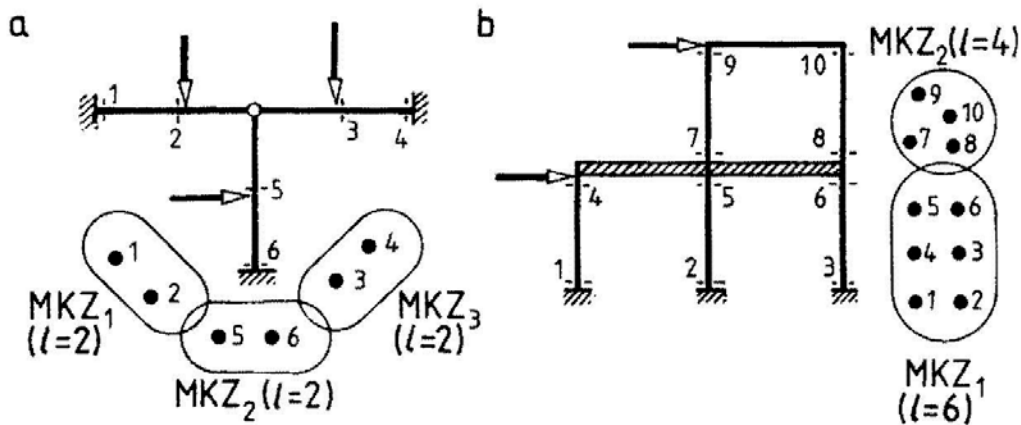
5.7. Series connections of minimum critical sets of decisive elements

Series connections of critical sets, every set is a parallel bundle of decisive elements - a mixed reliability model. The idea - Fig. 5.22.



Rys. 5.22. Schemat mieszanego modelu niezawodnościowego konstrukcji
A chain (series system) whose links are bundles of elements (parallel), all decisive elements in a single mechanism are parallel.

Structure in Fig. 5.23a exhibits 3 failure mechanisms, their elements (hinges) are separated for every mechanism (no common hinges).



Rys. 5.23. Schematy konstrukcji o mieszanym modelu niezawodnościowym

Every critical set denotes $l > 1$, following a **parallel** scheme, but all critical sets form a **series** – safety is determined by a weakest set.

Fig. 5.23b is a shear frame (rigid slabs + flexible columns), parallel elements (4 and 6) in each critical set, both are links of a series.

System resistance follows a series pattern, specifying the multi-element links, each link composed of parallel bundles of elements

$$N(\omega) = \min_{i=1}^n \left| \sum_j a_{ij} N_{ij}(\omega) \right|$$

$N_{ij}(\omega)$ – resistance of a j -th element in an i -th critical set,

a_{ij} – weight of a j -th element element in an i -th critical set,

i – the number of critical sets (mechanisms),

j – the number of elements in a critical set.

The following procedure to assess **resistance and reliability**:

- resistances N_i of each i -th critical set of elements, parallel manner

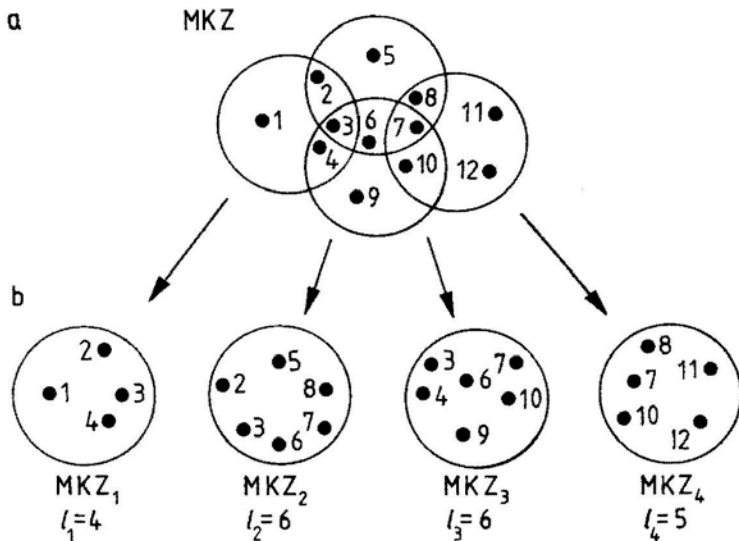
$$N_i(\omega) = \left| \sum_j a_{ij} N_{ij}(\omega) \right|$$

- partial safety p_i – preventing from particular mechanisms of failure

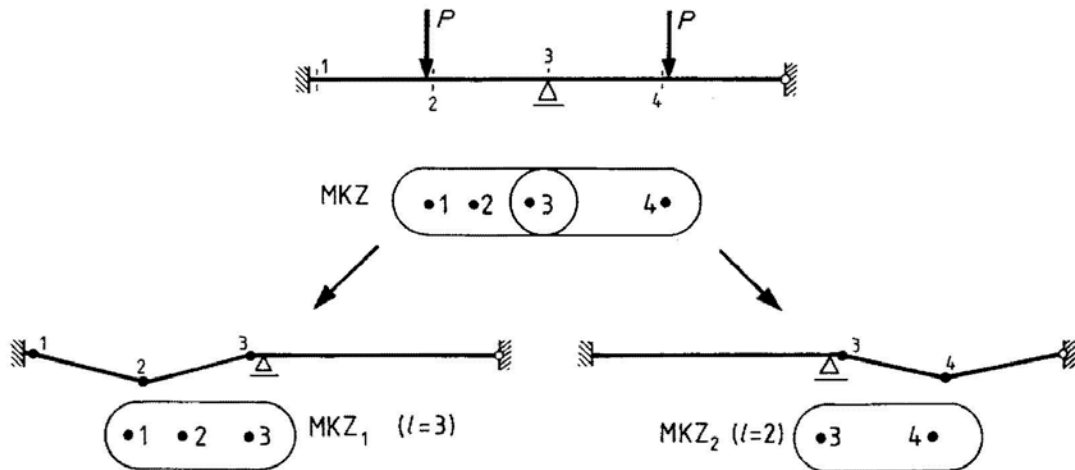
- resistance of a parallel system of N_i , R – reliability of a structure.

5.8. Models of critical sets including common elements

This case covers decisive elements common in system mechanisms. Thus the minimum critical sets of elements (MKZ_i) are parallel. The idea is shown in Fig. 5.25a, examples – in Figs. 5.26 and 5.27.



Rys. 5.25. Minimalne krytyczne zbiory konstrukcji o mieszanym modelu niezawodnościowym: a – z elementami wspólnymi, b – z elementami rozseparowanymi



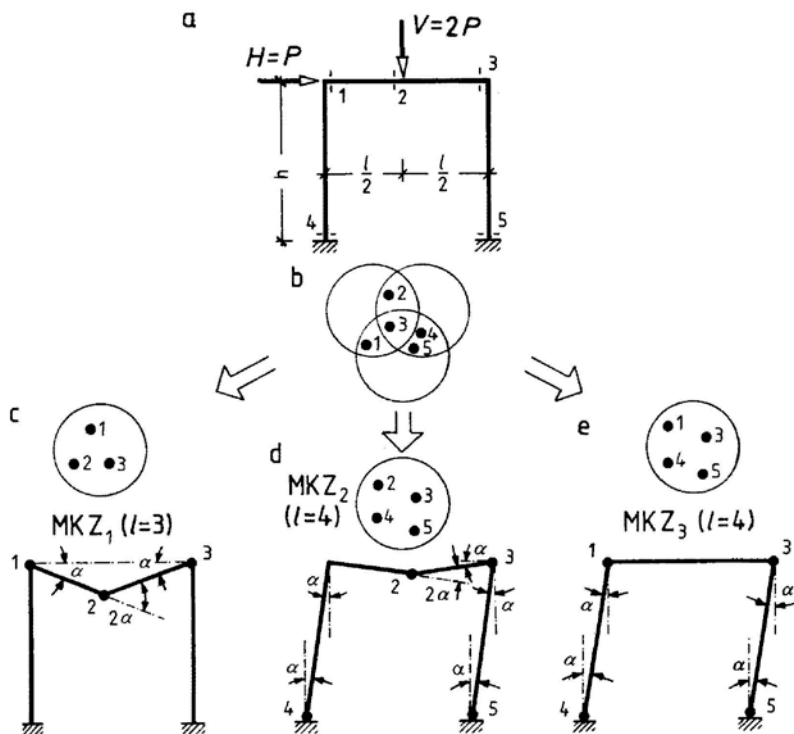
Rys. 5.26. Schemat belki o mieszanym modelu niezawodnościowym z elementem wspólnym

A continuous beam in Fig. 5.26 exhibits the following mechanisms

- MKZ_1 (critical set 1) involving elements (hinges) 1, 2 i 3;
- MKZ_2 (critical set 2) involving elements (hinges) 3, 4.

Both mechanisms MKZ_1 and MKZ_2 include a common decisive element – a hinge at the middle support (element 3).

A framed structure in Fig. 5.27 shows three critical sets of elements:



Rys. 5.27. Schemat ramy o mieszanym modelu niezawodnościowym z elementami wspólnymi

The common element in three mechanisms is a hinge 3.

Elements (hinges) 2 and 3 form MKZ_1 and MKZ_2 , elements 1 and 3 form MKZ_1 and MKZ_3 , elements 3, 4 and 5 form MKZ_2 and MKZ_3 .

Reliability assessment of a mixed model of common elements in critical sets is a considerable task, its complexity grows with the number of mechanisms and decisive elements.

Separation of mechanisms with common elements leads to the lower bound of structural resistance, the idea is shown in Figs 5.25a and b.

While the critical sets are separated each one contains less elements. A series system emerges, so its random capacity may be evaluated

$$N(\omega) = \min_{i=1}^n \left| \sum_j a_{ij} N_{ij}(\omega) \right| = \min_{i=1}^n N_i(\omega)$$

$N_{ij}(\omega)$ – random capacity of the j -th element in the i -th critical set,

a_{ij} – weight of the j -th element in the i -th critical set,

i – the number of critical sets (mechanisms),

j – the number of decisive elements in a critical set.