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Parallel and series connections of decisive elements

Parallel models match the structures which fail after all their decisive elements are disabled. A simple physical model is a bundle of parallel elements at tension. After the resistance of the decisive elements is exhausted, they still work (e.g. plastic hinges in bar elements), re-distributing cross-sectional forces along the system members. Their equilibrium paths exhibit a horizontal, plastic phase. Examples are illustrated in figures.

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The first case is a statically indeterminate beam. Failure means yield at two cross-sections, so they are decisive elements, the only mechanism involves both elements, so l = 2. The next two cases show systems of a greater number of decisive elements (cross-sectional yield) forming the only mechanism, they are l = 3 and l = 10, respectively. A single mechanism of l > 1 decisive elements is a parallel reliability model - structural failure means failure of all components. The figures denote a single kinematic mechanism of a number of decisive elements greater than one. The problem is much more complex in the case of multiple, non-separable mechanisms.

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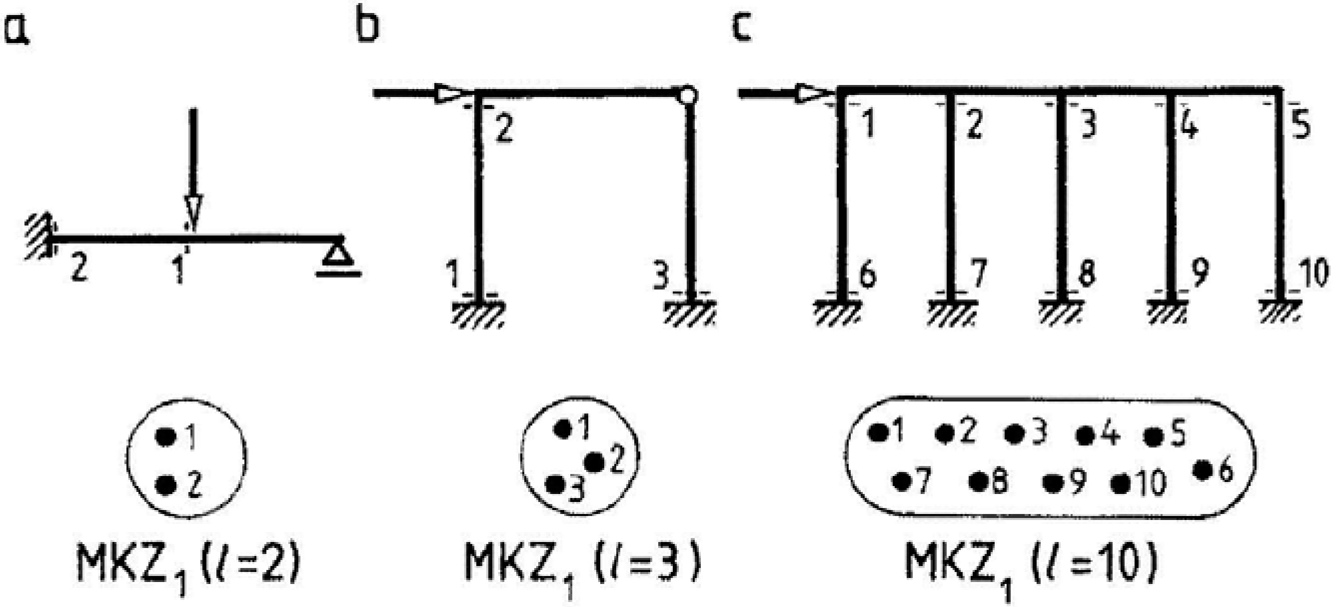

MKZ – minimal set of decisive elements (Polish abbreviation),

in fact a mechanism

The parallel models may be loaded incrementally, thus the weight resistances of elements are summing up. Random resistance $N(\omega)$ of a structure – parallel connection, is

$$N(\omega) = \sum_{i=1}^{n} a_i N_i(\omega) \qquad (1)$$

$N(\omega)$ – resistance of the *i*-th decisive element,
a_i – weight of the *i*-th decisive element,
n – the number of decisive elements.
Random resistance of the *i*-th decisive element is described by its mean value $\overline{N}_i$ and standard deviation s_{N_i}.
Mean value estimation of a parallel system resistance is

$$\overline{N} = \sum_{i=1}^{n} a_i \overline{N}_i \qquad (2)$$

Standard deviation of a parallel system is estimated by

$$s_N = \sqrt{\sum_{i=1}^{n} a_i^2 s_{N_i}^2} \tag{3}$$

Design resistance of a parallel system on a given significance level

$$N_0 = \overline{N} - t_0 s_N \tag{4}$$

An important effect due to parallel systems is a statistical reliability improvement – a conjugate action of component resistances.
Thus the increment of decisive elements in mechanisms improves the system reliability.
Design resistance is greater for a parallel system N_0 is greater than for n separate decisive elements nN_{0i}, taking constant t_0, as shown

$$\frac{\overline{N} - t_{0 s_N}}{n(\overline{N}_i - t_{0 s_{Ni}})} > 1 \tag{5}$$

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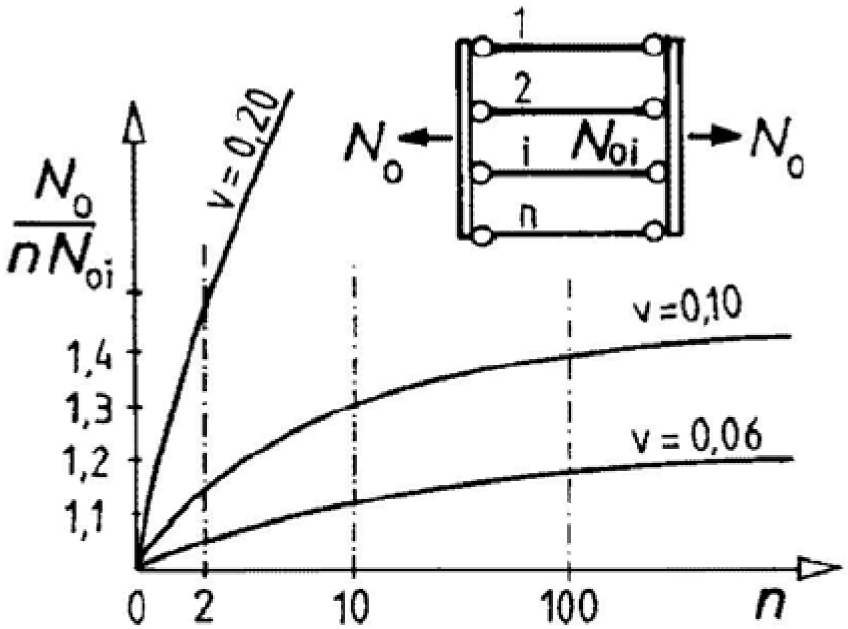
Conjugate action of component resistances of parallel systems is shown below. Ordinates denote the ratio (5) while abscissas - the number of decisive elements *n*, in logarithmic scale. The curves correspond to the coefficients of variation *v* = 0,06; 0,10; 0,20.



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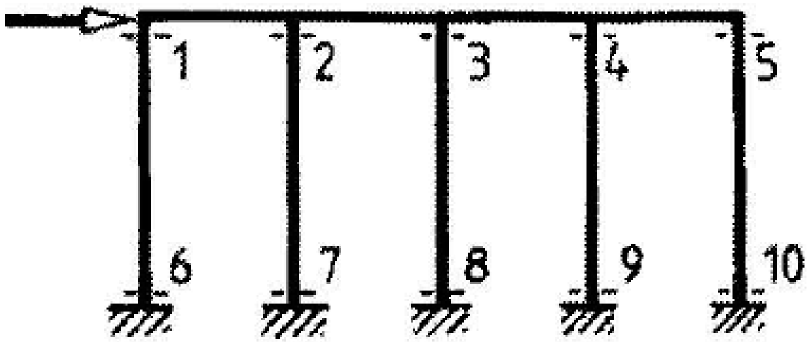
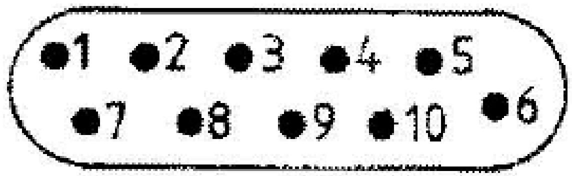
Problem.

A steel I200 section frame is shown below. Cross-sectional moment resistance is given its mean value $\bar{N} = 81.0\,kNm$ and standard deviation s_N = 9,26 kNm.

Estimate design resistance of a decisive element (single plastic hinge) due to a significance level $p(t) = p(3) = 0{,}99865$





MKZ_1 (l = 10)

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- mean value and standard deviation of a random system resistance, and design resistance N_0 due to significance *p(t) = p(3)* = 0,99865,
- the ratio: system resistance N_0 / / resistance of *n* decisive elements (hinges), each one denoted by a N_{0i} value,
- system reliability under random load $P(\omega)h = X(\omega)$ of $\bar{X}$ = 600 kNm and S_x = 50 kNm.

Design resistance of an element (single plastic hinge) for *p(3)* is

$$N_{0i} = \bar{N} - t_0 s_N = 81,0 - 3 \cdot 9,26 = 53,22 \text{ kNm}$$

Mean resistance of a system of 10 parallel elements (plastic hinges)

$$\bar{N} = \sum_{i=1}^{n} a_i \bar{N}_i = 10 \cdot 1 \cdot 81,0 = 810 \text{ kNm}$$

its standard deviation equals

$$s_N = \sqrt{\sum_{i=1}^{n} a_i^2 s_{N_i}^2} = \sqrt{10 \cdot 1^2 \cdot 9,26^2} = 29,28 \text{ kNm}$$

Design resistance of a system for a significance level *p(t)=p(3)* is

$$N_0 = \bar{N} - t_{0 s_N} = 810 - 3 \cdot 29,28 = 722,16 \text{ kNm}$$

The ratio N_0 / nN_{0i} is

$$\frac{N_0}{nN_{01}} = \frac{722,16}{10 \cdot 53,22} = 1,357$$

System reliability under random load of $\bar{X}$ = 600 kNm and s_x = 50 kNm is estimated by a reliability index

$$t = \frac{\bar{N} - \bar{X}}{\sqrt{s_N^2 + s_X^2}} = \frac{810 - 600}{\sqrt{29,29^2 + 50^2}} = 3,62$$

Standard Gaussian tables show the investigated reliability R (3,62) = 0,999853, a value corresponding to t = 3,62.

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Problem

Two beams 4 m long, resistant to lateral torsional buckling, made of I200 of moment resistance $\overline{N}$ = 81,0 kNm, s_N = 9,26 kNm. Task:

- design resistance of a simply supported beam, Fig 5.21a, in terms of a midspan force at a significance level $p(t) = p(3)$,
- reliability of a simply supported beam (Fig. 5.21a) due to a random midspan force of $\overline{X}_1$ = 45,0 kN, s_{X1} = 4,5 kN,
- resistance and reliability of a both sides clamped beam due to a random midspan force (Fig. 5.21 b) of $\overline{X}_2$ = 90,0 kN i s_{X2} = 9,0 kN.

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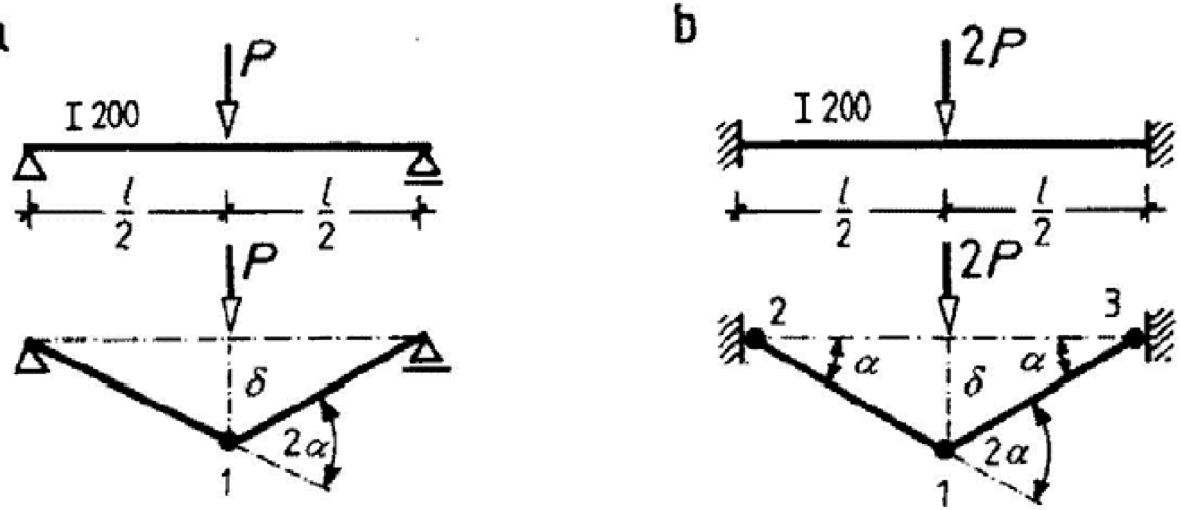

Mean value and standard deviation of a simply supported beam resistance, in terms of its midspan load ($P=P_1=N_1$, $M_{pl} = N$) corresponds to the failure mode shown in Fig. 5.21a

$$P_1\delta = 2\alpha M_{pl} \qquad \alpha = \frac{\delta}{l/2} = \frac{2\delta}{l} \qquad P_1 = N_1 = 2\frac{2}{l}N = \frac{4}{l}N$$

$$\bar{N}_1 = \sum_{i=1}^{n} a_1 \bar{N}_i = \frac{4}{l}\bar{N} = \frac{4}{4}\cdot 81,0 = 81 \text{ kN}$$

$$s_N = \sqrt{\sum_{i=1}^{n} a_i^2 s_{N_i}^2} = \sqrt{\left(\frac{4}{4}\right)\cdot 9,26^2} = 9,26 \text{ kNm}$$

Design resistance of a simply supported beam due to $p(3)$ equals

$$N_{01} = \bar{N}_1 - t s_{N_1} = 81,0 - 3\cdot 9,26 = 53,22 \text{ kNm}$$

Reliability of a simply supported beam due to random load $X_1(\omega)$ is estimated by a reliability index of a safety margin

$$t = \frac{\bar{N} - \bar{X}}{\sqrt{s_N^2 + s_X^2}} = \frac{81 - 45}{\sqrt{9,26^2 + 4,5^2}} = 3,498$$

A table of standard Gaussian CDF shows $p(3,498) = 0,999760$.
Mean value and standard deviation of a both sides clamped beam resistance, in terms of its midspan load ($2P = P_2 = N_2$, $M_{pl,i} = N$) *Fig.* 5.21b, are based on a limit state equation (weights of N applied)

$$P_2\delta = \alpha M_{pl,2} + 2\alpha M_{pl,1} + \alpha M_{pl,3}$$

$$\bar{N}_2 = \sum_{i=1}^{n} a_1 \bar{N}_i = \frac{4}{l}\left(0,5\bar{N} + \bar{N} + 0,5\bar{N}\right) = \frac{8}{l}\bar{N} = \frac{8}{4}\cdot 81,0 = 162 \text{ kN}$$

$$s_{N2} = \sqrt{\sum_{i=1}^{n} a_i^2 s_{N_i}^2} = \frac{4}{l}\sqrt{(0,5\cdot 9,26)^2 + 9,26^2 + (0,5\cdot 9,26)^2} = 11,34 \text{ kNm}$$

Design resistance of a clamped beam due to $p(3)$ equals

$$N_{02} = \bar{N}_2 - ts_{N_2} = 162 - 3\cdot 11,34 = 127,98 \text{ kNm}$$

Reliability of a clamped beam due to random load $X_2(\omega)$ is estimated by a reliability index t_2

$$t = \frac{\bar{N} - \bar{X}}{\sqrt{s_N^2 + s_X^2}} = \frac{162 - 90}{\sqrt{11,34^2 + 9,0^2}} = 4,972$$

A table of standard Gaussian CDF gives $p(4,972) = 0,9999996652$. Note, that loads in both cases result in equal deterministic safety, for

$$2\bar{X}_1 = \bar{X}_2, \quad \frac{\bar{X}_2 l}{8} = \frac{\bar{X}_1 l}{4}$$

Reliability of a simply supported beam of a series pattern is equal to 0,999760, less than the value for a clamped beam, of a parallel model, equal to 0,9999996652.
Plastic re-distribution of cross-sectional forces in a clamped beam results in its mean resistance twice the simply supported beam value

$$\bar{N}_2 / \bar{N}_1 = 2$$

Design resistances of both beams due to significance level $p(3)$ give

$$N_{02} / N_{01} = 127{,}98 / 53{,}22 = 2{,}4047$$

This effect means statistical improvement of a statically indeterminate system.

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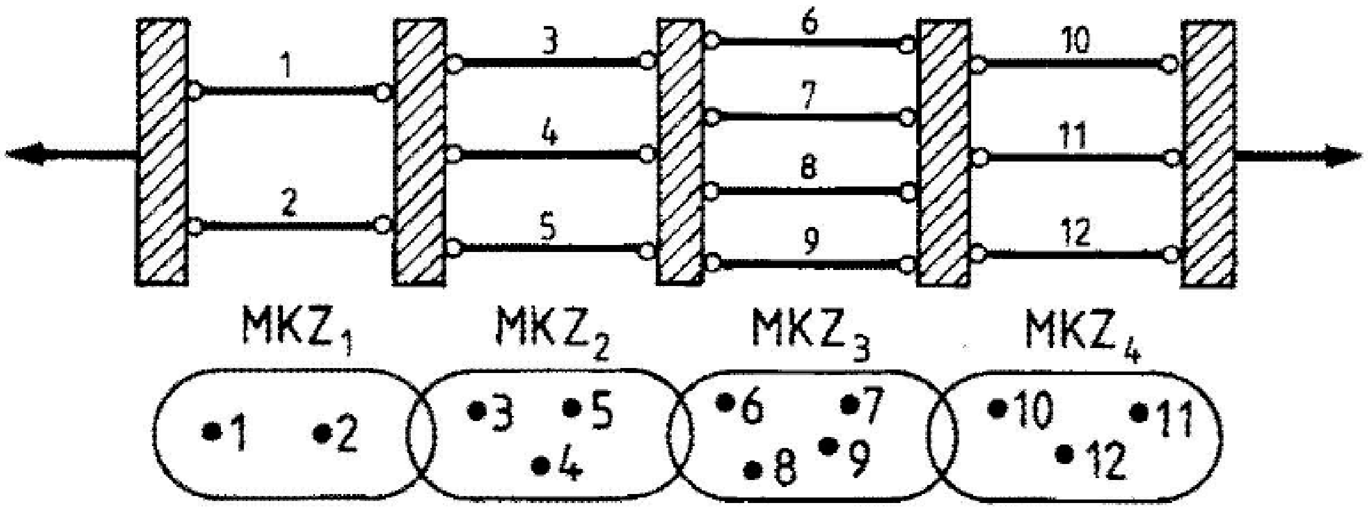
Series connections of minimum critical sets of decisive elements

Series connections of critical sets, every set is a parallel bundle of decisive elements - a mixed reliability model. The idea is presented below



A chain (series system) whose links are bundles of elements (parallel), all decisive elements in a single mechanism are parallel.

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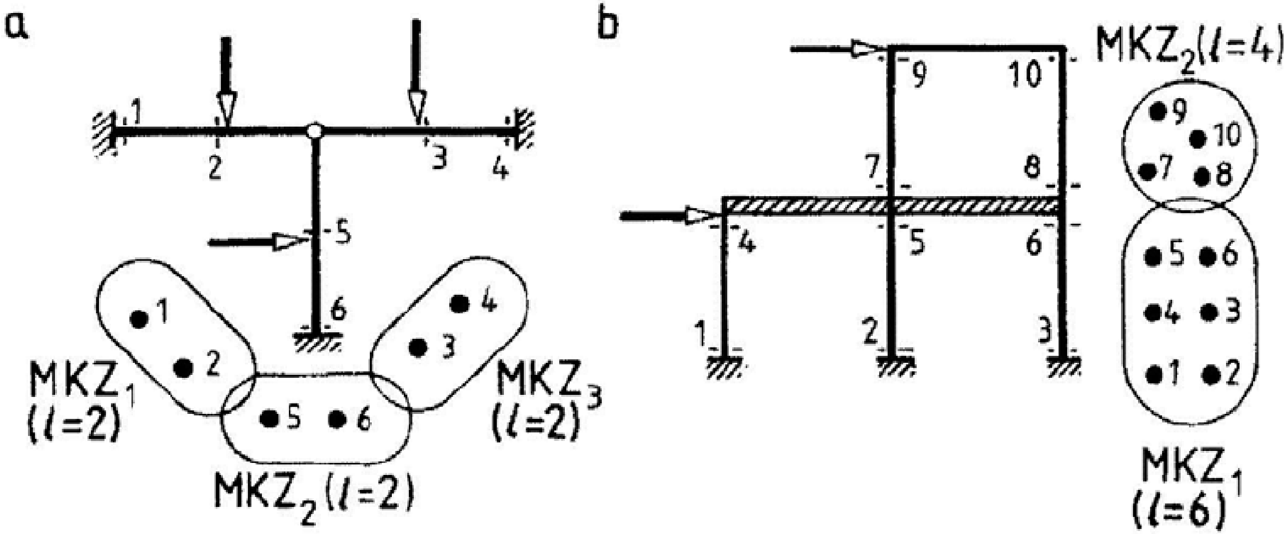
Structure in Fig. 5.23a exhibits 3 failure mechanisms, their elements (hinges) are separated for every mechanism (no common hinges).



Every critical set denotes $l > 1$, following a **parallel** scheme, but all critical sets form a **series** – safety is determined by a weakest set.

Fig. 5.23b is a shear frame (rigid slabs + flexible columns), parallel elements (4 and 6) in each critical set, both are links of a series.

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System resistance follows a series pattern, specifying the multi-element links, each link composed of parallel bundles of elements

$$N(\omega) = \min_{i=1}^{n} \left| \sum_j a_{ij} N_{ij}(\omega) \right|$$

$N_{ij}(\omega)$ – resistance of a *j*-th element in an *i*-th critical set,
a_{ij} – weight of a *j*-th element element in an *i*-th critical set,
i – the number of critical sets (mechanisms),
j – the number of elements in a critical set.

The following procedure to assess **resistance and reliability**:
- resistances N_i of each *i*-th critical set of elements, parallel manner

$$N_i(\omega) = \left| \sum_j a_{ij} N_{ij}(\omega) \right|$$

- partial safety p_i – preventing from particular mechanisms of failure
- resistance of a parallel system of N_i , R – reliability of a structure.

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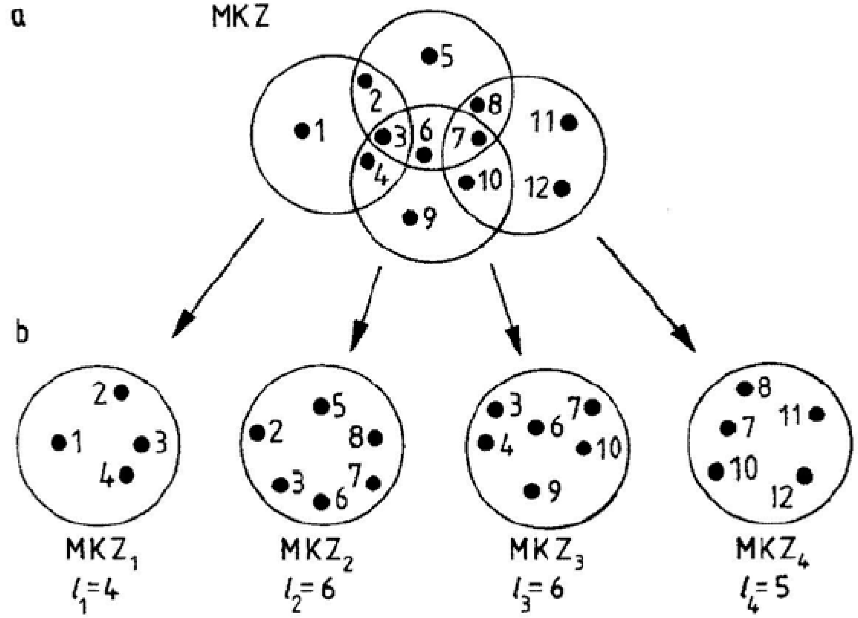
Models of critical sets including common elements

This case covers decisive elements common in system mechanisms. Thus the minimum critical sets of elements (Polish - MKZ*i*) are parallel.



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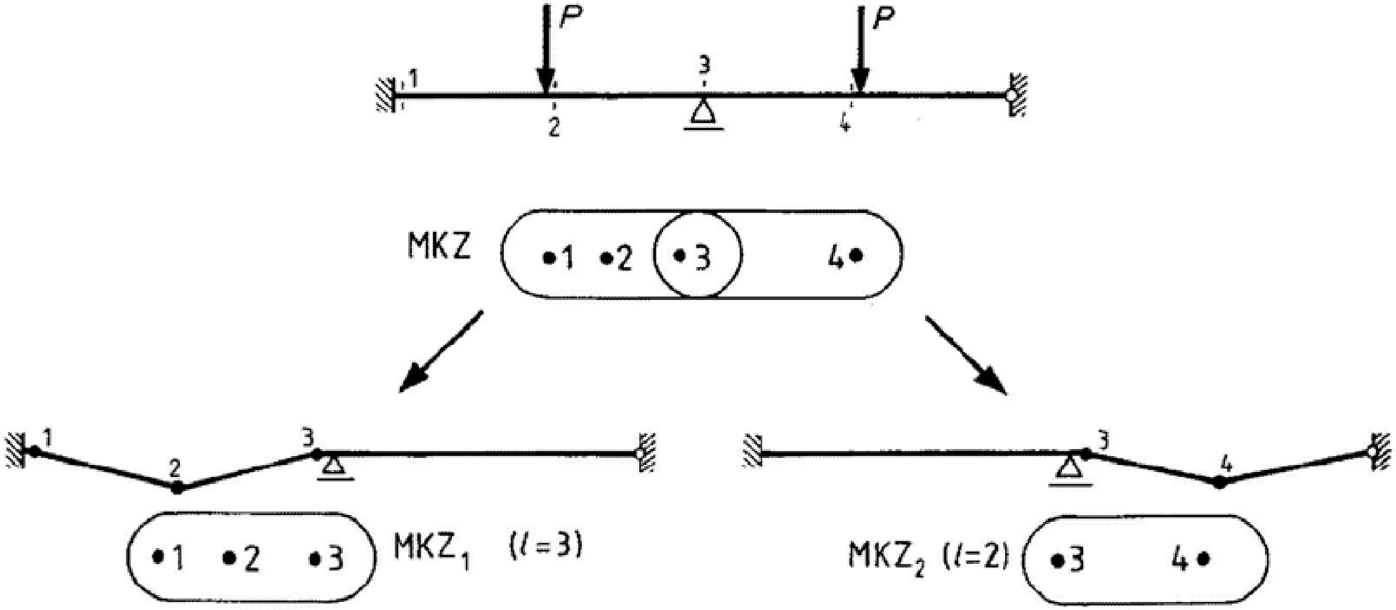

A continuous beam exhibits the following mechanisms - MKZ1 (critical set 1) involving elements (hinges) 1, 2 i 3; - MKZ2 (critical set 2) involving elements (hinges) 3, 4. Both mechanisms MKZ1 and MKZ2 include a common decisive element – a hinge at the middle support (element 3).

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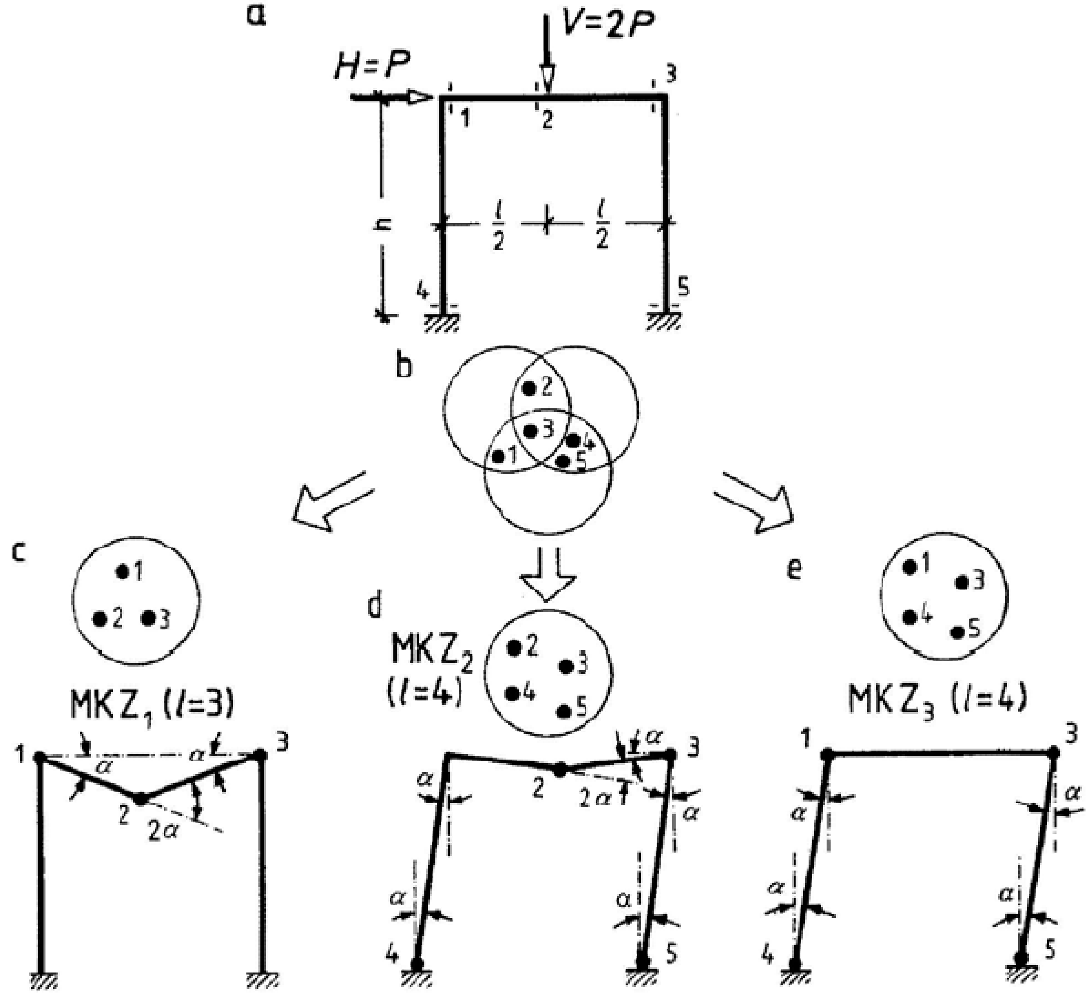

The framed structure shows three critical sets of elements.

The common element in every mechanism is a hinge 3.

Elements (hinges) 2 and 3 form MKZ_1 and MKZ_2, elements 1 and 3 form MKZ_1 and MKZ_3, elements 3, 4 and 5 form MKZ_2 and MKZ_3.

Reliability assessment of a mixed model of common elements in critical sets is a considerable task, its complexity grows with the number of mechanisms and decisive elements.

Separation of mechanisms with common elements leads to the lower bound of structural resistance, the idea is shown in Figs 5.25a and b.

While the critical sets are separated each one contains less elements. A series system emerges, so its random capacity may be evaluated

$$N(\omega) = \min_{i=1}^{n} \left| \sum_{j} a_{ij} N_{ij}(\omega) \right| = \min_{i=1}^{n} N_i(\omega)$$

$N_{ij}(\omega)$ – random capacity of the j-th element in the i-th critical set,

a_{ij} – weight of the j-th element in the i-th critical set,

i – the number of critical sets (mechanisms),

j – the number of decisive elements in a critical set.

References:

A. Biegus “Probabilistic analysis of steel structures” PWN Warszawa 1999

P. Thoft-Christensen, Y. Murotsu “Introduction to structural systems reliability theory” Springer 1986