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Probabilistic sensitivity of the limit states of structures.

The Monte Carlo simulation

Every problem of engineering design is formed by means of a set of input data, the so-called basic variables.

* actions, e.g. loads: dead, live, technological, wind and snow loads, non-static temperature impact, restraint deflections and initial imperfections
* structural parameters, e.g. Young’s modulus, yield stress, dimensions of structural members.

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It is a vital problem to assess the impact of specified basic variables on a chosen structural response objective.

Design sensitivity addresses variations of design parameters and develops procedures to compute structural behaviour variation that depends implicitly on design variables. In general, it is concerned with optimization – search for a combination of basic variables to maximize / minimize the objective function. Regarding variation of structural dimensions is examined, shape sensitivity is analysed. Deterministic structural sensitivity is a developed branch of the present-day mechanics

The advent of random philosophy of physical phenomena and progress of probability applications developed a probabilistic approach in technical sciences. A large group of civil engineering problems is analysed assuming their random nature. Research activity and codes of practice are given random implementation. Thus the deterministic approach to sensitivity may substantially enhanced.

Probabilistic sensitivity is bound to concern various branches, e.g. chemistry, biology, molecular physics, economical sciences. Multi-discipline applications are aimed at a rational framework to consider

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system uncertainty. An economical meaning is successfully implemented in various fields of engineering. It also concerns the impact of uncertainty on the design performance. One of the operational approaches introduces sensitivity factors accompanying the second-moment reliability methods.

The term stochastic sensitivity captures random functions representing input data and structural responses. Discretized structural models are usually applied to solve the engineering problems, therefore random variables are used instead. Uncertainty analysis can be performed on different levels. The so-called 2nd level probabilistic method employs the first and the second statistical moments of random parameters only, in this case perturbation method may be implemented. The 3^{rd} level incorporates probability density functions and cumulative distribution functions. Numerical methods are the most effective here. The Monte Carlo simulation method estimates probability distributions of random responses.

Structural failure is a very rare event. An imaginary probabilistic experiment assumes that a design variable can exceed its design value, leading to the limit state. Thus a new notion of probabilistic

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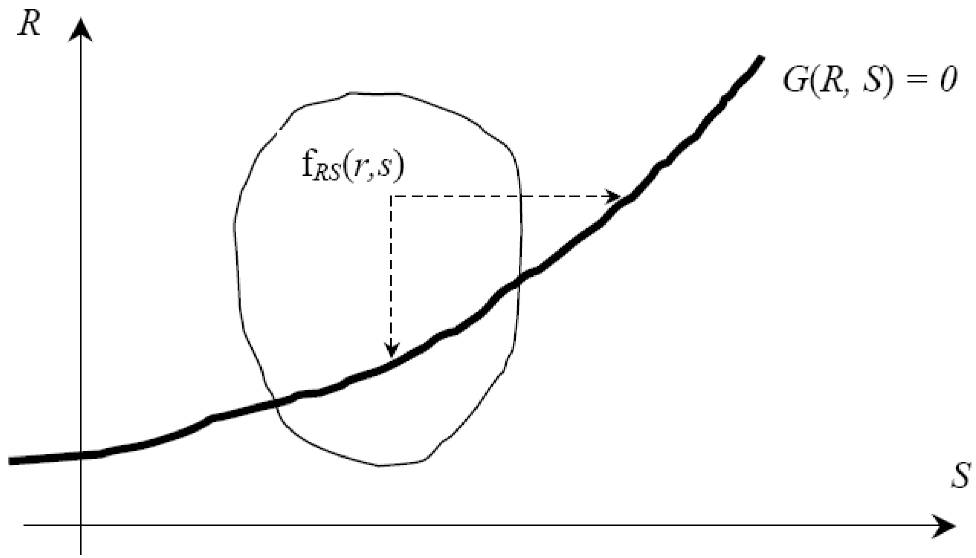
sensitivity of the limit state of the structure is possible with respect to the chosen design variable.

More complicated engineering structures require a dedicated Monte Carlo Simulation procedure. In this approach the computational means not only serve as a numerical tool, becoming part of the methodology. The probabilistic sensitivity problem applying the dedicated MCS procedure may be regarded the problem of computational science.

Consider a fundamental case of two basic variables only: demand (load effect) S and resistance R. It is a wide, practical definition. Variability region of S and R is specified, the probability density function $f_{RS}(r, s)$ and the limit state function $G(R, S)$ are given.



It is assumed that the random variable S can exceed the region of variability, the **load multiplier** λ_s, such that $G(R, \lambda_s S) = 0$, is introduced. The sample space of all failure events is defined, their probability is described by the random variable λ_s. The safety margin $G(R, S) = 0$ is $S = g_r(R)$, so $\lambda_s = \dfrac{g_r(R)}{S}$

In fundamental case, i.e. $G(R, S) = R - S$, it holds $\lambda_s = \dfrac{R}{S}$

The safe subregion $G(R, S) > 0$ provides $\lambda_s > 1$, the limit state $G(R, S) = 0$ gives $\lambda_s = 1$, failure $G(R, S) < 0$ yields $\lambda_s < 1$.

The probability density function of λ_s represents system probabilistic limit state sensitivity with respect to the load effect S.

The **resistance multiplier** λ_r takes $G(\lambda_r R, S) = 0$. The safety margin $G(R, S) = 0$ the form of $R = g_s(S)$, then $\lambda_r = \dfrac{g_s(S)}{R}$

While $G(R, S) = R - S$, so $\lambda_r = \dfrac{S}{R}$

Here $G(R, S) > 0$ implies $\lambda_r < 1$, the safety margin $G(R, S) = 0$ satisfies $\lambda_r = 1$ while $G(R, S) < 0$ yields $\lambda_r > 1$.

The probability density function of λ_r detects structural limit state probabilistic sensitivity with respect to resistance R.

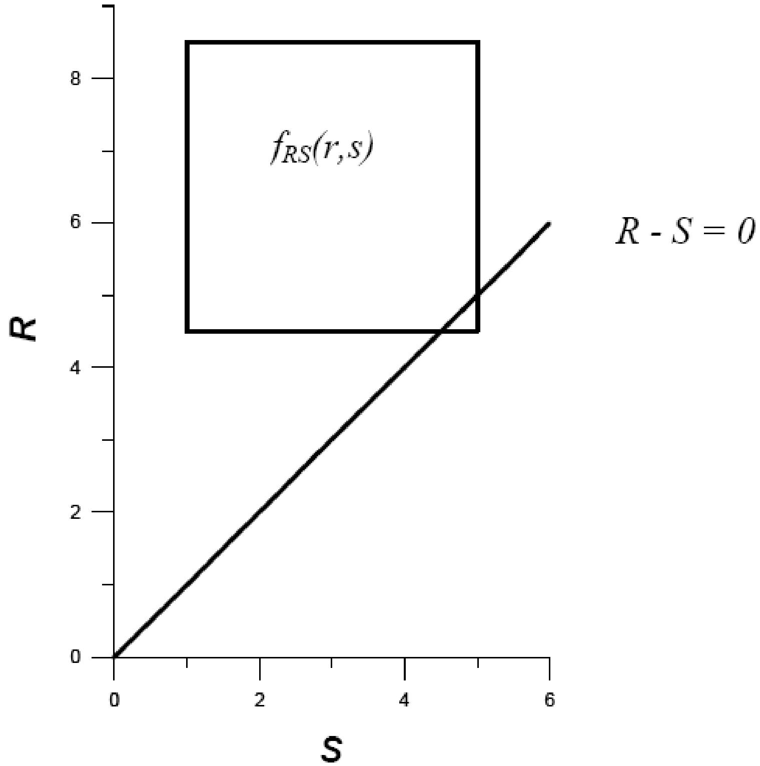
The fundamental example of distinct variables R and S is shown first. The multi-variable problems are solved by an algorithm, presented in the next section.

In the following example two random variables are taken:

* load effect S, uniformly in <1; 5>

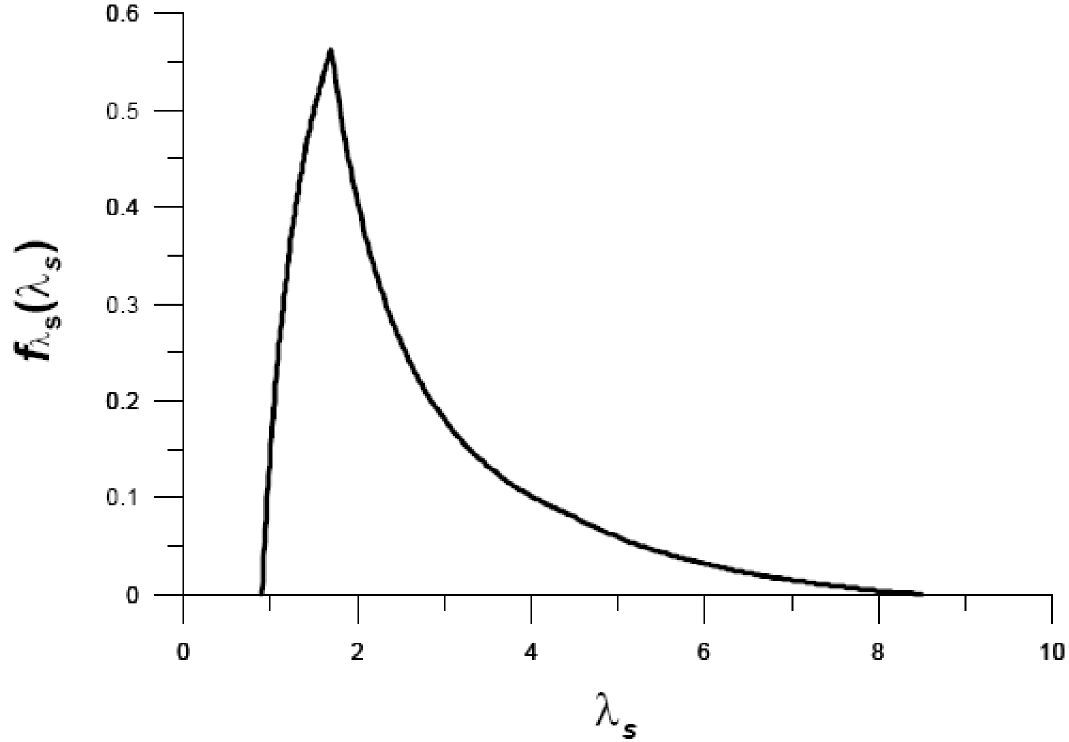
* resistance, uniform in <4.5; 8.5>.

Two-dimensional PDF of R and S is $f_{RS}(r,s) = 0.0625$. The function $G(R, S) = R - S$. Two-dimensional probability distribution of R and S, and the safety margin are presented



The limit load multiplier λ_s is a function of *R* and *S*. Its CDF may be computed analytically, by $F_{\lambda_s}\left(\lambda_s\right) = \iint\limits_{\frac{r}{s}<\lambda_s} f_{RS}\left(r,s\right) dr\, ds$, the PDF is

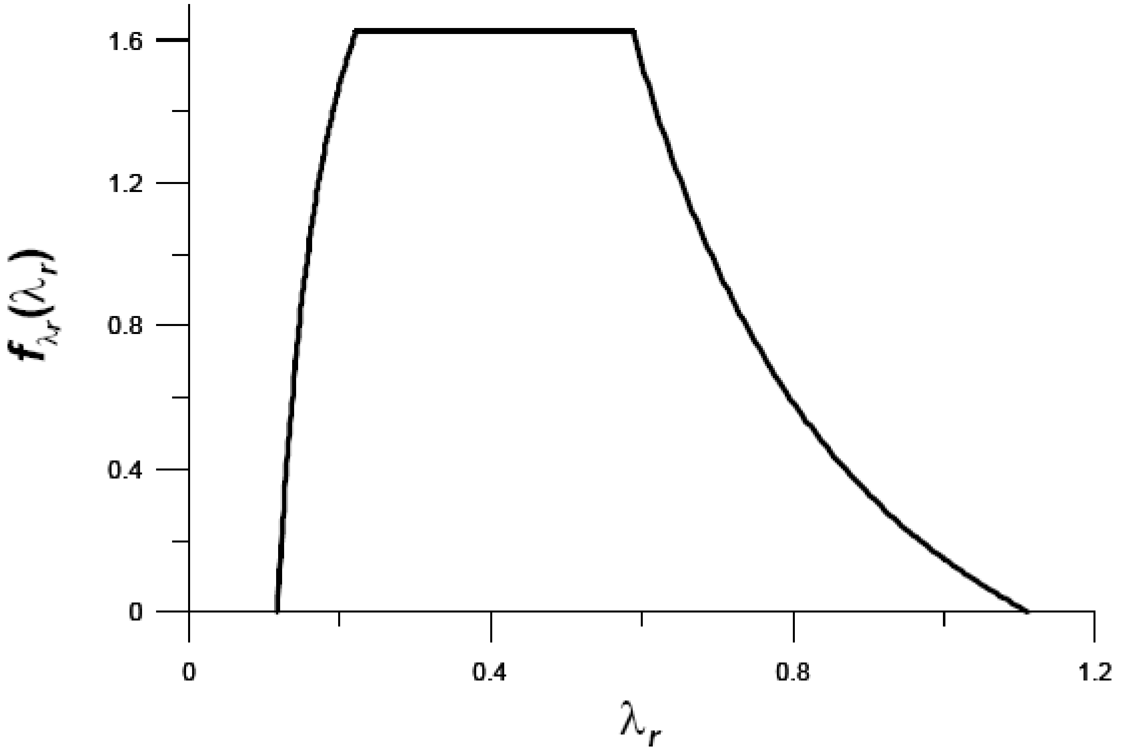
$$f_{\lambda_s}\left(\lambda_s\right) = \begin{cases} 0 & \lambda_s \le 0.9 \text{ } or \text{ } \lambda_s > 8.5 \\ \dfrac{1}{32}\left(25 - \dfrac{20.25}{\lambda_s^2}\right) & \lambda_s \in \left(0.9,\, 1.7\right] \\ \dfrac{1}{32}\cdot\dfrac{52}{\lambda_s^2} & \lambda_s \in \left(1.7,\, 4.5\right] \\ \dfrac{1}{32}\left(-1 + \dfrac{72.25}{\lambda_s^2}\right) & \lambda_s \in \left(4.5,\, 8.5\right] \end{cases}$$



Failure probability $p_f = \int_{0.9}^{1.0} \frac{1}{32}\left(25 - \frac{20.25}{\lambda_s^2}\right) d\lambda_s = 7.8125 \cdot 10^{-3}$

The CDF of the resistance multiplier $\lambda_r : F_{\lambda_r}\left(\lambda_r\right) = \iint\limits_{\frac{s}{r} > \lambda_r} f_{RS}\left(r, s\right) dr\, ds$

Its PDF is $f_{\lambda_r}\left(\lambda_r\right) = \begin{cases} 0 & \lambda_r \leq 0.1176 \text{ } or \text{ } \lambda_r > 1.1111 \\ \frac{1}{32}\left(72.25 - \frac{1}{\lambda_r^2}\right) & \lambda_r \in \left(0.1176, 0.2222\right] \\ \frac{52}{32} & \lambda_r \in \left(0.2222, 0.5882\right] \\ \frac{1}{32}\left(-20.25 + \frac{25}{\lambda_r^2}\right) & \lambda_r \in \left(0.5882, 1.1111\right] \end{cases}$



Failure probability: $p_f = \int_{1.0}^{1.1111} \frac{1}{32}\left(-20.25 + \frac{25}{\lambda_r^2}\right) d\lambda_r = 7.8125 \cdot 10^{-3}$

The major feature of the example is a fully analytical solution, due to the simplest reliability model G(R, S) = R – S of two basic variables only, possible to be applied an explicit, closed form.

Limit states of engineering structures

Failure cases form the range of geometrically and physically nonlinear problems, impossible to be solved analytically. The numerical solutions are based on the discretization of the system. Thus the structural limit state problem may be regarded a stochastically nonlinear problem.

It is impossible to define probabilistic sensitivity of the limit states of the structural systems in a straightforward, explicit way.

The computational methods may turn the most effective. The Monte Carlo Simulation (MCS) is one of the effective computational tools in stochastic analysis - as a tool of computational science. The probabilistic sensitivity of the limit states of structures may be defined here on the basis of a suitable MCS procedure. It is also possible to define the probabilistic sensitivity of the limit states, with respect to a chosen design parameter (or a group of parameters).

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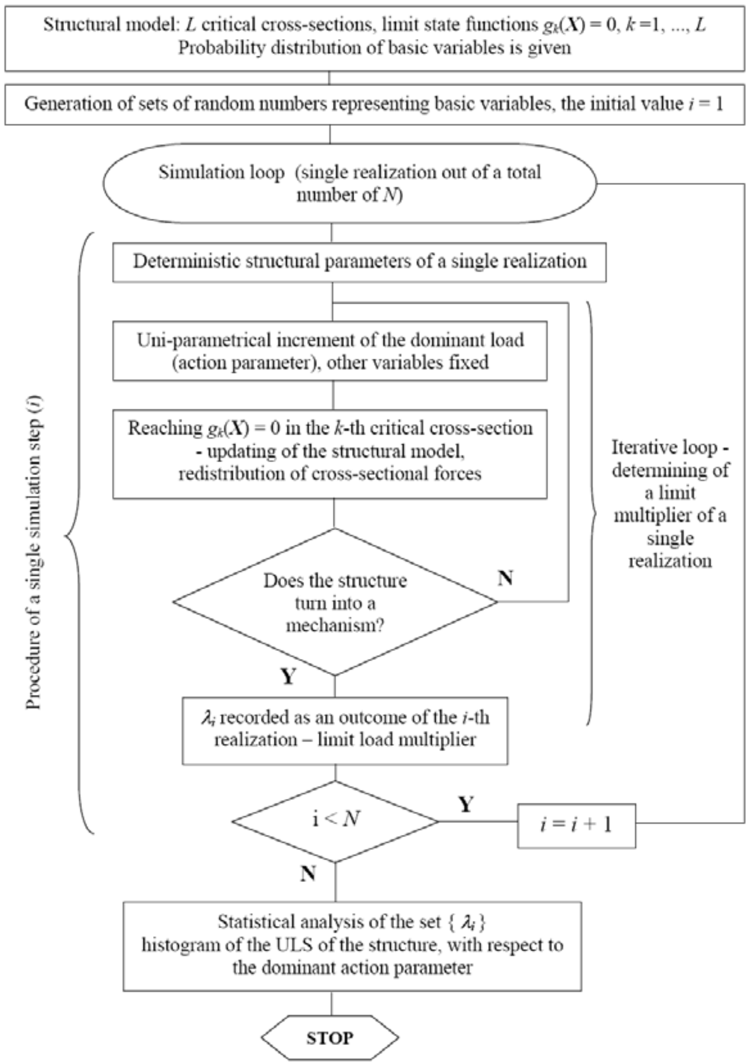
The MCS procedure to define the probabilistic sensitivity of the limit state is composed of a number of steps. The term “simulation step” is equivalent to the term “realization”. Every step in the MCS procedure is deterministic form, composed of two stages:

1) generation of all design parameters as a set of random variates,
2) the multiplier λ_p is introduced to the chosen (dominant) parameter, while all other parameters are fixed; incremental procedure leads to the limit state multiplier. The simulation result is a set of parameter multipliers, numerically

representing the random variable λ_p . The histogram estimating the probability density function of λ_p represents probabilistic sensitivity of the system with respect to the chosen parameter.

There are cases of structural parameter not to be uniquely classified into a group of actions or resistances. Structural dimensions affect cross-section load carrying capacity, but a larger cross-section produces a greater dead load. No direct geometrical interpretation is given here, reliability must be assessed the other way round.



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Engineering structure – a portal frame

A simple planar bar structure, made of welded steel I–bars is subjected to a combination of loads. Geometrical imperfection, a horizontal displacement v is taken into account.

Sensitivity of structural limit state with respect to selected design variables is assessed. Probability distribution of the limit state of the structure is investigated, different variables are dominant.

Deterministic parameters: steel modulus of elasticity $\bar{E} = 205\ GPa$

geometrical characteristics of the cross–sections: total area, web

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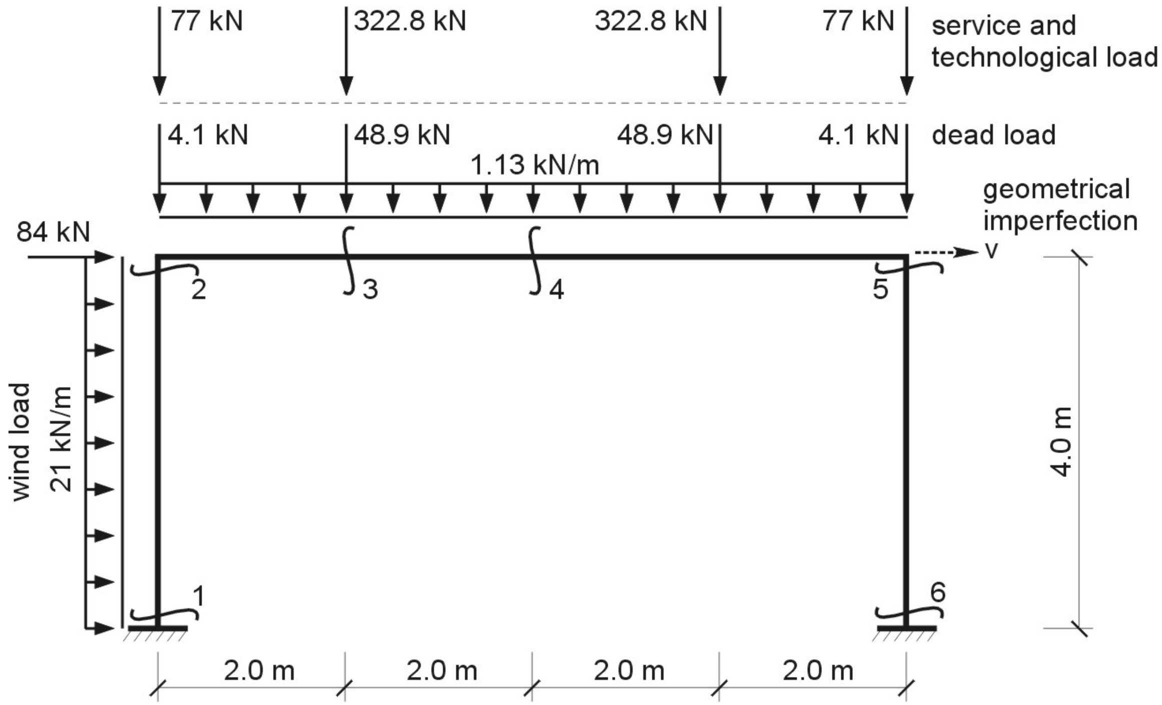
area, moment of inertia and section modulus of columns:

$$A_c = 193\,cm^2,\ A_{c_w} = 55.5\,cm^2, I_c = 120288\,cm^4, W_c = 4147.9\,cm^3$$

beam: $A_b = 205\,cm^2, A_{b_w} = 57.5\,cm^2,\ I_b = 137927\,cm^4, W_b = 4597.6\,cm^3$



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Random variables:

* live load: Gaussian (bounded) $\mu = 1.0$ $\sigma = 0.0667$ in (0.8; 1.2),

* wind load: Gumbel wind velocity variable (X): $u=0.2,$ $\alpha = 8.0$, wind pressure (Y) by $Y = X^2$, bounded by the range (0.0; 1.3)

* geometric imperfection: variable $VL(\omega)$ uniformly distributed in *(0.6, 1.0),* thus $v(\omega) = v_0 \cdot VL(\omega)$, where $v_0 = 0.0171\ m$

* yield stress of steel St3S - variable $NL(\omega)$ uniformly distributed in the range of *(0.9, 1.1)* – thus the random yield stress of steel is equal to $\sigma_0 \cdot NL(\omega)$, where $\sigma_0 = 235\,MPa$

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Six critical cross-sections are distinguished in the structure. In the incremental procedure of a single simulation step, the dominant variable increases, while other variables remain constant. In the step course limit state of a series of critical cross-sections is reached. T linearized limit state formula is applied

$$\frac{N_i(\omega)}{N_{Ri}(\omega)}+\frac{M_i(\omega)}{M_{Ri}(\omega)}+0.45\frac{T_i(\omega)}{T_{Ri}(\omega)}=1, \quad i\in\{1,2,3,4,5,6\}$$

here $N_i\left(\omega\right), T_i\left(\omega\right), M_i\left(\omega\right)$ are random cross-sectional forces. Their limit values at critical cross-sections 1, 2, 5 and 6 (i = 1, 2, 5, 6) are:

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$$N_{Ri}(\omega) = \overline{A}_c \cdot \sigma_0 \cdot NL(\omega) = 4535.5 \cdot NL(\omega)\ [kN]$$

$$M_{Ri}(\omega) = \overline{W}_c \cdot \sigma_0 \cdot NL(\omega) = 974.7 \cdot NL(\omega)\ [kNm]$$

$$T_{Ri}(\omega) = \overline{A}_{c_w} \cdot \sigma_0 \cdot NL(\omega) \cdot \frac{1}{\sqrt{3}} = 753.0 \cdot NL(\omega)\ [kN]$$

The limit values at critical cross-sections 3 and 4 (i = 3, 4) are:

$$N_{Ri}(\omega) = \overline{A}_b \cdot \sigma_0 \cdot NL(\omega) = 4817.5 \cdot NL(\omega)\ [kN]$$

$$M_{Ri}(\omega) = \overline{W}_b \cdot \sigma_0 \cdot NL(\omega) = 1080.4 \cdot NL(\omega)\ [kNm]$$

$$T_{Ri}(\omega) = \overline{A}_{b_w} \cdot \sigma_0 \cdot NL(\omega) \cdot \frac{1}{\sqrt{3}} = 780.1 \cdot NL(\omega)\ [kN]$$

Three probabilistic sensitivity variants of structural limit states are regarded, due to the following variables dominant: a) live load, b) wind load, c) yield stress of steel

a) dominant live load

A simulation procedure is performed, specific operations of every single simulation step are conducted, live load is the dominant design variable. The single simulation step operations are:

* design variables are generated: material and geometric properties, loads and geometric imperfection

* the live load - the chosen (dominant) variable is ordered a one-

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parametrical increment, the other design data are fixed. While limit state is achieved the limit live load multiplier is recorded.

A population of 100000 problem runs leads to limit state sensitivity due to live load. The histogram detects failure probability $p_f = P(\lambda_S < 1.0) = 0.0001$, reliability $R = 1 - p_f = 0.9999$ (more histogram parameters available, not mentioned here or further on)

Remark: 10000 realizations show only one value λ_S less than 1.0.

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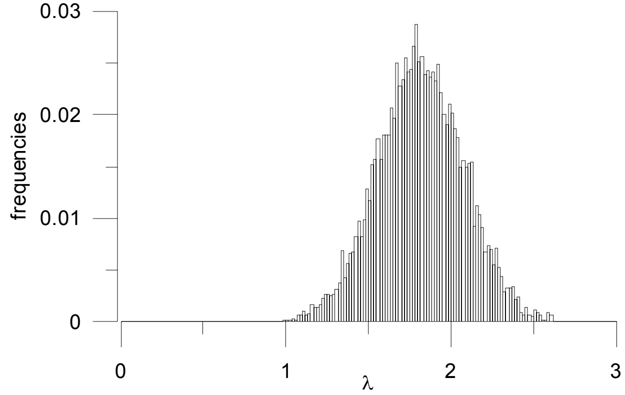

Histogram of the limit state, with respect to live loads

Dominant variable - wind load

Independent performance of the MCS algorithm is carried out, the wind load is regarded dominant now. A wind load multiplier is the outcome of the incremental procedure of each step. All simulation steps lead to histogram of wind load multipliers. Despite diverse

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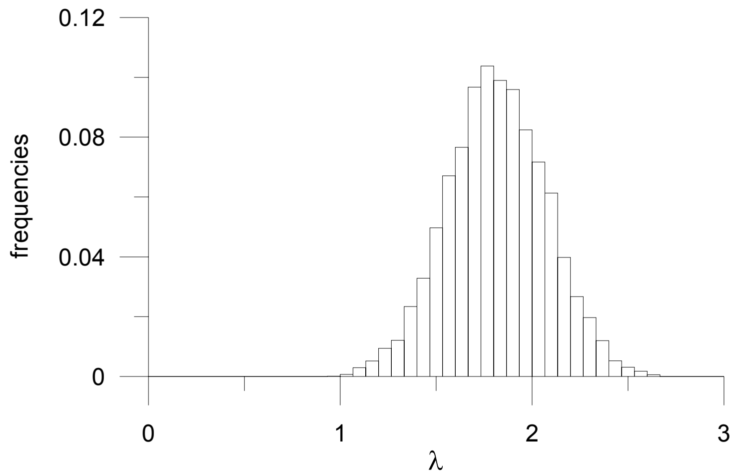
histogram parameters, failure probability $p_f = P(\lambda_S < 1.0) = 0.0001$,

reliability $R = 1 - p_f = 0.9999$.

Remark: 10000 realizations show only one value λ_S less than 1.0 too



Histogram of the limit state involving the wind load

Dominant variable - resistance parameter

The third variant of calculations with the steel yield stress as the dominant variable is presented below. Performing the operations of a single simulation step, the actions, i.e. the loads and the geometrical imperfection are fixed, whereas the resistance parameter is subjected to uni-parametrical increment.

The histogram of the resistance parameter multipliers is presented below. It illustrates structural limit state sensitivity related to the yield stress of steel. It is interpreted in a different way than in the previous variants referring to actions. Reliability and failure

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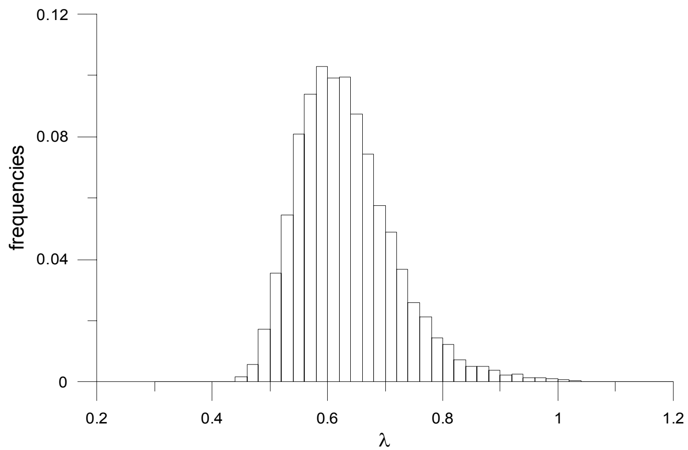
probability definitions are inverse. The estimated failure probability

$$p_f = P(\lambda_r > 1.0) = \frac{1}{10000} = 0.0001, \text{ reliability } R = 1 - p_f = 0.9999$$



Histogram of the limit state with respect to the yield stress of steel

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The idea of probabilistic sensitivity of limit states of structures is presented with respect to selected design variables. The definition of probabilistic sensitivity can be used in an analytical form only in the simplest structural cases. The probability density function of the multipliers of dominant variables are regarded sensitivity measures.

The dedicated Monte Carlo simulation procedure can be used to formulate the definition of probabilistic sensitivity of engineering structures. Thus the MCS procedure can be regarded a tool for computational science.

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Computational remark: The direct MCS procedures is the only category of simulation methods possible to solve a range of problems presented herein. An analysis of more complicated structures is bound to perform a huge amount of computational effort.