PROBABILISTIC METHODS APPLIED TO TIMBER STRUCTURE DESIGN

Ultimate and serviceability limit states are fundamental for engineering structure design. The limit state theory forms in a precise way the rules and theorems to estimate structural limit states. On the other hand the limit states are translated in the engineering standards into partial safety factor design.

The basic variables of structural design, i.e. loads, geometric and material parameters, geometric imperfections are subjected to inherent variation. The fully probabilistic structural approach assumes all or selected abovementioned parameters random variables of prescribed types of probability distribution functions.

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Consequently the structural limit state analysis may also be performed in probabilistic terms.

Theoretical background of probabilistic methods in mechanics have been developed in the half of the 20th century in analytical form. The progress in numerical methods and computational tools made it possible to conduct probabilistic analysis of complex systems, involving time consuming procedures. The research works were directed to computationally effective algorithms, with regard to their application in computational routines.

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Levels of reliability assessment

Throughout the centuries building engineering structures were designed on the basis of intuition and experience. Even in the rise of the 20th century a deterministic character of action and resistance parameters of civil engineering was manifested. The turning point in design philosophy development was marked by the progress in probability theory as an autonomous mathematical discipline in the 20s of the 20th century. The next decade brings the pioneering works formulating the structural design problem in probabilistic terms.

The research works conducted throughout decades distinguished the following three commonly denoted levels of probabilistic structural analysis (or reliability assessment only):

1. Level 1 – basic variables: loads, material and geometric data, geometric imperfections are noted an inherent scatter of values, but random variation is considered only in the form of partial factors – load and resistance factors. The limit state design, popular in engineering standards throughout the world represents the level 1 reliability assessment procedure. The computational design routine is deterministic, the only random impact is the assessment characteristic values (fractiles of variables) and calibration of partial safety factors
2. Level 2 – basic variables are represented by their statistical (sample) moments, estimating unknown moments of random variables: mean value and standard deviation. The common 2nd level reliability measure is the so-called reliability index. While all basic variables are assumed Gaussian and the limit state

function assumed linear the 2nd level methods estimate structural reliability (probability of failure) accurately enough for engineering design

3. Level 3 – direct estimation of failure probability. The analytical variant requires a multiple integral computation of a joint probability density function of a basic variable random vector. This routine is effective in the simplest cases only, of a low vector dimension. Numerical integration may be applied here, but the Monte Carlo simulation is the most effective computational means.

Complex engineering issues, not allowing for the explicit limit state criterion in the form of functions of basic variables require a problem-oriented Monte Carlo simulation procedure. This procedure results in the limit state probability distribution, estimated by a histogram. It further allows to estimate reliability of failure probability. This methodology is therefore qualified to the 3rd level method of reliability assessment.

Probabilistic limit state analysis of a system

The basic variables of the regarded problem – loads, geometric and material parameters, imperfections form a multidimensional random vector $\mathbf{X} = \{X_1 \quad X_2 \quad . \quad X_n\}$. The limit state function $g\{\mathbf{X}\}$ is defined in this space, subdividing it into the safety region ($g\{\mathbf{X}\} > 0$) and failure region ($g\{\mathbf{X}\} < 0$).

The condition $g\{\mathbf{X}\} = 0$ refers to the so-called limit state surface. The probability of failure (limit state) p_f may be obtained by integrating a joint probability density function $f(\mathbf{X})$ of variables **X** in the failure region. It is, however, a purely theoretical expression –

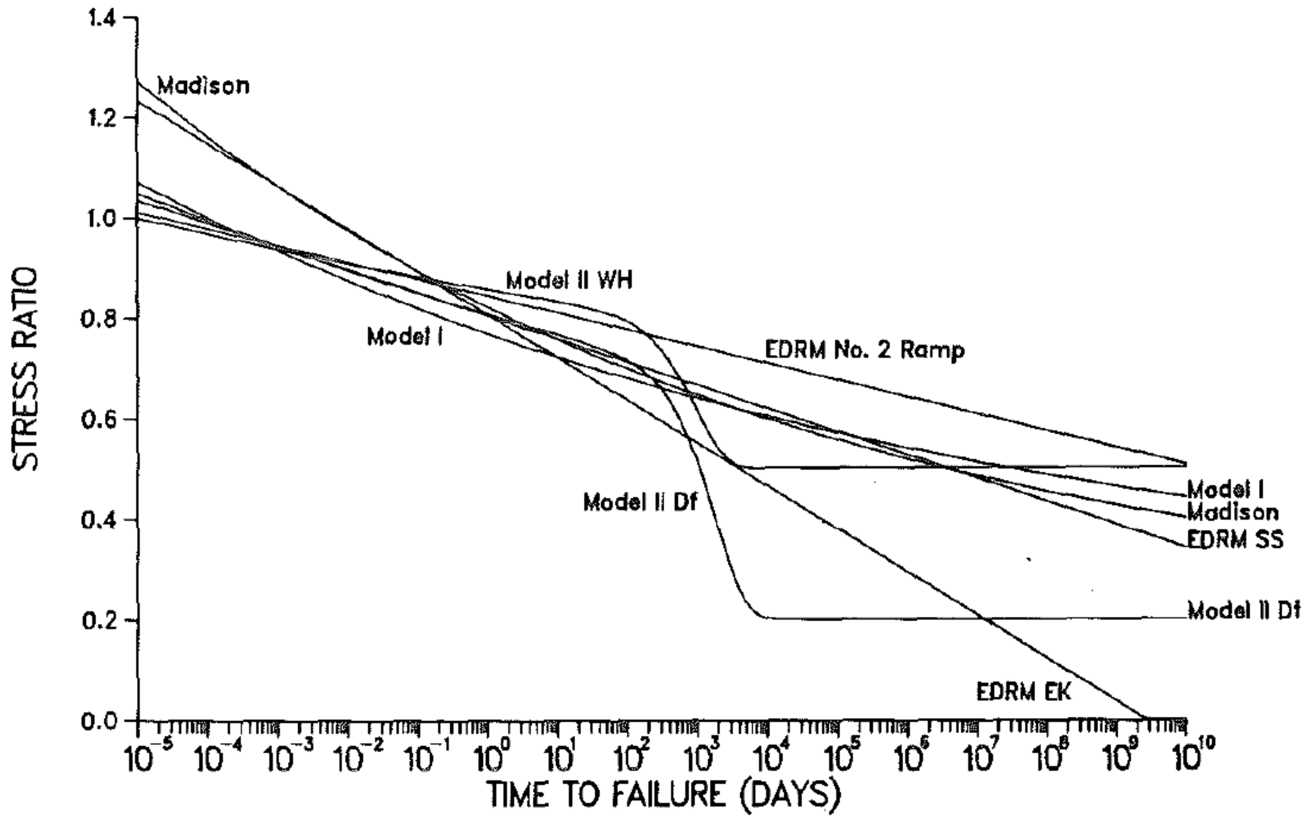
even in the simplest engineering cases it is impossible to use its analytical form, thus numerical methods to estimate failure probability are investigated, the Monte Carlo simulation is the prime numerical tool here. Referring to the 2nd level reliability methods based on the so-called reliability index β, assuming all basic variables Gaussian distributed, the reliability index may be expressed as $\beta = \Phi^{-1}\left(1 - p_f\right)$, here Φ is the Laplace function.

The so-called fundamental reliability problem may be formulated by means of two basic variables only: random load effect *S* and random resistance *R*, both presented in the same units. Therefore the limit state function takes the form $g\left\{R, S\right\} = R - S$, assuming *R*, *S* and *g* Gaussian variables. Thus the reliability index is a ratio of mean value and standard deviation of the function *g*, i.e. the reciprocal of its coefficient of variation.

Probabilistic issues of timber structures

Resistance of timber structural elements is affected by loading history, amplitude and location. In selected cases the random variable model proves insufficient, triggering the need to introduce the random process model. As an example, live load in its time-variant performance may be represented by a Poisson pulse process. Random, time-variant load acts upon the correction of timber resistance parameter, in the form of coefficients – relations between allowable stress level and time to failure. An example of such a relation is the Madison curve, shown in the figure, complemented by alternative relations. The ordinates show the so-called stress ratio i.e.

the ratio of actual and breaking stresses, the abscissas is time to failure in a logarithmic form. The damage accumulation concept is applied, here the damage extent is quantified by a variable $\alpha(t)$, of a zero initial value, equal unity in the failure instant. The time derivative $\alpha'(t)$ may be regarded failure rate (velocity). The Table shows possible models incorporated in the Figure, taken an appropriate form of $\alpha'(t)$. The corresponding constants in the formulae are calibrated on the experimental basis.


STRESS RATIO
TIME TO FAILURE (DAYS)
Madison
Model II WH
Model I
EDRM No. 2 Ramp
Model II Df
Model I
Madison
EDRM SS
Model II Df
EDRM EK

One of the possible forms to be implemented into computations is the exponential function of the stress degree, known as ERDM (exponential damage rate model).

Model (1)	Reference (2)	Damage rate (3)	σ_o (4)	A (5)	B (6)	C (7)	Material (8)
Madison Curve	20	$A(\sigma - \sigma_o)^B$	0.1803	$1.516 \times 10^4\ \text{day}^{-1}$	21.6	—	Douglas-fir Small clears
Model I	2	$A(\sigma - \sigma_o)^B \alpha^C$	0.2	$4.08 \times 10^6\ \text{day}^{-1}$	26.9	0	Douglas-fir Small clears
Model II DF	2	$A(\sigma - \sigma_o)^B + C\alpha$	0.2	$2.64 \times 10^7\ \text{day}^{-1}$	34.2	1.90×10^{-2}	Douglas-fir Small clears
Model II WH	7		0.5	$1.732 \times 10^{15}\ \text{day}^{-1}$	34	3.60×10^{-2}	Western Hemlock 2×6, No. 2
EDRM No. 2 Ramp	11	$\exp(-A + B\sigma)$	—	57.73 ln (day)	68.49	—	No. 2 Ramp
EDRM SS	11		—	40.00 ln (day)	49.75	—	Select Structural
EDRM EK	10		—	21.72 ln (day)	26.95	—	Douglas-fir, 2×4 edge knot lumber

Taking the symbols: T_L – predicted structural lifetime, T_f – time to failure, the limit state function may be expressed by one of the formulae: $g_1(\mathbf{X}) = T_f - T_L$ or $g_2(\mathbf{X}) = 1 - \alpha(T_L)$. It is possible to compare reliability measures of systems – reliability index β and failure probability p_f , with respect to the number of simulations, time model of load action and duration of impulses and breaks. In each case, excluding selected exceptions, the result convergence was satisfactory. Referring to the limit state method procedures it allows to calibrate the material factor, equal 0.75 and 0.9 with respect to the snow and live load action, respectively.

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Probabilistic structural mechanics, the research field to interpret standard design conditions is a highly developed discipline with regard to steel and concrete structures. Probabilistic approach to timber structures requires taking into account the time impact into loading history and strength / resistance, thus involving more advanced models. Several approaches are denoted here, the one, presented, is based on the damage accumulation concept and time variation of selected parameters. It is a discipline of ongoing research and development, illustrated by publications to show recent state-of-the-art progress.

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