$$\int_0^4 dx \int_2^3 (x-y^2)\,dy$$

$$k1 \quad \int_2^3 (x-y^2)\,dy = \left[xy - \frac{1}{3}y^3\right]\Big|_{y=2}^{3} = 3x - \frac{1}{3}\cdot 3^3 - \left(2x - \frac{1}{3}\cdot 2^3\right) =$$

$$= x - \frac{19}{3}$$

$$k2 \quad \int_0^4 \left(x - \frac{19}{3}\right) dx = \left[\frac{1}{2}x^2 - \frac{19}{3}x\right]\Big|_{x=0}^{4} = \frac{1}{2}\cdot 4^2 - \frac{19}{3}\cdot 4 - \left(\frac{1}{2}\cdot 0^2 - \frac{19}{3}\cdot 0\right) =$$

$$= 8 - \frac{76}{3} = \frac{24}{3} - \frac{76}{3} = -\frac{52}{3}$$

Odp.: $\int_0^4 dx \int_2^3 (x-y^2)\,dy = -\frac{52}{3}$

$$\int_{-1}^{2} dy \int_{0}^{3} (x+xy^2)\,dx$$

K1 $\int_{0}^{3} (x+xy^2)\,dx = \left[\frac{1}{2}x^2+\frac{1}{2}x^2y^2\right]\Big|_{x=0}^{3} = \frac{1}{2}\cdot 3^2+\frac{1}{2}\cdot 3^2 y^2-(0+0) =$

$= \frac{9}{2}+\frac{9}{2}y^2$

K2 $\int_{-1}^{2}\left(\frac{9}{2}+\frac{9}{2}y^2\right)dy = \left[\frac{9}{2}y+\frac{3}{2}y^3\right]\Big|_{y=-1}^{2} = \frac{9}{2}\cdot 2+\frac{3}{2}\cdot 2^3-\left(\frac{9}{2}\cdot(-1)+\frac{3}{2}\cdot(-1)^3\right) =$

$= 9+12-(-6) = 27$

Odp.: $\int_{-1}^{2} dy \int_{0}^{3} (x+xy^2)\,dy = 27$

$$\iint\limits_{[0,1]\times[-1,1]} x^2 y^2 \, dx\,dy = \int_0^1 x^2 dx \cdot \int_{-1}^{1} y^2 dy = \left(\frac{1}{3}x^3\Big|_{x=0}^{1}\right)\cdot\left(\frac{1}{3}y^3\Big|_{y=-1}^{1}\right) =$$

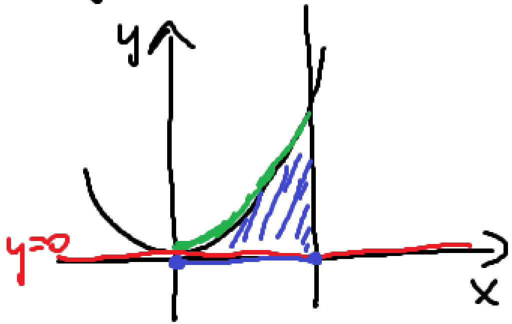
$$= \left(\frac{1}{3} - 0\right)\cdot\left(\frac{1}{3} - \left(-\frac{1}{3}\right)\right) = \frac{2}{9}$$

$$\iint\limits_{[-\frac{\pi}{4},\frac{\pi}{4}]\times[0,\frac{\pi}{4}]} \sin(x+y)\,dx\,dy = \iint\limits_{[-\frac{\pi}{4},\frac{\pi}{4}]\times[0,\frac{\pi}{4}]} (\sin x\cos y + \cos x \sin y)\,dx\,dy =$$

$$= \iint\limits_{[-\frac{\pi}{4},\frac{\pi}{4}]\times[0,\frac{\pi}{4}]} \sin x\cos y\,dx\,dy + \iint\limits_{[-\frac{\pi}{4},\frac{\pi}{4}]\times[0,\frac{\pi}{4}]} \cos x\sin y\,dx\,dy = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin x\,dx \cdot \int_0^{\frac{\pi}{4}} \cos y\,dy + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos x\,dx \cdot \int_0^{\frac{\pi}{4}} \sin y\,dy =$$

$$= 0 + \left(\sin x\Big|_{x=-\frac{\pi}{4}}^{\frac{\pi}{4}}\right)\cdot\left(-\cos y\Big|_{y=0}^{\frac{\pi}{4}}\right) = \left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right)\cdot\left(-\frac{\sqrt{2}}{2} + 1\right) = \sqrt{2} - 1$$

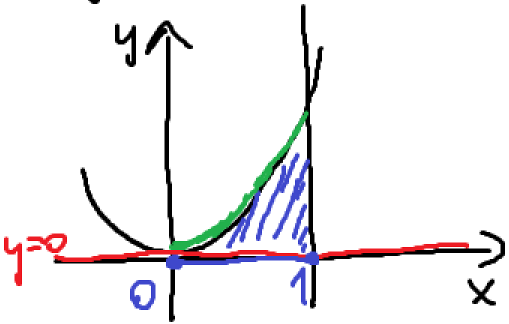
1) $y = 0$, $x = 1$, $y = x^2$

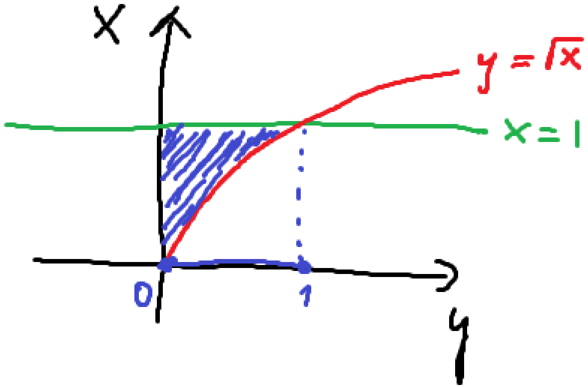


$x \in [0,1]$

$0 \leq y \leq x^2$

1) $y=0$, $x=1$, $y=x^2$





$x \in [0,1]$

$0 \leq y \leq x^2$

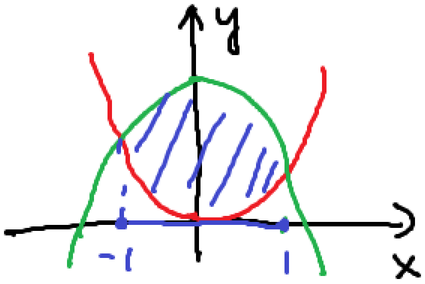
Obszar normalny względem Ox

$y \in [0,1]$

$\sqrt{x} \leq x \leq 1$

Obszar normalny względem Oy

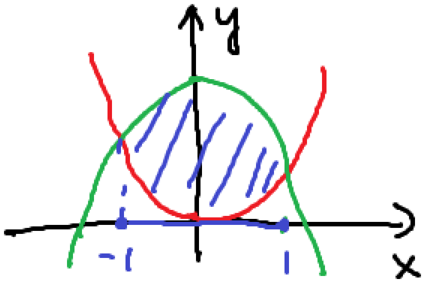
3) $y = -x^2 + 2 \quad , \quad y = x^2$

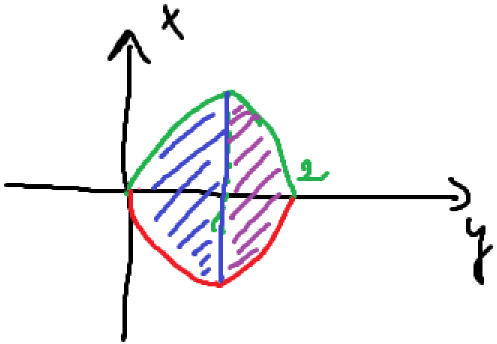


$x \in [-1, 1]$

$x^2 \leq y \leq -x^2 + 2$

3) $y = -x^2 + 2 \quad , \quad y = x^2$





$x \in [-1, 1]$

$x^2 \leq y \leq -x^2 + 2$

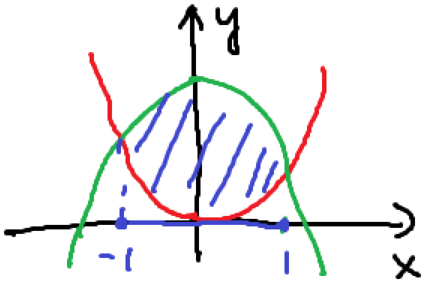
$y \in [0, 1]$

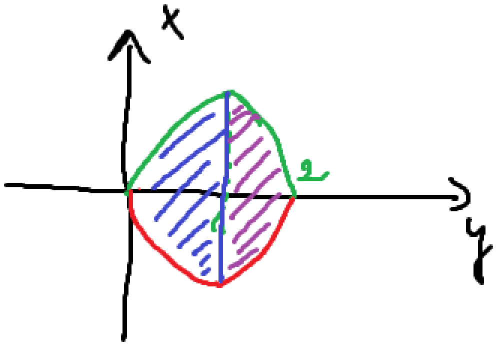
$-\sqrt{y} \leq x \leq \sqrt{y}$

$y \in [1, 2]$

$-\sqrt{2-y} \leq x \leq \sqrt{2-y}$

3) $y = -x^2 + 2$, $y = x^2$





$x \in [-1, 1]$

$x^2 \leq y \leq -x^2 + 2$

Obszar normalny względem Ox

$y \in [0, 1]$ $\qquad$ $y \in [1, 2]$

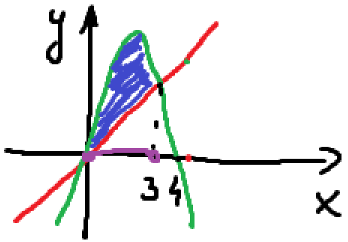
$-\sqrt{y} \leq x \leq \sqrt{y}$ $\qquad$ $-\sqrt{2-y} \leq x \leq \sqrt{2-y}$

Obszar normalny względem Oy

$$p(y) = \begin{cases} -\sqrt{y} ; & y \in [0,1] \\ -\sqrt{2-y} ; & y \in (1,2] \end{cases}$$

$$q(y) = \begin{cases} \sqrt{y} ; & y \in [0,1] \\ \sqrt{2-y} ; & y \in (1,2] \end{cases}$$

$$\iint_D (x^2 - xy^2)\,dxdy \qquad D = \{(x,y) \in \mathbb{R}^2 \,;\; y \geq x,\; y \leq 4x - x^2\}$$

D: 

$$\begin{cases} y = x \\ y = 4x - x^2 \end{cases} \Rightarrow x = 4x - x^2 \Rightarrow x^2 - 3x = 0$$

$$\Rightarrow x = 0 \text{ lub } x = 3$$

$x \in [0,3]$

$x \leq y \leq 4x - x^2$

$$\iint_D (x^2 - xy^2)\,dxdy = \int_0^3 \left[\int_x^{4x-x^2} (x^2 - xy^2)\,dy \right] dx$$

k1) $\displaystyle\int_{x}^{4x-x^2} (x^2 - xy^2)\,dy = \left[x^2 y - \frac{1}{3} x y^3\right]\Bigg|_{y=x}^{4x-x^2} =$

$= x^2(4x-x^2) - \frac{1}{3}x(4x-x^2)^3 - \left(x^2 \cdot x - \frac{1}{3}x \cdot x^3\right) =$

$= 4x^3 - x^4 - \frac{64}{3}x^4 + 16x^5 - 4x^6 + \frac{1}{3}x^7 - x^3 + \frac{1}{3}x^4 =$

$= \frac{1}{3}x^7 - 4x^6 + 16x^5 - 22x^4 + 3x^3$

k2) $\displaystyle\int_0^3 \left(\frac{1}{3}x^7 - 4x^6 + 16x^5 - 22x^4 + 3x^3\right)dx = \left[\frac{1}{24}x^8 - \frac{4}{7}x^7 + \frac{8}{3}x^6 - \frac{22}{5}x^5 + \frac{3}{4}x^4\right]\Bigg|_0^3 =$

$= -\frac{11421}{280}$