# PROCESS OF CONVERGENCE IN EU
# PRODUCTION FUNCTION – TOOL FOR ANALYSIS

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# KEY QUESTIONS

Why do we use **growth models**?

In order to 'discover':

1. What are the determinants of economic growth? → **theoretical models of economic growth (growth theories)**
2. What is the empirical validation of economic growth theories? → **empirical models of economic growth**

# THEORETICAL MODELS OF ECONOMIC GROWTH

- Describe the ***relationship(s)*** between economic growth/development (usually measured as a pace of growth of GDP per capita) and factors of growth (***growth determinants***)
- Used in order to understand ***why*** some countries/regions grow faster then others
- Allow us to ***isolate*** the impact of a given variable (potential growth determinant) on economic growth process
- Serve as a ***basics for empirical analysis*** of economic growth (based on real data)
- Each theory has some strengths and some weaknesses!

# THEORETICAL MODELS OF GROWTH

Link economic growth and factors (determinants) of growth in a form of theoretical equation:

$$\mathbf{Y=f(X)}$$

where

Y- a measure of economic growth

X – a set of factors influencing growth /development f – link function

# THEORY: DEFINES MAIN COMPONENTS OF ECONOMIC GROWTH

- **Capital Accumulation, investments** in physical and human capital
- Growth in population and **labor force**
- **Technological progress**

# PRODUCTION FUNCTION

The production function represents the transformation of inputs (labor (*L*), capital (*K*)) into final outputs (goods and services for a certain time period

- **Basic form:** output (Y) depends on two **factors of production** – capital (K) and Labor (L) which are allowed to change through time:

- $$Y(t) = F\big(K(t), L(t)\big)$$

$Y(t)$ is the total amount of production of the good at time t

$K(t)$ is the capital stock at time t

$L(t)$ is the labour force at time t

# Production function per worker

**Production function can be easily transformed into:**

$$Y/L = f\left(K/L\,,1\right) \text{ or } y = f(k)$$

***y=Y/L***
**product per person (employed) = output per worker**

***k=K/L***
**capital to labor ratio (typically higher in advanced economies)**

# TYPES OF RETURNS TO SCALE

- **Constant returns to scale –** increase in all factors of production by the same proportion causes equiproportional increase in output

- When $F(\alpha K, \alpha L) = \alpha F(K, L)$ → constant return to scale
- When $F(\alpha K, \alpha L) > \alpha F(K, L)$ → increasing return to scale
- When $F(\alpha K, \alpha L) < \alpha F(K, L)$ → decreasing return to scale

$\alpha$ is always > 1

# COBB-DOUGLAS PRODUCTION FUNCTION (1)

$$Y = F(K, L) = K^{\alpha} L^{\beta}$$

**α** and **β** are the **output elasticities** of labor and capital C-D: **0<α<1 and 0<β <1**

[Output elasticity measures the responsiveness of output to a change in levels of either labor or capital used in production, *ceteris paribus*]

C-D: assumption that **α+β=1**
→ **constant returns to scale**

# COBB-DOUGLAS PRODUCTION FUNCTION – ASSUMPTIONS (1)

**1. α+β=1 (constant returns to scale)**

if for example L and K are each increased by 20%,
then Y increases by 20%

The C-D production function can therefore be rewritten as:

$$Y = F(K, L) = K^{\alpha} L^{1-\alpha}$$

# COBB-DOUGLAS PRODUCTION FUNCTION – ASSUMPTIONS (2)

→1. **Positive and Diminishing Marginal Product of capital and labor**

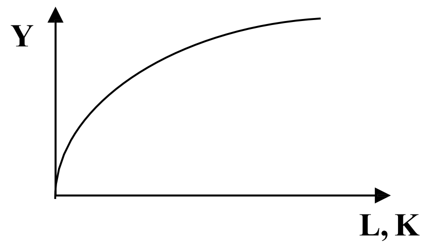
$$\frac{\partial F}{\partial K} > 0 \qquad \frac{\partial^2 F}{\partial K^2} < 0$$

$$\frac{\partial F}{\partial L} > 0 \qquad \frac{\partial^2 F}{\partial L^2} < 0$$

$$\frac{\partial F}{\partial K} > 0$$

$$\frac{\partial F}{\partial L} > 0$$

Positive first derivative
→curve is increasing



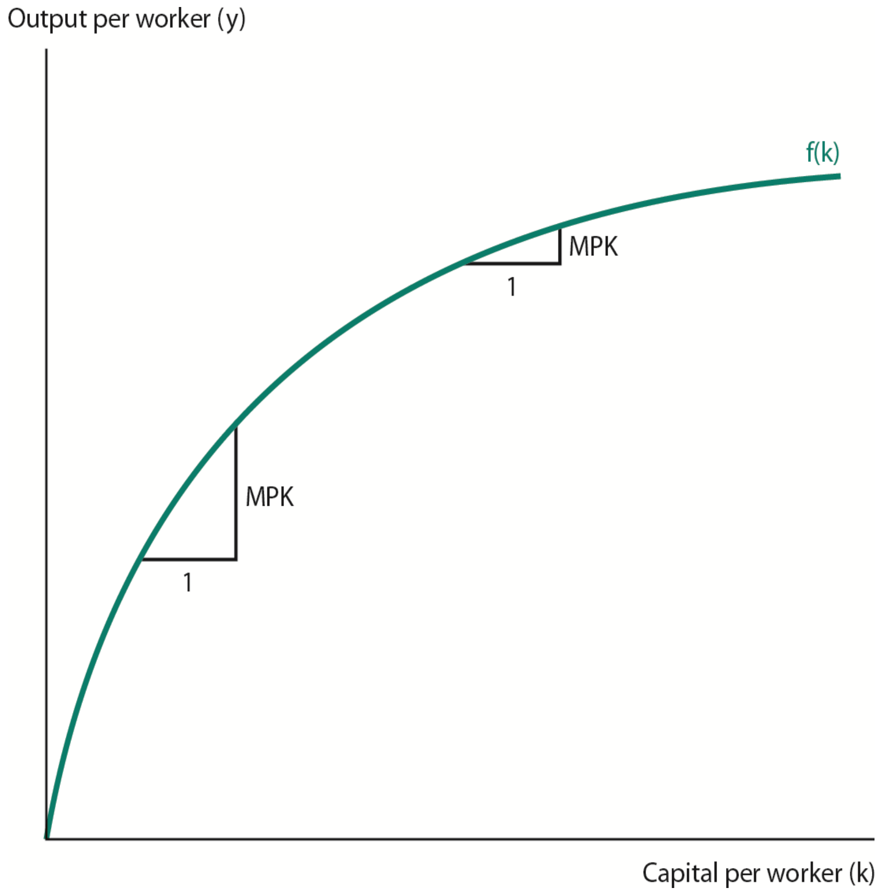

Law of diminishing returns implies **diminishing marginal productivity of capital and labour**

$$\frac{\partial^2 F}{\partial K^2} < 0$$

$$\frac{\partial^2 F}{\partial L^2} < 0$$

Negative second derivative→curve has inverted U shape

# A PRODUCTION FUNCTION WITH DIMINISHING MARGINAL PRODUCT OF CAPITAL




The production function shows how the amount of capital per worker $k$ determines the amount of output per worker $y = f(k)$.

The **slope of the production function** is the marginal product of capital per worker: if $k$ increases by 1 unit, $y$ increases by $MP_K$ units.

# COBB DOUGLAS PRODUCTION FUNCTION - SUMMARY

$$Y = F(K, L) = K^{\alpha} L^{1-\alpha}$$

- **α** elasticity of production with respect to capital

  Interpretation: If K increases by 1% then production Y increases by α%, *ceteris paribus*

- **(1- α )** elasticity of production with respect to labour Interpretation: If L

  increases by 1% then production Y increases by (1-α)%, *ceteris paribus*

*Weights:* α and (1- α) reflect capital and labour share in output (income)

For constant returns to scale: **0<α<1 [α +(1- α)=1]**

# FACTORS SHARE

Factor share: **how much income a factor of production (for example capital or workers) receive as a proportion of income.**

For example, in an economy there are 100 workers, each earns 700€ (wage=700€) and 50 units of capital (machinery), rental rate is 600€ per unit of machinery

Production=Income=100,000€

Factor shares for workers =(100 ×700)/100,000 =0.7

Factor shares of capital = (50 × 600)/100,000 = 0.3

# FACTOR SHARES IN COBB-DOUGLAS PRODUCTION FUNCTION

1. Production factors' payments:

w - wage = price of each unit of labour

r -interest rate, „price" of each unit of capital

2. Profit= Income – Total costs

$$\pi = Y - (wL + rK) = K^{\alpha}L^{1-\alpha} - (wL + rK)$$

profit is maximized when the marginal revenue product of labor is equal to the marginal cost of labor (the wage) and the marginal revenue product of capital is equal to the marginal cost of capital (the rent)

$$\frac{\delta\pi}{\delta L} = 0 \quad \Rightarrow \quad w = MPL$$

$$\frac{\delta\pi}{\delta K} = 0 \quad \Rightarrow \quad r = MPK$$

3. Marginal product of labor (MPL) and marginal product of capital (MPK)

$$MPL = \frac{\delta Y}{\delta L} = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}$$

$$MPK = \frac{\delta Y}{\delta K} = (\alpha)\left(\frac{L}{K}\right)^{1-\alpha}$$

4. Factor shares: fraction of income spent on a resource as the expenditure on the resource (price times amount of resource used) divided by the total income Y

Labor's share =(wL)/Y (3)
Capital's share=(rK)/Y (4)

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# 5. SUBSTITUTIONG EQ. 1 IN EQ. 3 AND EQ. 2 IN EQ 4 AND USING THE FORMULA FOR COBB-DOUGLAS PRODUCTION FUNCTION WE OBTAIN:

$$Labor's\ share = \frac{\left(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}\right)L}{K^{\alpha}L^{1-\alpha}} = (1-\alpha)$$

$$Capital's\ share = \frac{\left((\alpha)\left(\frac{L}{K}\right)^{1-\alpha}\right)K}{K^{\alpha}L^{1-\alpha}} = \alpha$$

Conclusion: regardless of the amount of labor or capital used, or their relative prices, **the share of income spent on labor and capital are constant.**

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