PROCESS OF CONVERGENCE IN EU
GROWTH DECOMPOSITION AND CONVERGENCE IN THE SOLOW MODEL

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*Slides adapted from teaching materials produced by dr hab. Joanna Wolszczak-Derlacz

Solow decomposition, growth accounting

Growth decomposition - growth accounting:

- procedure which **decomposes** a country's growth rate of output and shows to what extent it can be attributed to the accumulation of factors of production (physical capital, human capital) and to what extent – to the technological progress
- A technique which allows us to **derive the growth rate of productivity** (technological progress – directly unobservable!)

Solow decomposition – output growth (Y)

$$Y = AK^{\alpha}L^{1-\alpha}$$

$$\ln Y = \ln A + \alpha \ln K + (1-\alpha)\ln L$$

$$\frac{d\ln Y}{dt} = \frac{d\ln A}{dt} + \alpha\frac{d\ln K}{dt} + (1-\alpha)\frac{d\ln L}{dt}$$

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}$$

$$g_Y = g_A + \alpha g_K + (1-\alpha)g_L$$

Solow decomposition – output (Y) growth

$$Y = AK^{\alpha}L^{1-\alpha}$$

Level of production depends on capital stock, number of people employed and the level of technology

$$g_Y = g_A + \alpha g_K + (1-\alpha) g_L$$

Rate of growth of production depends on:

- the rate of growth of A (thus on the pace of **technological progress**)
- the rate of growth of capital
- the rate of growth of labor (→ population growth)
- parameter α- showing how capital and labor combine to produce output

Solow decomposition – output *per worker* (y) growth

$$y = Ak^{\alpha}$$

$$\ln y = \ln A + \alpha \ln k$$

$$\frac{d \ln y}{dt} = \frac{d \ln A}{dt} + \alpha \frac{d \ln k}{dt}$$

$$\frac{dk}{dt} = \dot{k}, \frac{dA}{dt} = \dot{A}$$

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + \alpha \frac{\dot{k}}{k}$$

$$g_y = g_A + \alpha g_k$$

Solow decomposition – output *per worker* (y) growth

$$y = Ak^{\alpha}$$

<u>Level</u> of production *per worker* depends on capital stock *per worker* and the level of technology

$$\downarrow$$

$$g_y = g_A + \alpha \cdot g_k$$

<u>Rate of growth</u> of production per worker (y) depends on:

- the rate of growth of A (thus on the pace of **technological progress**)
- the rate of growth of capital per worker
- parameter α- showing how capital and labor combine to produce output

Growth accounting with two types of capital (physical and human capital) (1)

$$y=\frac{Y(t)}{L(t)}=\frac{AF(K,H,L)}{L}=AF(\frac{K}{L,},\frac{H}{L},\frac{L}{L})=A\left(\frac{K}{L}\right)^{\alpha}\left(\frac{H}{L}\right)^{1-\alpha}=Ak^{\alpha}h^{1-\alpha}$$

Taking logs and differentiating:

$$\frac{\dot{y}}{y}=\frac{\dot{A}}{A}+\alpha\frac{\dot{k}}{k}+(1-\alpha)\frac{\dot{h}}{h}\longrightarrow\boxed{g_y=g_A+\alpha\cdot g_k+(1-\alpha)g_h}$$

Growth accounting with two types of capital (physical and human capital) (2)

$$\boxed{g_y = g_A + \alpha \cdot g_k + (1-\alpha) g_h}$$

<u>Rate of growth of output per worker</u> depends on:

- the rate of growth of A (thus on the pace of **technological progress**)
- the rate of growth of *physical* capital per worker (→capital accumulation)
- the rate of growth of *human* capital per worker (→ increase in knowledge, capacities ...)
- Parameter α- showing how capital and labor combine to produce output

Solow residual and TFP (1)

- Using real statistical data we can calculate rates of growth of output, capital and population (~labour force) and alpha parameter (= capital's share of income)
- Thanks to the decomposition of Solow we can calculate „the rest" - the so called **'Solow residual'** – rate of growth of ***TFP*** (*Total Factor Productivity*)

Solow residual and TFP (2)

$$\frac{\dot{y}}{y} = \boxed{\frac{\dot{A}}{A}} + \alpha \frac{\dot{k}}{k}$$

Calculated with real statistical data

$$\text{TFPgrowth} = \frac{\dot{A}}{A} = \frac{\dot{y}}{y} - \alpha \frac{\dot{k}}{k}$$

$$\frac{\dot{Y}}{Y} = \boxed{\frac{\dot{A}}{A}} + \alpha \frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}$$

Calculated with real statistical data

$$\text{TFPgrowth} = \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} - \alpha \frac{\dot{K}}{K} - (1-\alpha)\frac{\dot{L}}{L}$$

Solow decomposition - results

Table 3.5 The Solow Decomposition (average annual growth rates)

***(b)* 1987–1997**

	GDP	Contribution of inputs	Residual
France	2.0	1.1	1.0
Germany*	1.4	0.2	1.2
Netherlands	2.9	1.8	1.1
United Kingdom	2.2	1.4	0.7
United States	3.0	2.5	0.5
Japan	2.7	1.4	1.3

* 1991–1997

***(c)* 1997–2006**

	GDP	Contribution of inputs	Residual
France	2.2	1.3	1.0
Germany	1.4	0.6	0.8
Netherlands	2.3	1.4	0.9
United Kingdom	2.7	1.7	1.0
United States	3.0	1.9	1.1
Japan	1.2	0.1	1.1

Source: Burda&Wyplosz (2009), Macroeconomics A European Text, p.76

Solow model dynamics – sort run and long run effects

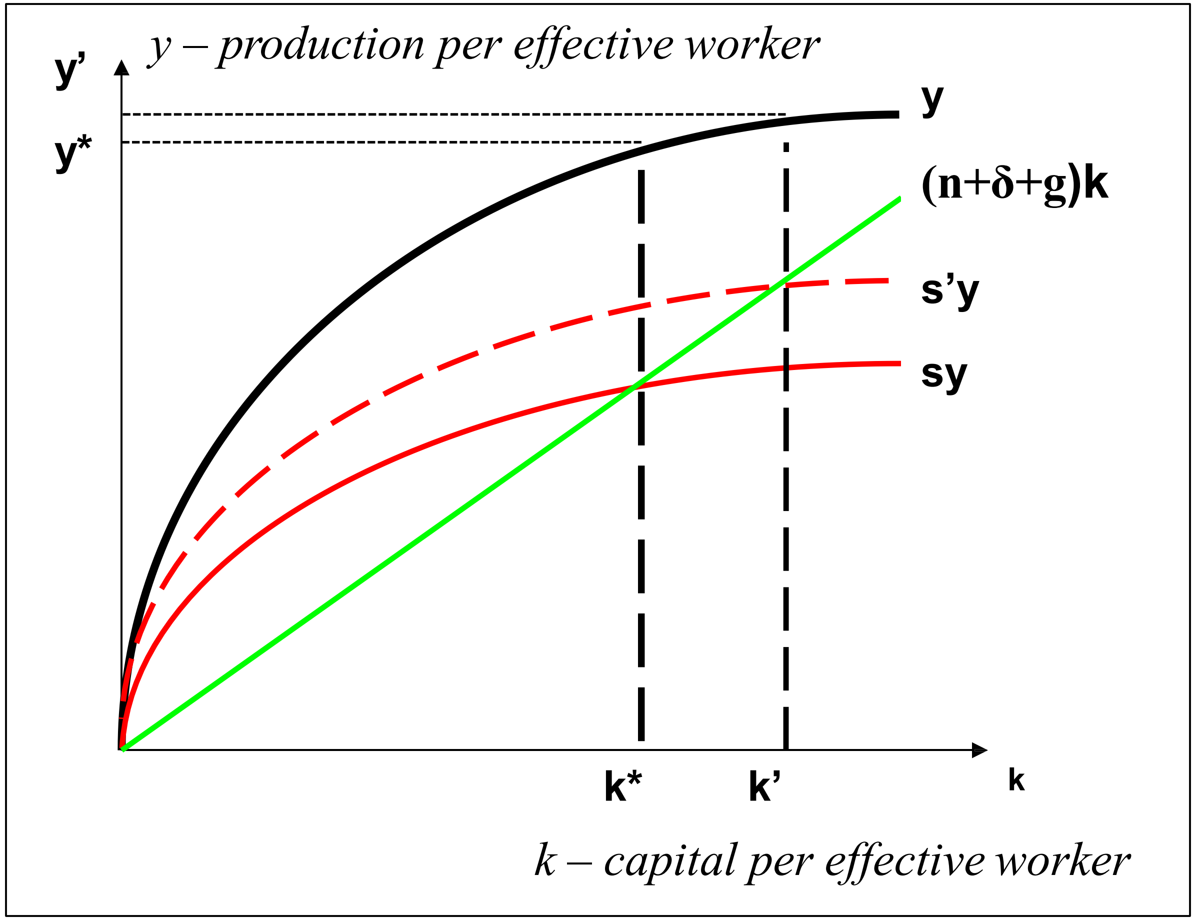
- Some factors influence **short run** rise in the <u>level</u> of output per worker; different factors influence the <u>growth</u> rate of output per worker in the **long run**

- <u>Exogenous</u> and <u>endogenous</u> factors

The effects of and increase in savings (→investment rate)

- *Rember! Increase in investment rate = increase in saving rate*

- Increasing investment rate, influences *levels* of k and y in the steady state



Short run implications of the Solow model

- Policy measures like tax cuts or investment subsidies can affect the steady state level of *output* but not the long-run growth rate.
- Growth is affected only in the short-run as the economy converges to the new steady state output level.
- The rate of growth as the economy converges on the steady state is determined by the rate of capital accumulation.
- This is in turn determined by the savings rate (the proportion of output used to create more capital rather than being consumedd) and the rate of capital depreciation.

Long run implications of the Solow model

- The economy cannot forever grow from capital accumulation - eventually it settles down in the steady-state
- At the long run, growth rate is independent of the savings rate – a higher savings rate means a higher steady-state level of income, and an increase in the savings rate increases the economy's growth rate temporarily. But the long run, growth rate is not affected.
- In neoclassical growth models, the **long-run rate of growth** is *exogenously* determined - in other words, it is determined outside of the model.
- A common prediction of these models is that an economy will always converge towards a steady state rate of growth, which depends only on the rate of technological progresss

Convergence rate

- Important property of the Solow model – allows to analyse the **speed of convergence** (thus how quickly economies move towards the steady state)

- Empirical analysis based on Solow model showed that speed of convergence towards the steady state is around 2% (this is the pace at which the gap in output levels is being closed)

Conclusions (1)

- When the initial capital stock is low (positive but below k_{ss}), the marginal product of capital is high relative to the effective depreciation rate. Therefore, the economy will be growing.
- When the initial capital stock is very large (above k_{ss}), the marginal product of capital falls below the effective depreciation rate. Therefore, the capital stock will diminish over time and the economy will be shrinking.
- The closer we get to the steady-state from below k_{ss} , the slower the economy will grow

Conclusions (2)

- if the countries have the same steady state, the absolute convergence hypothesis should hold: poor countries (further below its steady state) should grow faster than the rich countries.
- If the countries have different steady states then we observe relatve convergence
- There can be different scenario of catching up

Key terms

- ❑ Solow decomposition
- ❑ Solow residual
- ❑ Total Factor of Production
- ❑ Short run implication of Solow model
- ❑ Long run implication of Solow model

Sources

- Weil D., Economic Growth, (2009, 2013) Pearson International Edition
- Burda&Wyplosz (2009), Macroeconomics A European Text