NONLINEAR REGRESSION

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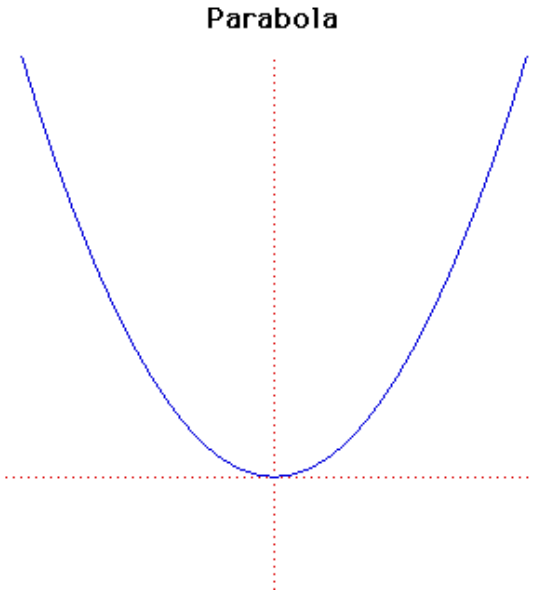
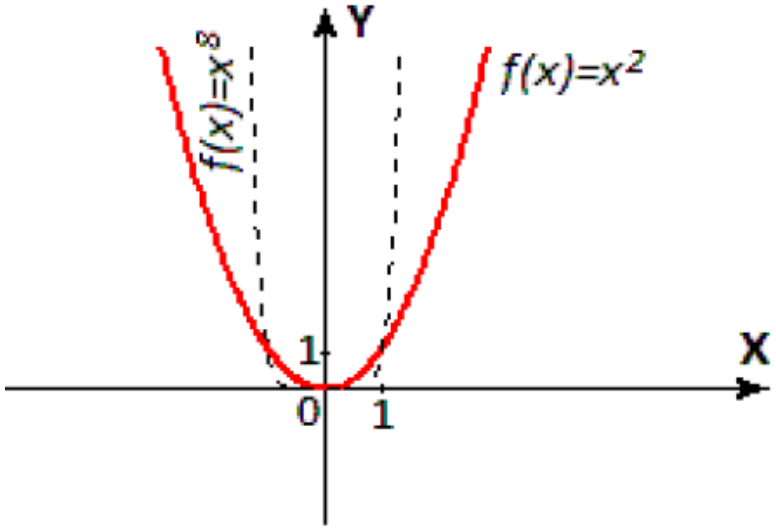
NONLINEAR FUNCTIONS

I. Quadratic function (parabola)
II. Hyperbolic function (hyperbola)
III. Polynomial function
IV. Logarythmic function
V. Exponential function
VI. Power function

QUADRATIC FUNCTION (PARABOLA)

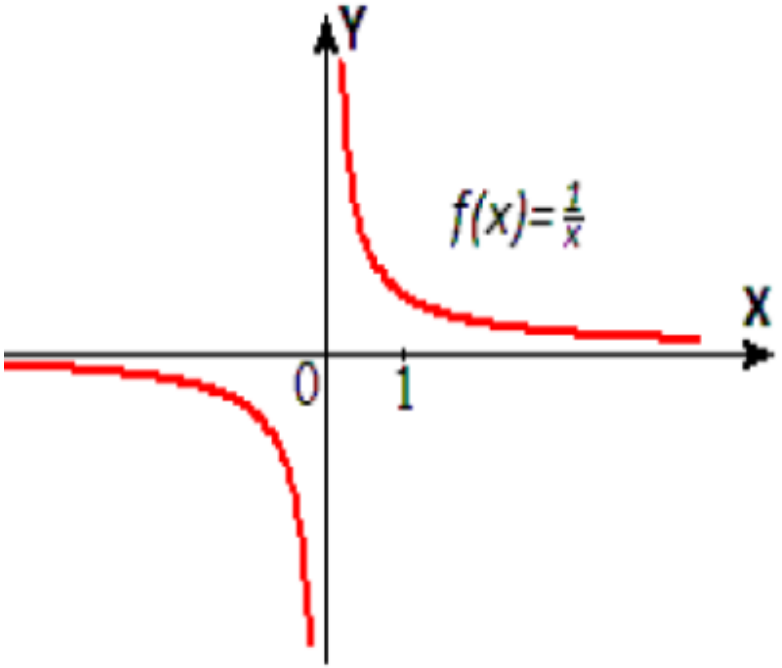
$$y=a+bx+cx^2$$





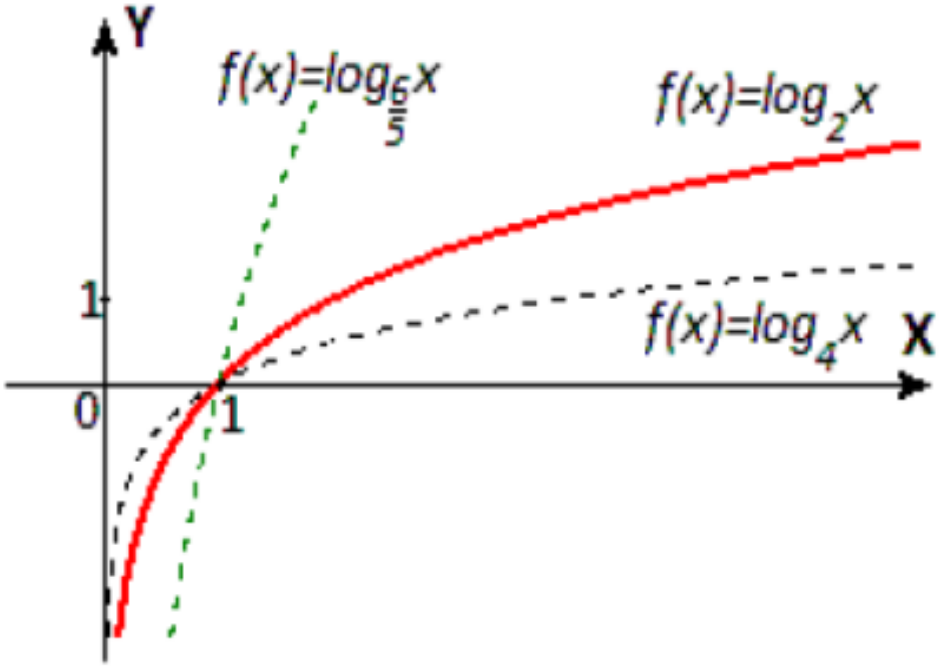
HYPERBOLA

$y=1/x$



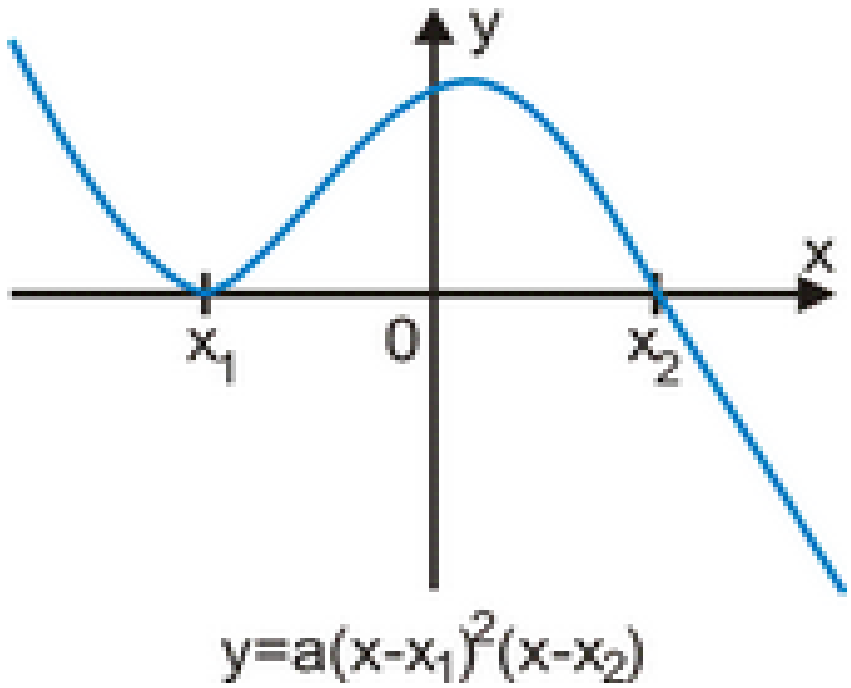
LOGARYTHMIC FUNCTION

$y=a+b\ln x$



POLYNOMIAL FUNCTION

$$y=a_0+a_1x+a_2x^2+a_3x^3$$



TASK. 1.

A real estate agent would like to predict the **selling price** of single-family homes. After careful consideration, he concludes that the variables likely to be most closely related to selling price is **the size of the house** (in 100s ft^2). As an experiment, he takes a random sample of fifteen recently sold houses and records the selling price (in $ 1,000s). These are shown in the accompanying table. Find and interpret the:

I. **linear regression model,**
II. **qudratic nonlinear regression model ,**
III. **hyperbolic nonlinear regression model ,**
IV. **logarythmic nonlinear regression model.**

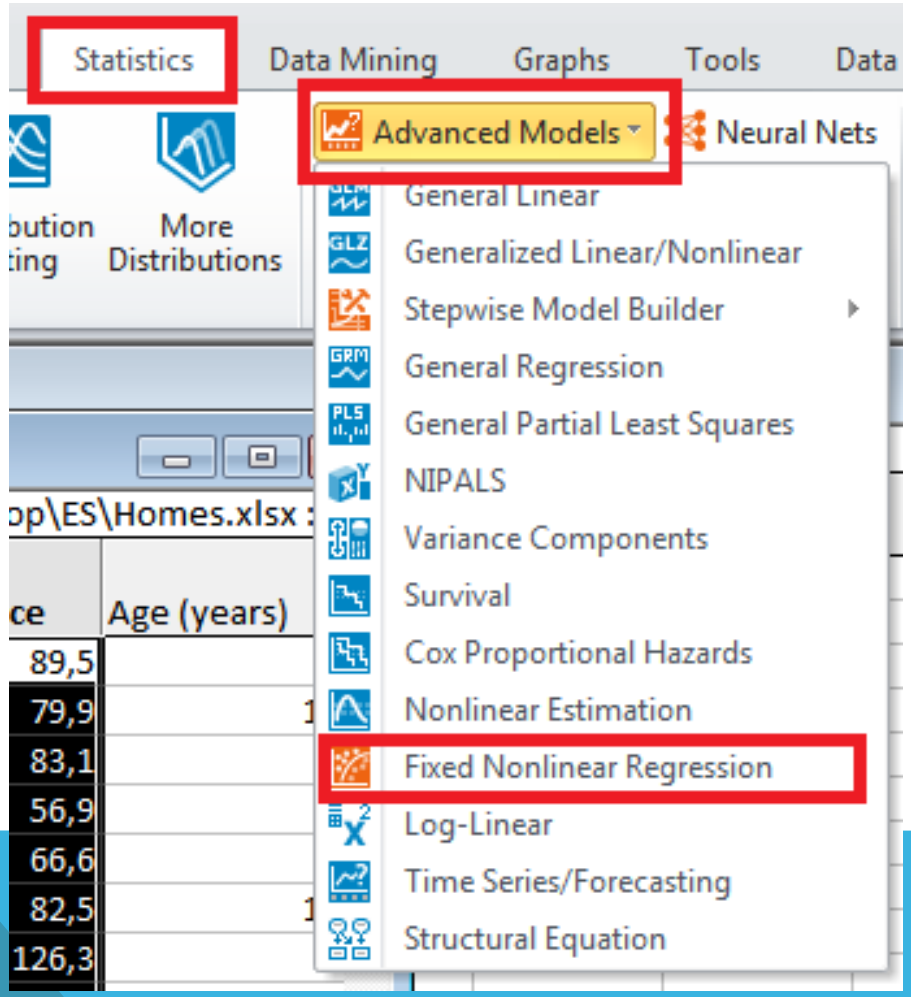
TASK. 1.

House size	Selling Price
20	89.5
14.8	79.9
20.5	83.1
12.5	56.9
18	66.6
14.3	82.5
27.5	126.3
16.5	79.3
24.3	119.9
20.2	87.6
22	112.6
19	120.8
12.3	78.5
14	74.3
16.7	74.8

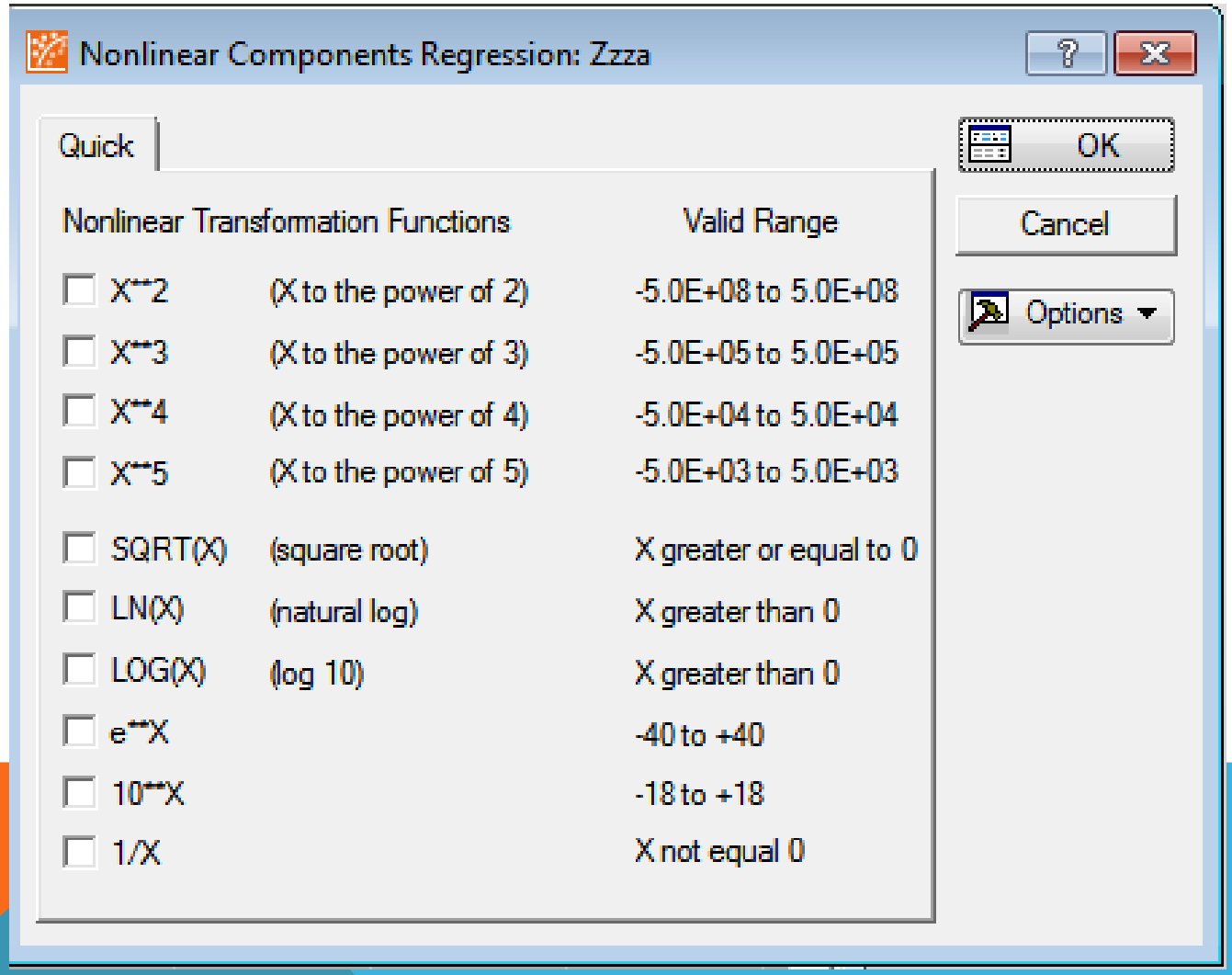
LINEAR REGRESSION

N=15	Regression Summary for Dependent Variable: Selling Price (Zzza) R= ,80470742 R2= ,64755403 Adjusted R2= ,62044280 F(1,13)=23,885 p<,00030 Std.Error of estimate: 12,997					
	b*	Std.Err. of b*	b	Std.Err. of b	t(13)	p-value
Intercept			18,35380	14,80774	1,239473	0,237080
House size	0,804707	0,164655	3,87855	0,79361	4,887237	0,000297

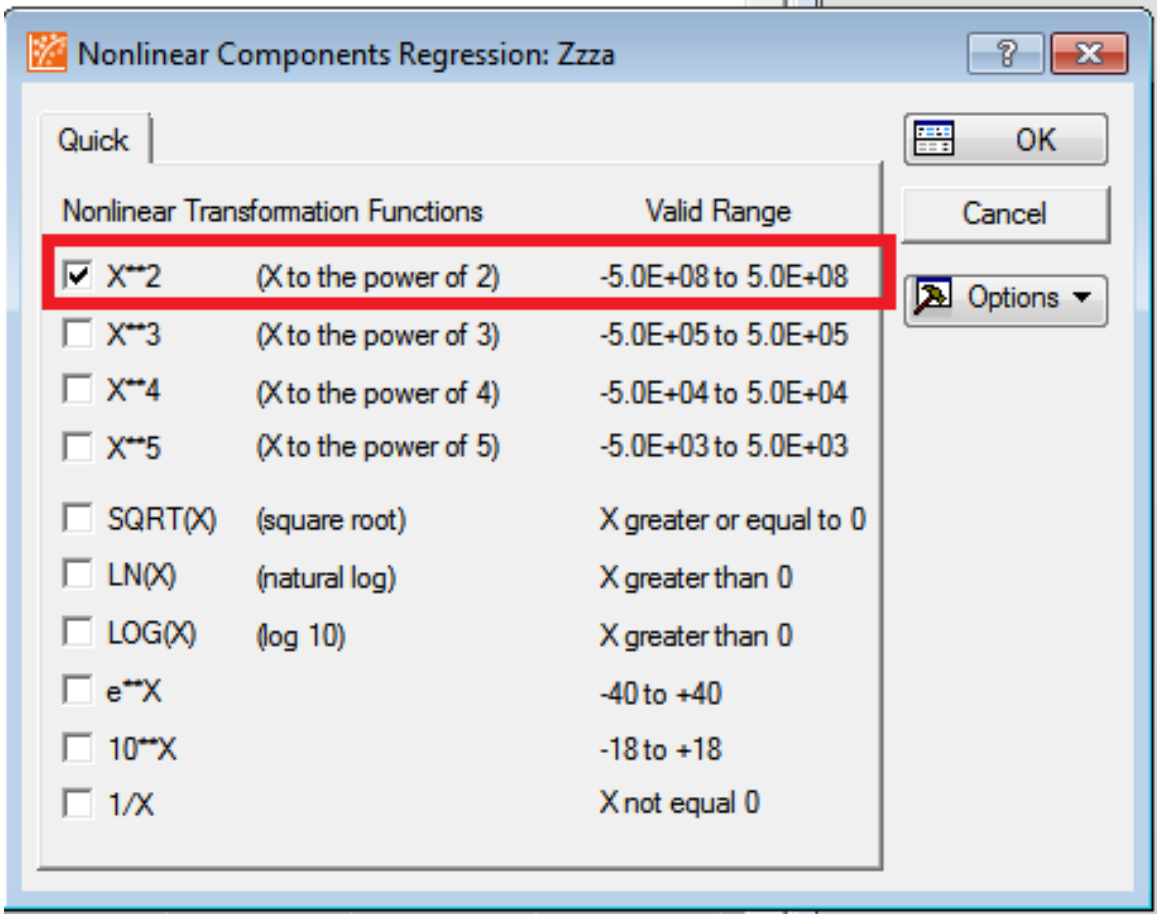
FIXED NONLINEAR REGRESSION

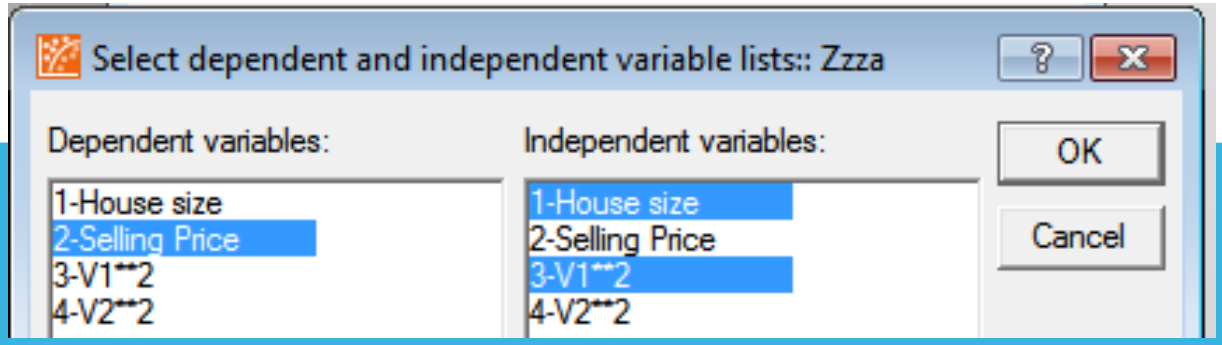


FIXED NONLINEAR REGRESSION



PARABOLA

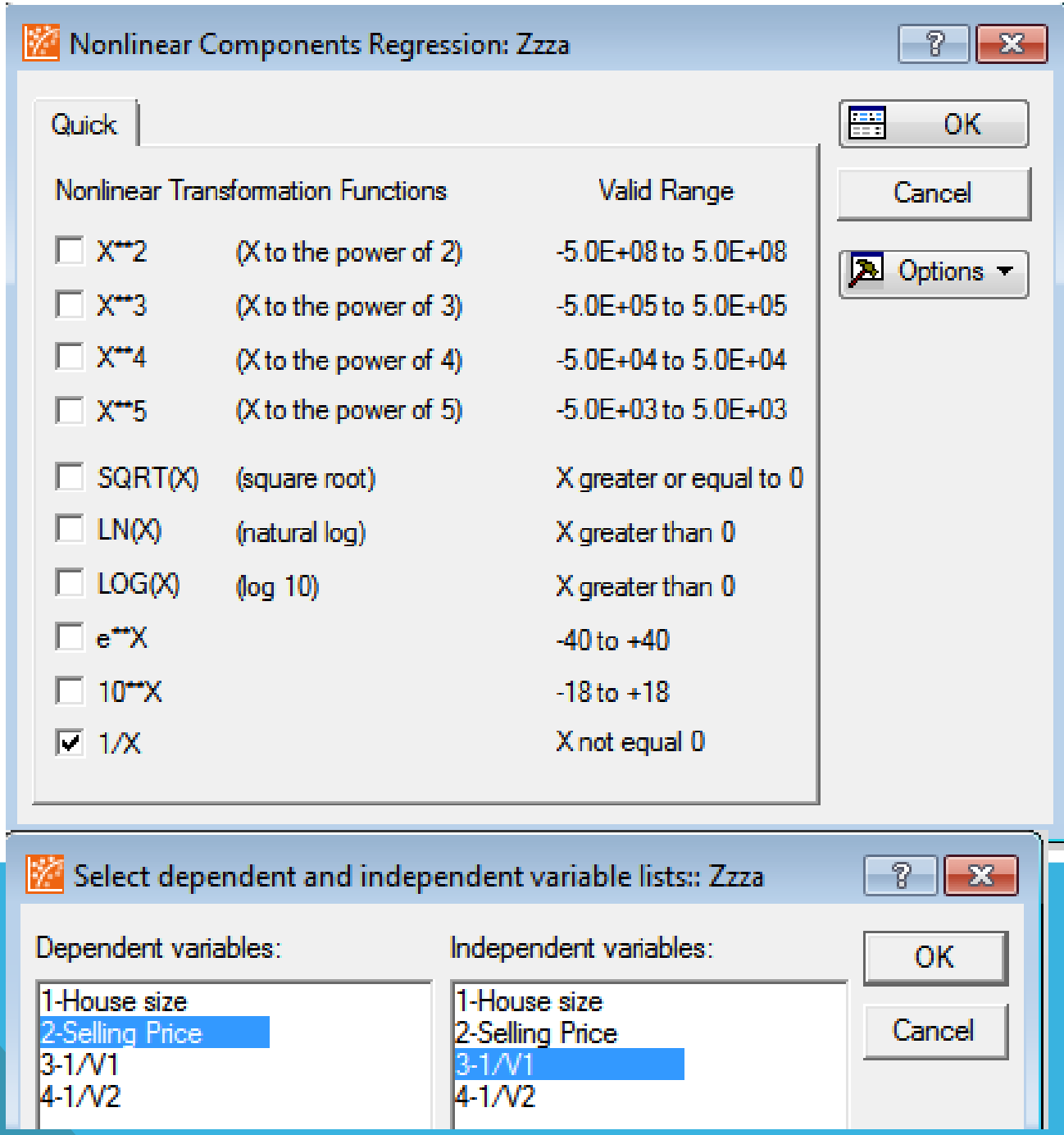

Nonlinear Components Regression: Zzza
Quick
OK
Cancel
Options
Nonlinear Transformation Functions
Valid Range
X**2 (X to the power of 2) -5.0E+08 to 5.0E+08
X**3 (X to the power of 3) -5.0E+05 to 5.0E+05
X**4 (X to the power of 4) -5.0E+04 to 5.0E+04
X**5 (X to the power of 5) -5.0E+03 to 5.0E+03
SQRT(X) (square root) X greater or equal to 0
LN(X) (natural log) X greater than 0
LOG(X) (log 10) X greater than 0
e**X -40 to +40
10**X -18 to +18
1/X X not equal 0


Select dependent and independent variable lists:: Zzza
Dependent variables:
Independent variables:
OK
Cancel
1-House size
2-Selling Price
3-V1**2
4-V2**2
1-House size
2-Selling Price
3-V1**2
4-V2**2

PARABOLA

N=15	Regression Summary for Dependent Variable: Selling Price (Zzza) R= ,81226759 R2= ,65977864 Adjusted R2= ,60307508 F(2,12)=11,636 p<,00155 Std.Error of estimate: 13,291 b*	Std.Err. of b*	b	Std.Err. of b	t(12)	p-value
Intercept			57,30011	61,21403	0,936062	0,367698
House size	-0,081556	1,360157	-0,39309	6,55572	-0,059961	0,953174
V1**2	0,893133	1,360157	0,11111	0,16921	0,656640	0,523805

HYPERBOLA

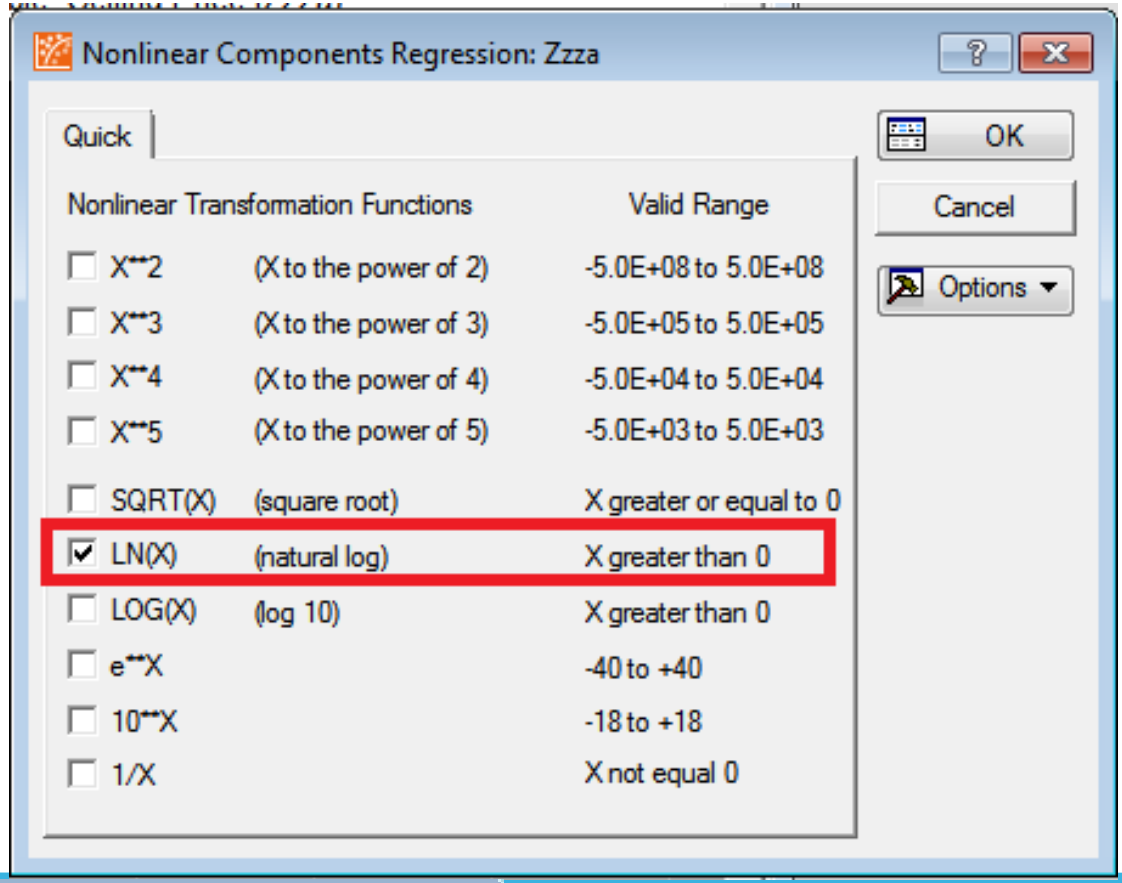

Nonlinear Components Regression: Zzza
Quick
OK
Cancel
Options
Nonlinear Transformation Functions
Valid Range
X**2 (X to the power of 2) -5.0E+08 to 5.0E+08
X**3 (X to the power of 3) -5.0E+05 to 5.0E+05
X**4 (X to the power of 4) -5.0E+04 to 5.0E+04
X**5 (X to the power of 5) -5.0E+03 to 5.0E+03
SQRT(X) (square root) X greater or equal to 0
LN(X) (natural log) X greater than 0
LOG(X) (log 10) X greater than 0
e**X -40 to +40
10**X -18 to +18
1/X X not equal 0
Select dependent and independent variable lists:: Zzza
Dependent variables:
Independent variables:
OK
Cancel
1-House size
2-Selling Price
3-1/V1
4-1/V2
1-House size
2-Selling Price
3-1/V1
4-1/V2

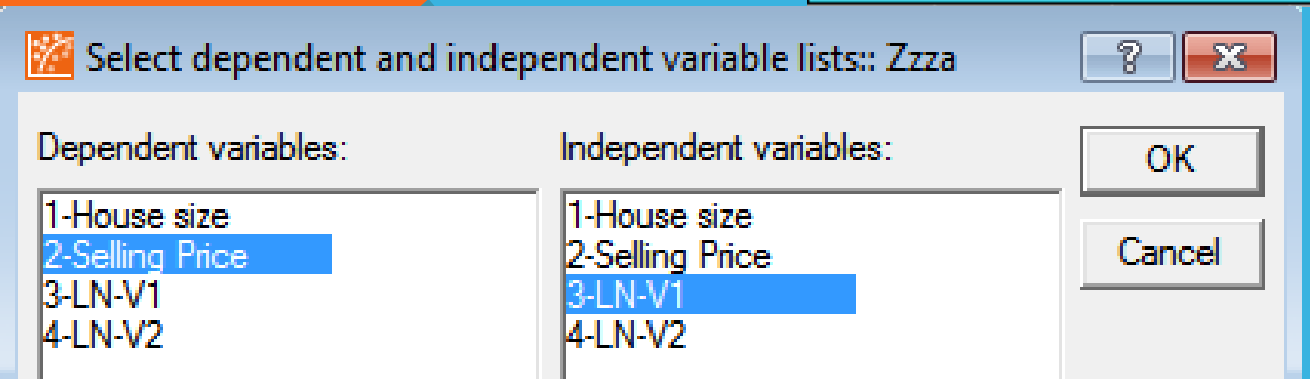
HYPERBOLA

Regression Summary for Dependent Variable: Selling Price (Zzza)
R= ,75344633 R2= ,56768137 Adjusted R2= ,53442609
F(1,13)=17,070 p<,00118 Std.Error of estimate: 14,394

N=15	b*	Std.Err. of b*	b	Std.Err. of b	t(13)	p-value
Intercept			156,28	16,7410	9,33528	0,000000
1/V1	-0,753446	0,182360	-1162,58	281,3841	-4,13164	0,001181

LOGARITHMIC FUNCTION





LOGARITHMIC FUNCTION

N=15	Regression Summary for Dependent Variable: Selling Price (Zzza) R= ,78417346 R2= ,61492802 Adjusted R2= ,58530710 F(1,13)=20,760 p<,00054 Std.Error of estimate: 13,585					
	b*	Std.Err. of b*	b	Std.Err. of b	t(13)	p-value
Intercept			-110,575	43,90718	-2,51838	0,025682
LN-V1	0,784173	0,172107	69,401	15,23194	4,55631	0,000539

MODELS' COMPARISON

Function	R square	Models' significance	Coefficients' significance	Standrad error of estimate
Linear	0.65	p<0.0003	No	12.997
Parabola	0.66	p<0.002	No	13.291
Hyperbola	0.57	p<0.0018	Yes	14.39
Logarithmic function	0.61	p<0.0005	Yes	13.585

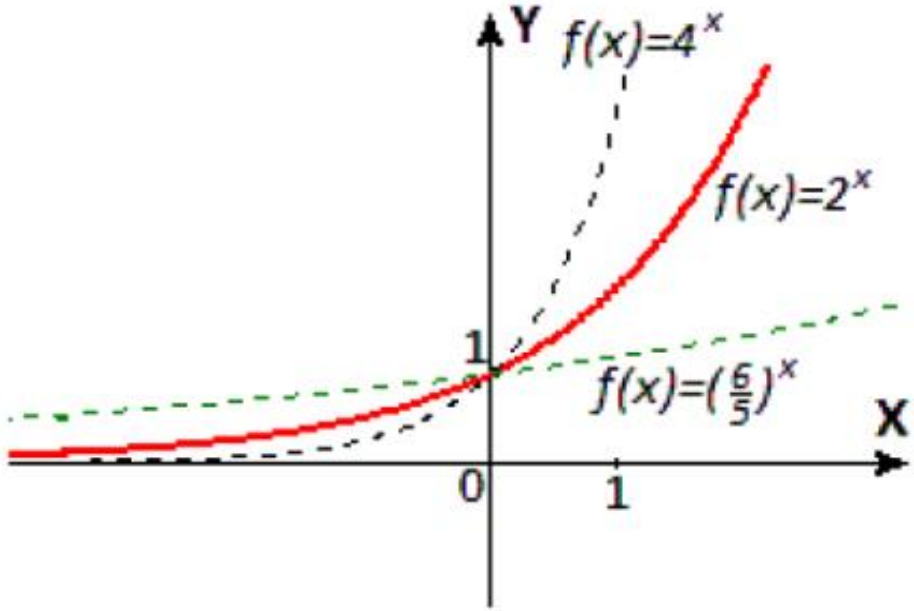
LINEARIZATION

1. Exponential function
2. Power function

EXPONENTIAL FUNCTION

$y=ab^x$

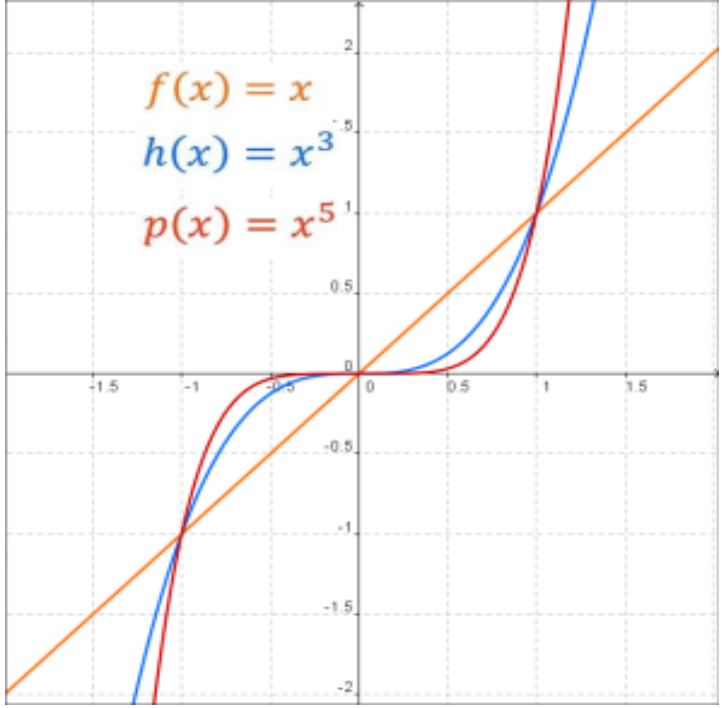
$\ln y=\ln a+x\ln b$

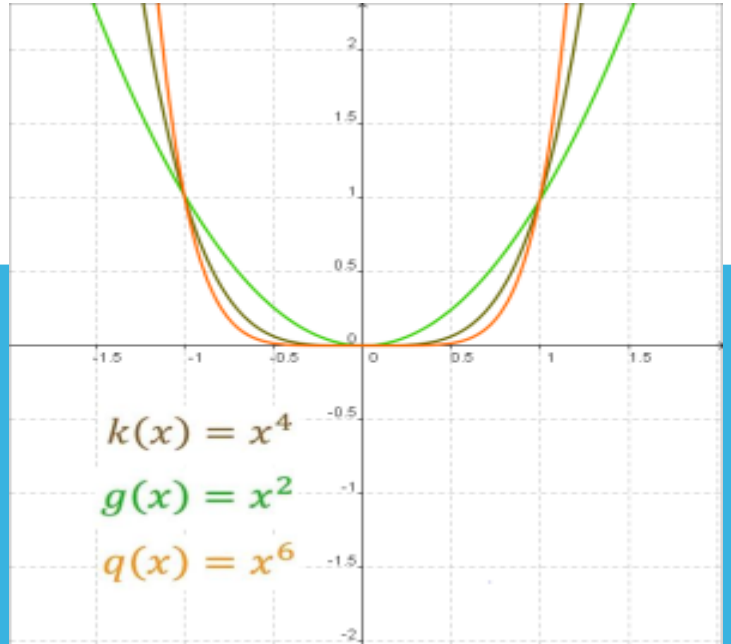


POWER FUNCTION

$y=ax^b$

$\ln y=\ln a+b\ln x$





TASK. 2.

A real estate agent would like to predict the **selling price** of single-family homes. After careful consideration, he concludes that the variables likely to be most closely related to selling price is **the size of the house** (in 100s ft^2). As an experiment, he takes a random sample of fifteen recently sold houses and records the selling price (in $ 1,000s). These are shown in the accompanying table. Find and interpret the:

I. **linear regression model,**

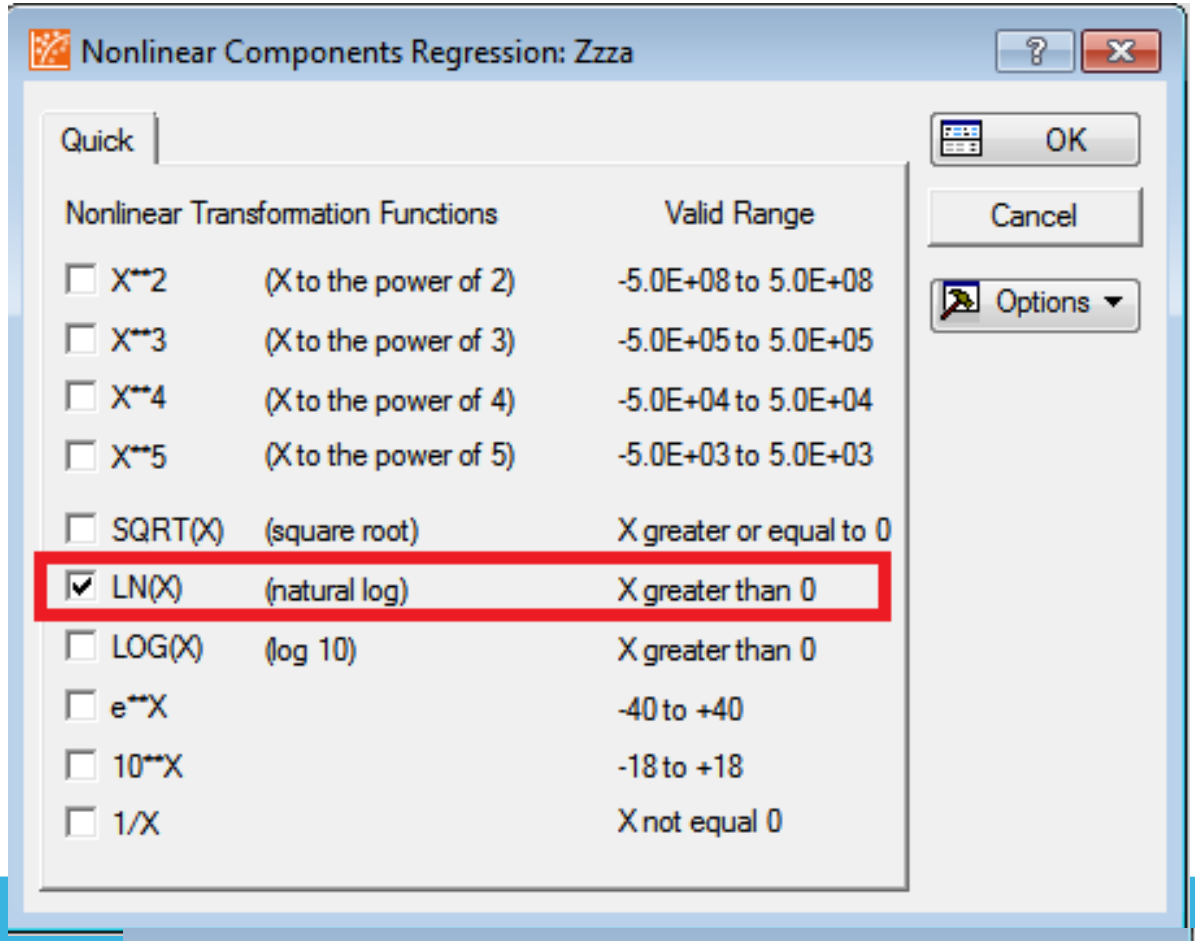
II. **exponential function model ,**

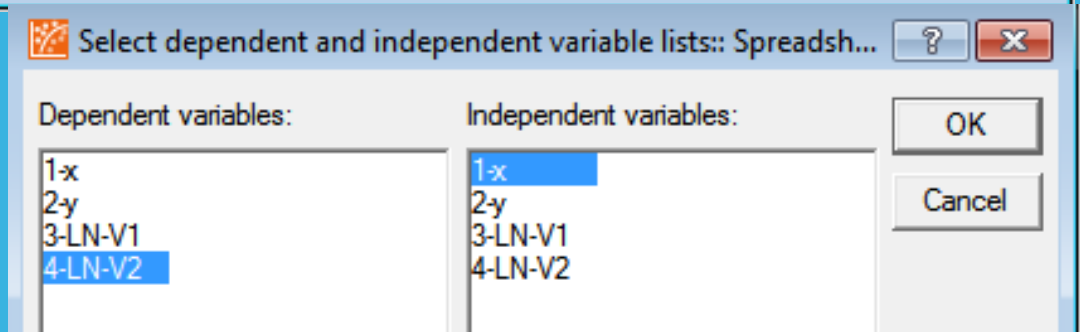
III. **power function model .**

TASK. 2.

House size	Selling Price
20	89.5
14.8	79.9
20.5	83.1
12.5	56.9
18	66.6
14.3	82.5
27.5	126.3
16.5	79.3
24.3	119.9
20.2	87.6
22	112.6
19	120.8
12.3	78.5
14	74.3
16.7	74.8

EXPONENTIAL FUNCTION





EXPONENTIAL FUNCTION

N=15	b*	Std.Err. of b*	b	Std.Err. of b	t(13)	p-value
Regression Summary for Dependent Variable: LN-V2 (Spreadsheet1) R= ,79441378 R2= ,63109326 Adjusted R2= ,60271582 F(1,13)=22,239 p<,00040 Std.Error of estimate: ,14563						
Intercept			3,699401	0,165931	22,29488	0,000000
x	0,794414	0,168456	0,041938	0,008893	4,71585	0,000404

$$\ln y = 3.7 + 0.04x$$

$\ln a$ $\ln b$

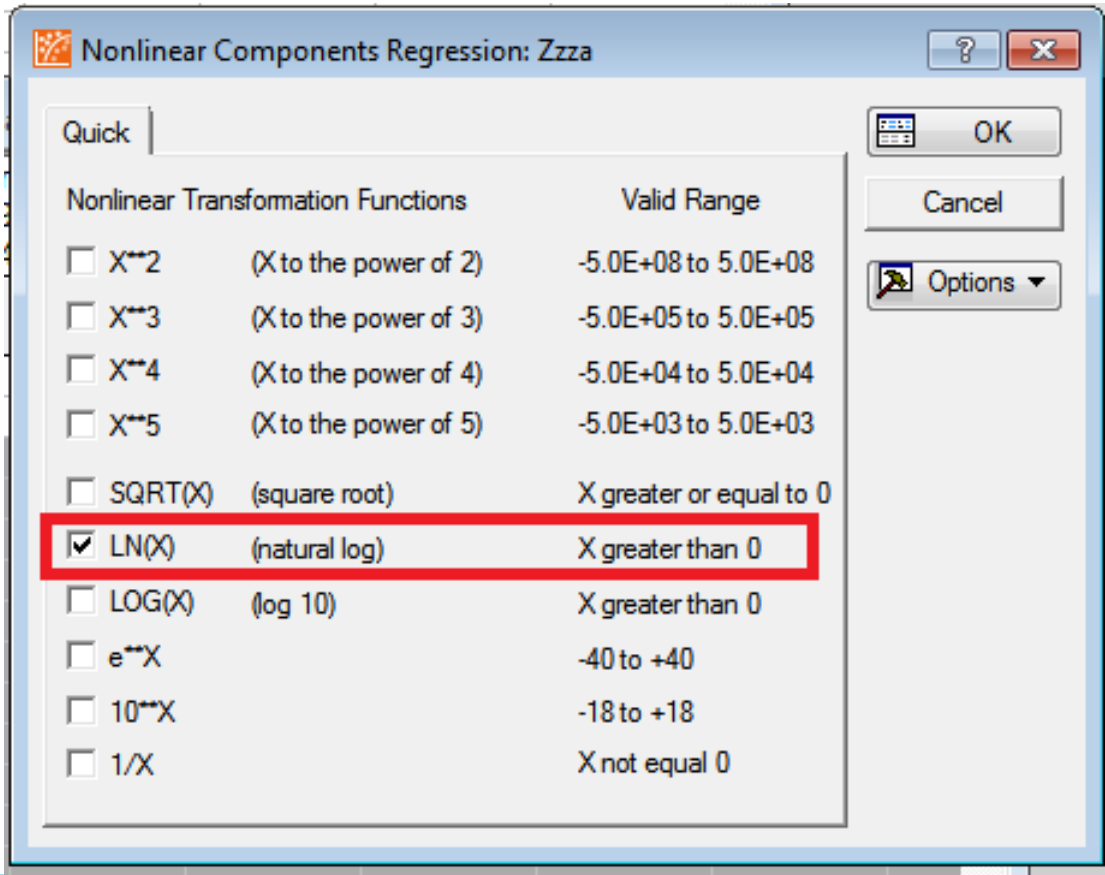
$\ln a = 3.7 \rightarrow a = e^{3.7} \rightarrow a = 40.43$

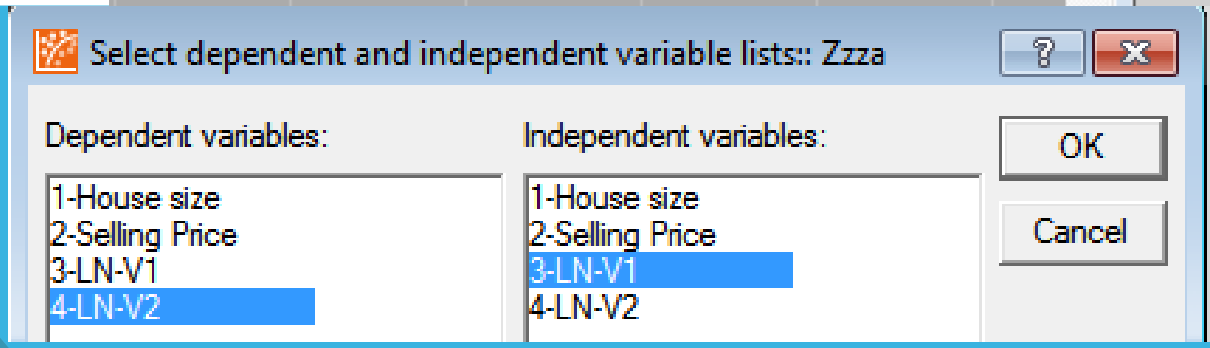
$\ln b = 0.04 \rightarrow b = e^{0.04} \rightarrow b = 1.04$

$$y = 40.43 * 1.04^{x}$$

If the size of the house increases by 1 unit, the selling price increases by (1.009-1)*100%=0.9%, on average.

POWER FUNCTION





POWER FUNCTION

Regression Summary for Dependent Variable: LN-V2 (Zzza)
R= ,78122672 R2= ,61031519 Adjusted R2= ,58033943
F(1,13)=20,360 p<,00058 Std.Error of estimate: ,14968

N=15	b*	Std.Err. of b*	b	Std.Err. of b	t(13)	p-value
Intercept			2,285599	0,483779	4,724473	0,000397
LN-V1	0,781227	0,173135	0,757283	0,167829	4,512238	0,000584

$$\ln y = 2.285 + 0.757 x$$

$\ln a$

$$\ln a = 2.285 \rightarrow a = e^{2.285} \rightarrow a = 9.823$$

$$y = 9.823 x^{0.757}$$

If the size of the house increases by 1%, the selling price increases by 0.757%, on average.

MODELS' COMPARISON

Function	R square	Models' significance	Coefficients significance	Standrad error of estimate
Exponential	0.63	$p<0.0004$	Yes	0.145
Power	0.61	$p<0.0005$	Yes	0.15

THANK YOU FOR YOUR ATTENTION