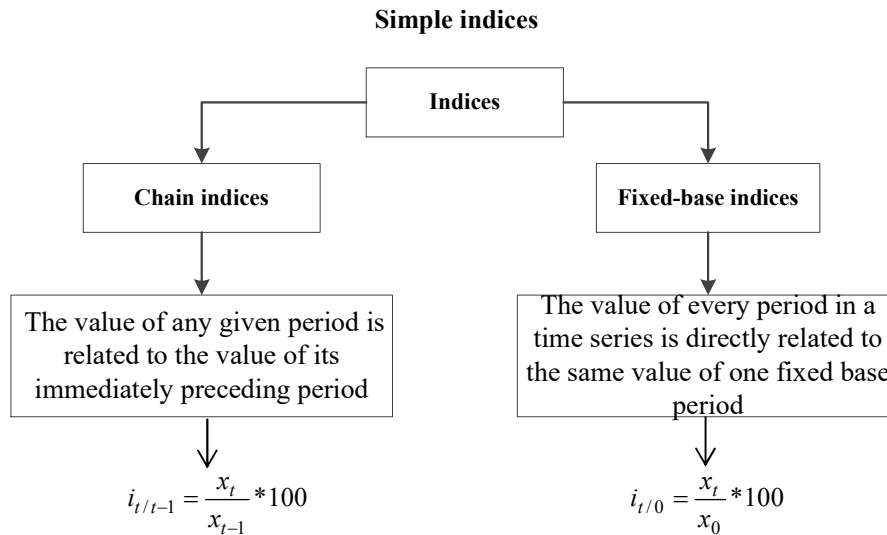# Index numbers

Index number- a ratio, expressed as a percentage, that relates to two or more time periods, one of which is designated as a base period. The data to be compared may be prices, quantities, money values. Index numbers measure the changes over time of particular time series data.

The basic function of an index number is to describe the change in some variable over time, in as straightforward a manner as possible.

Simple indices

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                          ┌──────────────┐
                          │   Indices    │
                          └──────────────┘
            ┌──────────────┐        ┌──────────────────┐
            │ Chain indices│        │ Fixed-base indices│
            └──────────────┘        └──────────────────┘
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Chain indices	Fixed-base indices
The value of any given period is related to the value of its immediately preceding period	The value of every period in a time series is directly related to the same value of one fixed base period

$$i_{t/t-1} = \frac{x_t}{x_{t-1}} * 100$$

$$i_{t/0} = \frac{x_t}{x_0} * 100$$

Average rate of change

$$\bar{i} = \sqrt[n-1]{\frac{x_1}{x_0} * \frac{x_2}{x_1} * \frac{x_3}{x_2} * \frac{x_4}{x_3} * ... * \frac{x_n}{x_{n-1}}} * 100 = \sqrt[n-1]{\frac{x_n}{x_0}} * 100$$

$$G = \bar{i} - 100$$

EXAMPLES

Example 1. The data are presented in the table 1.

Year	Average hourly earnings ($)
1950	1,335
1955	1,710
1960	2,090
1965	2,450
1970	3,220
1975	4,700
1980	6,150

a) Construct the fixed-base index of average hourly earnings of private nonagricultural workers from the data presented in the table 1. Select 1960 as the base year.

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Year	Average hourly earnings ($)	Earnings Index	Calculations
1950	1,335	63,876	1,335/2,090*100
1955	1,710	81,818	1,710/2,090*100
1960	2,090	100,000	100 Base Index
1965	2,450	117,225	2,450/2,090*100
1970	3,220	154,067	3,220/2,090*100
1975	4,700	224,880	4,700/2,090*100
1980	6,150	294,258	6,150/2,090*100

b) Rebase the earnings index from 1960 to 1950. Find the average rate of change.

Year	Average hourly earnings ($)	Earnings Index (1960 Base)	Calculations (1960 Base)	Earnings Index (1950 Base)	Calculations (1950 Base)
1950	1,335	63,876	1,335/2,090*100	100	100 Base Year
1955	1,710	81,818	1,710/2,090*100	128,090	81,818/63,876*100
1960	2,090	100,000	100 Base Year	156,554	100/63,876*100
1965	2,450	117,225	2,450/2,090*100	183,521	117,225/63,876*100
1970	3,220	154,067	3,220/2,090*100	241,199	154,067/63,876*100
1975	4,700	224,880	4,700/2,090*100	352,060	224,880/63,876*100
1980	6,150	294,258	6,150/2,090*100	460,674	294,258/63,876*100

Average rate of change: .

$$\bar{i} = {}_{n-1}\sqrt{\frac{x_n}{x_0}} * 100 = \sqrt[6]{460,674} * 100 = 215,2$$

$$G = \bar{i} - 100 = 215,2 - 100 = 115,2$$

c) Construct the chain index of average hourly earnings of private nonagricultural workers from the data presented in the table 1.

Year	Average hourly earnings ($)	Earnings Index (Chain)	Calculations (Chain)
1950	1,335	-	-
1955	1,710	128,090	1,710/1,335*100
1960	2,090	122,222	2,090/1,710*100
1965	2,450	117,225	2,450/2,090*100
1970	3,220	131,429	3,220/2,450*100
1975	4,700	145,963	4,700/3,220*100
1980	6,150	130,851	6,150/4,700*100

Task 1. The data are presented in the table.

a) Construct the fixed-base index of natural gas price ($) from the data presented in the table 1. Select 1972 as a base year.
b) Find the average rate of change.
c) Construct the chain index of natural gas price ($) from the data presented in the table 1.

Year	Natural Gas Price ($)
1970	16
1971	17
1972	18
1973	19
1974	22
1975	30
1976	45
1977	58
1978	79
1979	91
1980	118
1981	159
1982	198
1983	246
1984	259

Task 2. A historical record of the consumer price index contains the values shown in the accompanying table.

a) Rebase the index to 1961.
b) Rebase the index to 1971.
c) Construct the chain index.

Year	Index
1961	89,6
1962	90,6
1963	91,7
1964	92,9
1965	94,5
1966	97,2
1967	100
1968	104,2
1969	109,8
1970	116,3
1971	121,3
1972	125,3
1973	133,1
1974	147,7
1975	161,2
1976	170,5
1977	181,5
1978	195,4
1979	217,4
1980	246,8